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# Modeling and control of nonlinear systems

Presented by

**Zineb BOULAARAS**

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# Dedication

I dedicate doctoral dissertation work to my loving parents and my husband, whose words of support and push for persistence continue to reverberate in my ears. A special thanks goes out to my sisters, who have never left my side and to whom I dedicate this dissertation. My friends and colleagues who did not stingy in their prayers for me to succeed and excel.

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# Abstract

Known the suspension types Three categories passive suspension, and classified of a semi-active suspension, and active suspension are used to categorize suspension systems. In the first decades of the 20th century, the phrase "quarter car model" first appeared. It is regarded as the most effective method for examining the efficiency of vehicle stability. The mathematical model represents a spring mass (Quarter of the chassis) and an unsprung mass (the wheel), with two degrees of freedom (2-DOF) system characterized by a pair of differential equations. This dissertation discusses the modeling and control of a nonlinear active suspension system for a quarter car. Finding a control method that will give superior performance in terms of sprung displacement, sprung mass velocity, suspension deflection, peak overshoot, and setting time is the goal of this effort because the conventional PID and FOPID did not produce satisfactory results, the active control of the suspension system is accomplished using PID and Fractional-Order PID (FOPID) modified using Particle Swarm Optimization algorithms (PSO algorithms) and Genetic Algorithms (GA). On the other hand, we manage the system using the Linear Quadratic Regulator (LQR) or other words optimal control. MATLAB/Simulink is used to design and simulate the results. In comparison to Passive suspension, intelligent PID, and intelligent FOPID controller, with LQR control. It has been found that the LQR controller provides greater ride comfort by lowering the RMS error and vibration of various types of road conditions.

# Résumé

Types de suspension connue Trois catégories de suspension passive, de suspension semi-active et de suspension active sont utilisées pour classer les systèmes de suspension. Dans les premières décennies du 20e siècle, l'expression "modèle de quart de voiture" est apparue pour la première fois. Elle est considérée comme la méthode la plus efficace pour examiner l'efficacité de la stabilité du véhicule. Le modèle mathématique représente une masse élastique (le quart du châssis) et une masse non suspendue (la roue), avec un système à deux degrés de liberté (2-DOF) caractérisé par une paire d'équations différentielles. Cette thèse traite de la modélisation et du contrôle d'un système de suspension active non linéaire pour un quart de voiture. Trouver une méthode de contrôle qui donnera des performances supérieures en termes de déplacement suspendu, de vitesse de masse suspendue, de déviation de la suspension, de dépassement de crête et de temps de réglage est l'objectif de cet effort car le PID et le FOPID conventionnels n'ont pas produit de résultats satisfaisants, le contrôle actif de le système de suspension est réalisé à l'aide de PID et de PID d'ordre fractionnaire (FOPID) modifiés à l'aide d'algorithmes d'optimisation d'essaim de particules (algorithmes PSO) et d'algorithmes génétiques (GA). D'autre part, nous gérons le système à l'aide du régulateur quadratique linéaire (LQR) ou d'autres termes, un contrôle optimal. MATLAB/Simulink est utilisé pour concevoir et simuler les résultats. En comparaison avec la suspension passive, le PID intelligent et le contrôleur FOPID intelligent, avec contrôle LQR. Il a été constaté que le contrôleur LQR offre un plus grand confort de conduite en réduisant l'erreur RMS et les vibrations de divers types de conditions routières.

# الملخص

أنواع التعليق المعروف تستخدم ثلاث فئات وهي التعليق السلبي ،التعليق شبه النشط والتعليق النشط. في العقود الأولى من القرن العشرين ، ظهرت عبارة "طراز ربع السيارة" لأول مرة حيث تعتبر الطريقة الأكثر فعالية لفحص فعالية ثبات السيارة. تم تمثيل نموذج طراز ربع السيارة رياضيا ، بنظام درجتين من الحرية (*DOF-2*) يتميز بزوج من المعادلات التفاضلية. تتناول هذه الرسالة النمذجة والتحكم في نظام التعليق النشط غير الخطي لربع سيارة، و الهدف من هذا العمل هو إيجاد طريقة تحكم تحقق أداءً فائقًا من حيث إزاحة النوابض ، وسرعة الكتلة النابضة ، وانحراف التعليق ، والتجاوز ووقت الاستقرار ، لأن أنظمة *PID* و *FOPID* التقليدية لم تسفر عن نتائج مرضية في التحكم النشط باستخدام *PID* والمتحكم ذو الترتيب الكسري (*FOPID*) المعدلة باستخدام خوارزميات تحسين سرب الجسيمات (خوارزميات *PSO*) والخوارزميات الجينية (*GA*). من ناحية أخرى ، قمنا بإدارة النظام باستخدام المتحكم الخطي التربيعي (*LQR*) أو بعبارة أخرى ، التحكم الأمثل. إستخدمنا برنامج *MATLAB / Simulink* لتصميم ومحاكاة النتائج. بالمقارنة مع التعليق السلبي ، *PID* الذكي وجهاز التحكم الذكي *FOPID* ، مع التحكم في *LQR*. تم التوصل الى أن أداة المتحكم *LQR* يوفر قدر أكبر من الراحة أثناء القيادة عن طريق تقليل خطأ *RMS* والاهتزاز لأنواع مختلفة من ظروف الطريق.

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# Acronyms

***DOF**: Degrees of Freedom*

***2-DOF**: Two Degree of Freedom*

***GA**: Genetic Algorithms*

***PSO**: Particle Swarm Optimization*

***MPC**: Model Predictive Control*

***PID**: Proportional Integrator Derivative Control*

***FOPID**: Fractional Order PID Control*

***RMSE**: Root Mean Square Error*

***LQR**: Linear Quadratic Regulator*

***Iter**: Iteration*

***Maxiter**: Maximum Iteration*

***ITAE**: Integral Time of Absolute Error*

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## Chapter III: Controller Design and Metaheuristic Optimization



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# General Introduction

# General Introduction

Nonlinear systems are a complex and fascinating part of the real world. These systems, which can be found in nature, technology, economics, and engineering applications, involve equations that cannot be solved with simple linear algebraic techniques. Nonlinear systems require specialized methods for analysis and control due to their unpredictable behavior. This unpredictability is what makes them so interesting to study in the first place [1, 2].

In nature, nonlinear dynamical systems can be seen everywhere, from weather patterns to population growth trends. The most famous example is chaos theory, where even small changes in initial conditions can lead to drastically different outcomes over time; this phenomenon has been observed in many natural phenomena such as climate change or predator-prey relationships [3]. In economic applications, nonlinearity arises due to feedbacks between markets or other economic variables; these feedbacks often cause sudden shifts or instability that traditional linear models fail to capture accurately enough to predict future market movements successfully. When it comes down engineering applications like robotics there are several examples of nonlinear dynamics at play: from robot arms manipulating objects with varying mass distributions (nonlinearly changing inertia), to autonomous vehicles navigating unknown environments (with uncertain terrain properties) [4]. All these algorithms capable of dealing with unexpected events while still maintaining stability incorporating sophisticated mathematical tools designed specifically for analyzing non-liner dynamic [5].

Overall, it’s clear why understanding how Nonlinear Systems work is essential if we want make sense out of our increasingly interconnected world – whether its predicting chaotic weather patterns, trading stocks efficiently on Wall Street or controlling robots performing various tasks autonomously - all these problems have one thing common: they all rely heavily on us being able understand how Nonlinear Systems behave [6].

The ability to govern non-linear systems has grown in significance throughout time. This is because many dynamic systems in real life are inherently nonlinear and need sophisticated control methods to maintain their smooth operation. For conventional linear controllers like PID controllers, nonlinear system dynamics can be particularly complicated and tricky to adequately model. Over the past ten or so years, there has therefore been an increased emphasis on creating new techniques for controlling these kinds of systems. Model predictive control (MPC) is one strategy that has lately gained popularity [7]. Using optimization algorithms and a predictive model of the system's dynamics, MPC finds the best paths through state space while taking input and output limitations into account. Compared to conventional linear controller designs, this approach has a number of benefits, such as increased robustness against disturbances and increased flexibility when managing many objectives or constraints at once. Additionally, MPC has the ability to incorporate sensor feedback into its computations, which enables it to respond swiftly to changing operational conditions or unanticipated events. For the purpose of controlling nonlinear systems, metaheuristic algorithms are effective tools for optimizing PID and FOPID controllers. Compared to conventional optimization methods like gradient-based approaches, these algorithms offer a variety of benefits [8]. They can be used, in particular, to swiftly locate viable answers in intricate search spaces with numerous local optima. Metaheuristics are also more resilient to changes in system characteristics or operating conditions than other methods because they require fewer assumptions about the problem structure [9].

The ability of metaheuristic algorithms to efficiently explore large search spaces with multiple local optima without becoming stuck at any one point due to premature convergence on suboptimal solutions or parameter settings that might not produce optimal results overall is one of their main advantages. In contrast to gradient-based techniques, which frequently struggle with highly nonlinear systems or complex cost functions that have several minima throughout their surface, this enables them to discover near-optimal controller settings considerably more quickly [10]. The adaptability of metaheuristics is another benefit. Most recent implementations come preconfigured out of the box, so all you need to do is describe your objective function(s) and some basic restrictions, then let it work its magic. In contrast to most conventional numerical solvers, which typically require time-consuming manual adjustment before each run [11].

The use of metaheuristic algorithms to modify the gain parameters of the PID and FOPID controllers for regulating the active suspension system of the quarter-car model is the first contribution made by this thesis. The proposed method's framework consists of two methods. We begin by presenting the feedback loop controllers' optimal gain parameters. A brief comparison between PSO and GAs is presented in order to look at the effectiveness of optimization techniques. In the second method, we will be controlling the active suspension system for the quarter car model using the LQR control, where LQR utilizes an optimization algorithm to determine an optimal control input that minimizes a cost function that takes account both system performance and energy consumption. This method has been successfully used in automotive control systems [12].

The first step in using the LQR approach is to define the state variables describing the system dynamics. These can be either continuous or discrete depending on whether or not it's necessary to consider time delays between inputs and outputs of the system being controlled. The second step involves specifying a cost function that will be minimized by finding an optimal solution for controlling this particular non-linear system [13], this includes defining weights associated with different aspects such as stability requirements versus energy consumption needs when selecting parameters for implementation within LQR algorithms. Finally, the third step requires solving equations derived from linearizing around desired operating points so that they can be solved using numerical techniques such as gradient descent. This is the method adopted in this thesis.

This thesis is composed of forth chapters organized as follows:

**In the First Chapter** we present the general motion of dynamic model of nonlinear systems, linearization, and stability.

**In the Second Chapter** we present an overview of suspension systems with all classification.

**In the Third Chapter** we explain the metaheuristic algorithms and the theory of controllers' methods (PID, FOPID, and LQR) with theory of optimization methods used in the present study.

**In the Fourth Chapter** we present the results of control the active suspension system for quarter car model with steps of optimization of the controllers' parameters for PID, FOPID on one hand, and on another hand calculate the parameters of LQR control. The results show a comparison between the passive suspension and active suspension.

# Chapter I

## Overview of Nonlinear Systems

### I.1. Introduction

An algebraic, functional, ordinary differential, partial differential, integrative, or a combination of these nonlinear equations make up a nonlinear system [14].

Dynamic systems are now utilized as a substitute for nonlinear systems when the system's nonlinear equation represents the evolution of a solution over time. The system may be dependent on certain factors. Although a nonlinear system may not be applicable to mechanics, the term "dynamic system" originated from the known equation that governs the motion of a system of particles. [15].

We can consider it to be a feedback loop in a nonlinear system if an element's output differs from its inputs. a nonlinear system that is used to explain a wide range of phenomena in engineering, earth sciences, and social life sciences. Applications of the theory of nonlinear systems include issues with economics, population growth, and gene spread. Numerous more phenomena include elasticity, chemical reactions, regulating heartbeat, neurophysiology, and many others. [16].

We are typically interested in permanent, and other, steady states when using nonlinear systems theory. As a result, the static answers to the governing equations are particularly intriguing [17]. Engineers must comprehend and be able to use a wide variety of nonlinear analysis techniques when they study and develop nonlinear dynamics systems for use in electrical circuits, mechanical systems, control systems, and other engineering disciplines [18]. In this chapter, a nonlinear system with an analysis approach for nonlinear systems' stability is introduced.

As every model of nature or human behavior falls under nonlinearity, all systems in the real world are nonlinear. To solve it, one can either assume linear variables to prevent issues in nonlinear systems, which decreases vibrations or confusion in modeling systems in various social domains. It is possible to assume an idealized linear system, whose mathematical analysis is well-established, well-known, and typically results in closed-form symbolic solutions where the state of the system at any given moment is a function of the system parameters. Additionally, towards equilibrium, the linear part of the system that can be derived from a mathematical model via polynomial expansion takes precedence over the nonlinear parts [19]. The definition of nonlinear systems and general facts about them are covered in this chapter. Additionally, this chapter is regarded as the foundation of his thesis, which examines the control of nonlinear systems with a variety of controllers, such as classical and artificial intelligence-based ones, and terminology from the control literature.

### I.2. Definition of Nonlinear systems

A nonlinear system is one in which the change in output is not proportional to the change in input in mathematics and science [20]. Engineers, biologists, and physicists are all interested in nonlinear problems [21], due to the fact that most systems are intrinsically nonlinear, mathematicians [22] as well as many other scientists.

Contrasting with considerably simpler linear systems, nonlinear dynamical systems which represent changes in variables over time might look chaotic, unpredictable, or counterintuitive [23].

The behavior of a nonlinear system in mathematics is usually described by a nonlinear system of equations, which is a set of simultaneous equations in which unknowns (or unknown function in the case of differential equations) appear as variables of polynomial degree greater than one or in a function argument not a first- order polynomial [24]. In other words, in a nonlinear system of equations, the equations to be solved cannot be written as a linear set of unknown variables or function that appear in it. Systems can be defined as nonlinear regardless of whether or not known linear function appear in the equations, so in particular, a differential equation is linear if the terms of the unknown function and its derivatives is linear, even if it is nonlinear in terms of other variables that appear in it [25].

### I.3. Nonlinear Systems Analysis

The analysis methods do not always reveal the true type of nonlinearity; however, the methods will give the reader an idea of how nonlinearity works in a linear system environment [26]. Phase space is a multidimensional abstraction space is used to graphically represent everything that is possible states of a dynamical system [27], if it was the actual number of variables that govern the behavior the dynamical system is unknown, then the phase space diagrams they are reconstructed by late embedding over time, which is based on the concept of taken theory [28].

According to the theory, if only one variable of the system, let's say $(x)$, is accessible (i.e., only one dimension can be measured), then it is possible to reconstruct the full dynamics of the system of the only observed variable $(x)$ by plotting its values against itself a specific number of times over a predetermined period of time [29].

A linear function $f(x)$ in mathematics is one that meets both of the requirements listed below:

- Additivity or superposition principle: $f(x+y) = f(x) + f(y)$;
- Homogeneity: $f(\alpha x) = \alpha f(x)$.

We called linear system if $f(x)$ is a linear function ($f(x)$ is a mathematics model for physics system) and nonlinear otherwise [30].

### I.3.1. Nonlinear Recurrence Relations

Each term that follows in a series is a nonlinear function of the terms that came before it, according to a nonlinear recurrence relation. Deseret nonlinear models that describe a large class of nonlinear recurrence relations include the NARMAX (Nonlinear Moving Average Autoregressive with Exogenous Input) model and related nonlinear system identification and analysis tools. A wide range of complicated nonlinear behaviors in the time-frequency, spatial, and temporal domains can be studied using these methods [31].

### I.3.2. Nonlinear Differential Equations

In the case of the unknown function and its derivatives, a nonlinear differential equation is not a linear equation. Very few methods exist for fully resolving nonlinear differential equations, and those that do depend on the symmetries included in the problem [32].

It is a characteristic of chaos that nonlinear differential equations can display extremely complicated behavior over long periods. Nonlinear differential equations pose fundamental questions regarding their uniqueness and scalability of solutions, yet if a differential equation is a well-defined representation of a significant physical process, one would expect it to have a solution [33].

Nonlinear equations are frequently approximated by linear differential equations. These approximations are only applicable in limited circumstances; for instance, the harmonic oscillator equation is a small-amplitude approximation of the nonlinear pendulum equation [34].

## I.4. Linearization of Nonlinear System

In this section, we show how to perform the linearization of systems described by nonlinear differential aquations. The presented procedure is Taylor dependent Expansion series and knowledge of nominal system paths and input of system [35].

Finding a function's linear approximation at a specific point is what mathematicians mean by linearity. The initial Taylor expansion about a point of interest is the function's linear approximation. An approach for assessing the local stability of the equilibrium point for a set of nonlinear differential equations or discrete dynamical systems is used in the study of dynamical systems. This approach is employed in a variety of disciplines, including engineering, physics, economics, and environmental studies [36].

### I.4.1. Linearization of Nonlinear Function

Linearization of a function means the area around the point is approximated by the tangent line to the function at that point. Local linearity is the term used to describe the relationship between the tangent and the graph at the point tangency [37]. The linearization of a differential equation is derived from an original nonlinear equation by treating each dependent variable as consisting of the sum of an imperturbation from that mean [38].

The linearization process, often called the method of small perturbation, results in a linear differential equation with perturbations of the original dependent variables as the new dependent variables. It has been successfully used to solve problems in nonlinear systems, this relationship between referred to as local linearity [39].

Linearization of a differential equation is the process of taking the gradient of a nonlinear function concerning each variable and converting it into a linear representation at that location. This technique is necessary for some types of analysis, including stability analysis, replanning trellis solutions, and putting the model into linear state space form [40]. Figure (I-1) gives a general view of linearization [40].

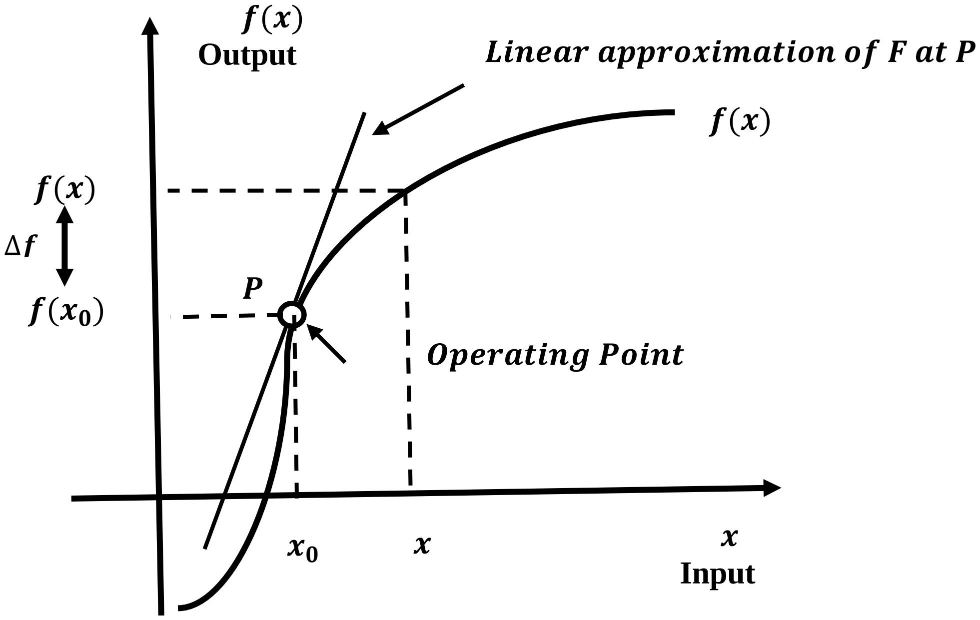



**Figure (I-1):** A General view for linearization

In other words, we can say a standard method for dealing with a nonlinear system is to linearize, that is means transforming it suck that its state equation is linear. A linearized model is typically only valid in some neighborhoods of the state -space This neighborhood is selected by choosing an operating point $x_0$ used in the linearization process [41].

We use three considerations when choosing an operating point:

1. That implied by the name, it should be in a region of state space where the state will stay throughout a system's operation.

2. The validity of the model near the operating point;

Due to the fact that nonlinear system tends to be more linear equilibria, the second consideration frequently suggests we choose one as an operating point: $x_0 = \bar{x}$ [42].

**I.4.2. Taylor Series Expansion**

A Taylor series extension of the operational point in equation (I.1) No, to fields for a nonautonomous system. polynomial terms for a quadratique lineare, etc. In $X$ and $U$ If we simply retain the linear terms and create fresh input and state variables:

$$\dot{X} = X - X_0 \text{ and } \dot{U} = U - U_0 \tag{I-1}$$

We get a linear state equation:

$$\frac{d\dot{X}}{dt} = A\dot{X} + B\dot{U} \tag{I-2}$$

Where the matrix components are given by:

$$A_{ij} = \left.\frac{df_i}{dx_j}\right|_{X_0,U_0} \quad \text{and} \quad B_{ij} = \left.\frac{df_i}{dx_j}\right|_{X_0,U_0} \tag{I-3}$$

These is a first derivative matrices are generally called Jacobian Matrices; this result also applies to autonomous aquations if we drop the $B\dot{U}$ term [43].

**I.4.3. Example of Linearization of Non-Linear Systems**

Consider a 2-Dimensional first order autonomous system $\dot{Y} = f(y)$, we can be written as [44]:

$$\begin{cases} \dot{X} = f(X,Y) \\ \dot{Y} = f(X,Y) \end{cases} \tag{I-4}$$

Suppose that this system has an equilibrium point $y_e = (x_e, y_e)$. Near this equilibrium, the system can be approximated by $\dot{Z} = JZ$ and written as follows:

$$\begin{bmatrix} \dot{Z}_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x}(x_e, y_e) & \frac{\partial f}{\partial y}(x_e, y_e) \\ \frac{\partial g}{\partial x}(x_e, y_e) & \frac{\partial g}{\partial x}(x_e, y_e) \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} \quad \textbf{(I-5)}$$

Where $Z_1 = x - x_e$, and $Z_2 = y - y_e$. The system with $Z$ is called the linearized system, and $J$ is called the Jacobian matrix.

### I. 5. Control of Nonlinear System

Always bear in mind that there are a variety of approaches to obtain models of dynamical systems. These strategies may be summed up into two categories: the first is based on systems recognition, and the second is constructive. The system's inputs and outputs are measured in the first experimental part, which also aims to identify the best model that can provide these inputs and outputs [45].

The models obtained in this way do not have subtle dynamics that cannot be noticed at the exits, that is mean the internal system remains unknown. The structure method in identifying systems starts with relying on the physical laws that govern the system and the theoretical construction of the system as the models obtained in this way also contain information about the internals of the system and the hidden internal motion that cannot be observed [46].

There are many techniques available for designing Controls for dynamic systems. The classical methods or other words traditional methods to the theory of automatic Control are designed for linear systems and model the control as an application of a linear operator on the system's Current Phase state, both close to and far from the required. Terminal state, this approach's shortcomings are clear the degree of the control decreases as it gets closer to the final state, leaving certain control possibilities unrealized [47].

The control Process's time is strictly speaking limitless because the phase state can only asymptotically tend to the terminal state as the time approaches infinity. On the other hand, when the control is far from the terminal state, the control magnitude can surpass the restrictions frequently placed on the control, making it difficult or even impossible to account for the constraints given by linear techniques. Additionally, conventional methods based on linear models are typically inapplicable to nonlinear systems, and even when they are, their applicability must be rigorously tested [48].

The theory of optimal control's methods, which account for the variety of restrictions placed on the control and thought with significant difficulties, on the state Variables, can be applied to nonlinear systems. A dynamical system is optimally brought to a specified terminal state using optimal control techniques. Developing the ideal control for a nonlinear system is a challenging topic for which there is rarely an explicit answer. The creation of a feedback optimum control for a nonlinear system is very challenging, especially for systems with few degrees of freedom and even the aid of modern computers [49].

There exist a number of other general methods of control including the feedback linearization method, the method of systems with changeable structures, and their various generalizations. Unfortunately, the limitations imposed on the Control and state variables are typically not taken into consideration by these methods, and the structure of fundamental equations of motion as well as other particular characteristics by these methods since they are so generic a different Control [50].

### I.5.1. Theory of Classical Control

In classical control theory, a branch of control theory that studies the behavior of dynamic systems with inputs and how feedback impacts that behavior, the Laplace Transform is a fundamental modeling technique [51].

A common goal of control theory is to alter a system, also known as the plant, such that its output complies with the desired control signal, also known as the reference, which may have a fixed value or fluctuate. A controller is developed in order to accomplish this, and it constantly examines the output and compares it to the reference in an effort to lessen the error signal and bring the actual output closer to the reference. The discrepancy between the desired and actual output is applied as feedback to the system's input [52].

It is possible because traditional control theory emphasizes linear time-invariant systems with signal-input, signal-output (SISO) relationships. The Laplace algorithm must be used to transform the input and output signals of such a system. The input and output Laplace transformations are connected by the transfer function [53].

The limitation of the open-loop controller is overcome using the feedback element of classical control theory. A closed loop controller uses feedback to control the state or outputs of a dynamical system; its name is taken from the system's low-level information for instance, a process input like the voltage delivered to an elective motor affects the process outputs like the speed or torque of the motor, which are monitored using a sensor and processed by the controller. The outcome that completed the loop is feedback as input to the process [54].

The response of a physical system can be depicted in the time domain, where it depends on its numerous inputs, previous system values, and the passage of time. The system's condition and responsiveness change throughout time. However, high-order differential equations are frequently employed to represent time-domain models of systems, and in some circumstances, even the most effective vehicle computer systems find it hard to solve these equations [55].

A normal algebraic polynomial in the frequency domain and an ordinary differential equation (ODE) in the time domain are employed in classical control theory to solve the problem. Once a system has been converted into the frequency domain, it may be managed more readily [56].

### I.5.2. Intelligent Control

Intelligent control is a computationally efficient way to direct a complex system toward a predetermined goal when it is working with incomplete instructions, inadequate representation, and in an unreliable environment. Intelligent control typically combines planning and online error correction, and it also necessitates learning about the system and its surroundings. The most important aspect is that intelligent control often employs combinatorial search as their primary operator and generalization-focused attention as their primary operator, resulting in a multiscale structure [57].

As a definition, we can say that an intelligent Controller is a computer-based controller that can generate the required control actions by emulating the thought processor of a human expert in the relevant field [58]. Artificial intelligence (AI) techniques are employed here for the purposes of information acquisition knowledge representation and the generation of control decisions via inappropriate reasoning mechanisms. There has been a lot of interest in using AI techniques to manage complicated processes as a result of the field's Consistent advancements, particularly in relation to the creation of useful expert systems or knowledge systems. So intelligent control is used by complex engineering systems to deal with conditions when traditional control techniques are ineffective [59].

### I.5.3. Comparison Study of Classical Control and Intelligent Control

This Chapter presented classical or (Traditional) control system and intelligent control schemes. In this section we will take a comparison between a classical control and intelligent control, with taking some examples of these controllers.

The traditional control methods must use a model of the system being controlled, but intelligent control may address control issues in non-typical system. This is the major destination between intelligent control and traditional control [60].

In a time when calculation had to be done by hand, classical control theory flourished, manually resolving differential equations is laborious, and Laplace Transforms were excellent at streamlining calculations. Furthermore, one might construct controllers in a tractable manner thanks to methods like the Nyquist criterion and root locus design. This is possible for systems with large state space, especially when some poles predominate over the majority of others, when a computer wasn't an option in practical applications, classical control theory provided a useful toolkit for manually. creating a functional controller [61].

The benefit of modern control (or Intelligent Control) theory is that it offers excellent structural insight into control system design. It is the proper approach to approaching control The main issue is that manual computations are impossible, to determine if a system is stable, we would need to perform a laborious mathematical collation. Additionally, if made you constantly adjust the controller and manually calculate the results every time, we would become unpractical [62]. So modern control theory is a more effective tool than classical control theory.

The figure (I-2) gives some examples of classical control and intelligent control [63]:

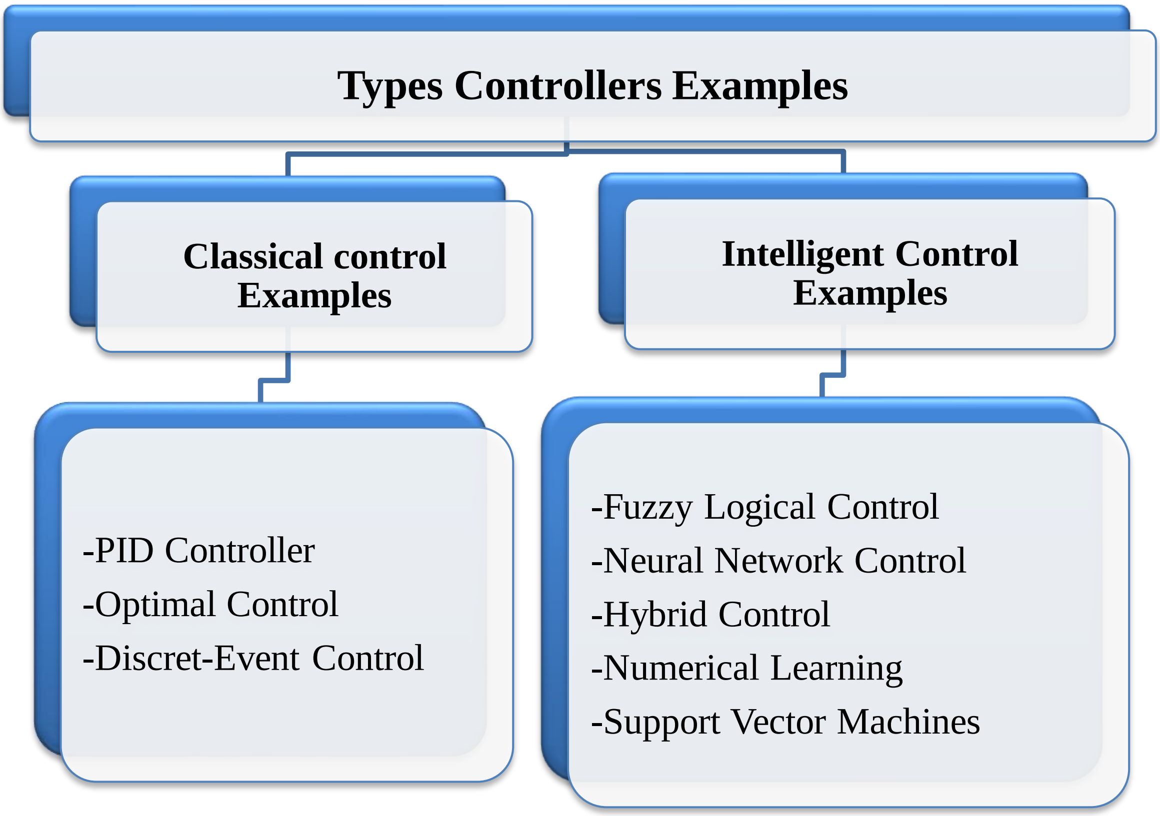



**Figure (I-2)**: Diagram comparison between classical and intelligent controllers' examples

### I.6. Nonlinear System Stability Analysis

Stability theory is essential for both dynamics and control in nonlinear system stability in system engineering, particularly in the field of automation and control systems. Whether or not control and disturbance inputs are present, a dynamical system must be stable in order to be helpful, especially in the majority of real-world applications. Bounded-input/bounded-output stability is the condition in which the system's outputs and internal signal are both constrained within permissible bonds. More precisely, asymptotic stability is the condition in which the system's outputs tend to an equilibrium state of interest [64].

The stability of a system with respect to its equilibria, the orbital stability of a system's output trajectory, and the structural stability of a system itself are the main concerns in nonlinear dynamics and control systems. Conceptually, there are various types of stability, but there are fundamental ideas [65]. The study of a mechanical system's equilibrium state, which got its start with E. Torricelli's investigation of the equilibrium of a rigid body under the force of gravity in 1644, is where the basic idea of stability has its origins. The most well-known conclusion on the stability of conservative mechanical systems is G. Lagrange's classical stability theorem, which was developed in 1788. This rule indicates that if the system's potential energy is at a low value, an equilibrium that may be subject to a simple restriction is stable [66]. The overall problem of motion stability was solved by A. M. Lyapunov in 1892, although the development of the core concepts of system and trajectory stabilities had a long history and had undergone numerous beneficial developments [67]. Control systems are solidly built to withstand a variety of disturbances, from noise to unusual dynamics. Numerous difficult models have been presented in recent study and publications. Current, thorough evaluations, however, support the development of closed-loop system stability solutions. In this thesis, we demonstrate the significance of stability and argue that it is a crucial consideration when examining the behavior of a nonlinear system [68].

Nonlinear systems behave linearly close to their stable state. Actually, the Taylor sense expansion's central concept is this. If you zoom in enough, any function will seem as a straight line [69]. Figure (I-3) provides an illustration. Nonlinear equation stability [70]:

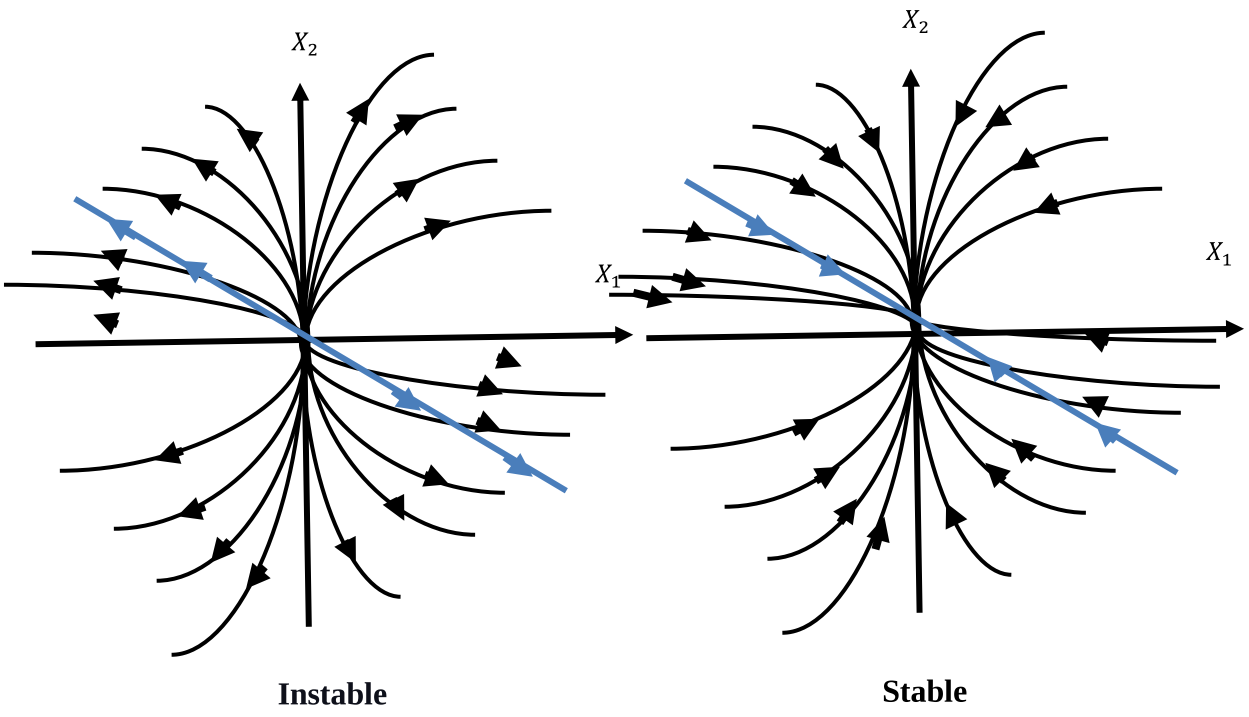



**Figure (I-3):** Example of stability for nonlinear equation

For this section we conclude that it is for find the stability of study state of nonlinear system must first linearize the equation by Taylor series expansion, put the linearized system into matrix format, and find the eigenvalues of the matrix of coefficients (Jacobian Matrix) [71]:

$$J = \begin{bmatrix} (\partial f_1/\partial y_1) & (\partial f_1/\partial y_2) & (\partial f_1/\partial y_3) \\ (\partial f_2/\partial y_1) & (\partial f_2/\partial y_2) & (\partial f_2/\partial y_3) \\ (\partial f_3/\partial y_1) & (\partial f_3/\partial y_2) & (\partial f_3/\partial y_3) \end{bmatrix} \quad \textbf{(I-6)}$$

### I.7. Conclusion

In this chapter, we have shown that the behavior of dynamic nonlinear systems can be described using differential equations. Linear equations show that an equation can represent some operations. However, the evolution of large processes that can be found in the environment or industries does not follow a linear path. So, we introduce nonlinear dynamical systems and give some tools that allow us to better analyze their behavior (stability and control nonlinear system). We have shown that the evolution of these dynamical systems can have chaotic behavior. The car suspension system, one of the non-linear systems that will be the focus of the second chapter, is the system that this thesis is researching, so this chapter is regarded as the introduction to the dissertation.

# Chapter II

## Literature Review on Vehicle Suspension Systems

## II. 1. Introduction

The performance and safety of an automobile are greatly influenced by its suspension, which also affects how well it rides, handles, and even consumes fuel. The suspension system is made up of a variety of components that work together to provide stability at high speeds, dampen shock from road irregularities, and avoid body roll during cornering maneuvers. These components can become worn down over time due to improper tuning and maintenance of the suspension, which can lead to poor handling characteristics or, worse yet, an accident from a lack of control while driving. This chapter examines the various forms of suspension and defines each with the goal of improving ride comfort for both drivers and passengers [72, 73].

## II. 2. Definition of Suspension Cars

The vibrations and concussions caused by the road's elevations and protrusions during the vehicle's trip made the vehicle waver, endangering the safety of the passengers or the freight. To reduce road shocks, absorb them, and prevent them from being transmitted to the body of the vehicle as much as possible, shock absorbers have been placed between the wheel axle and the body of the vehicle [74]. The suspension system adds grit to the road, which helps improve the roads. Suspension system carries a load bearing body and maintains its body. Figure (II-1) illustrate the components of suspension systems [75]:

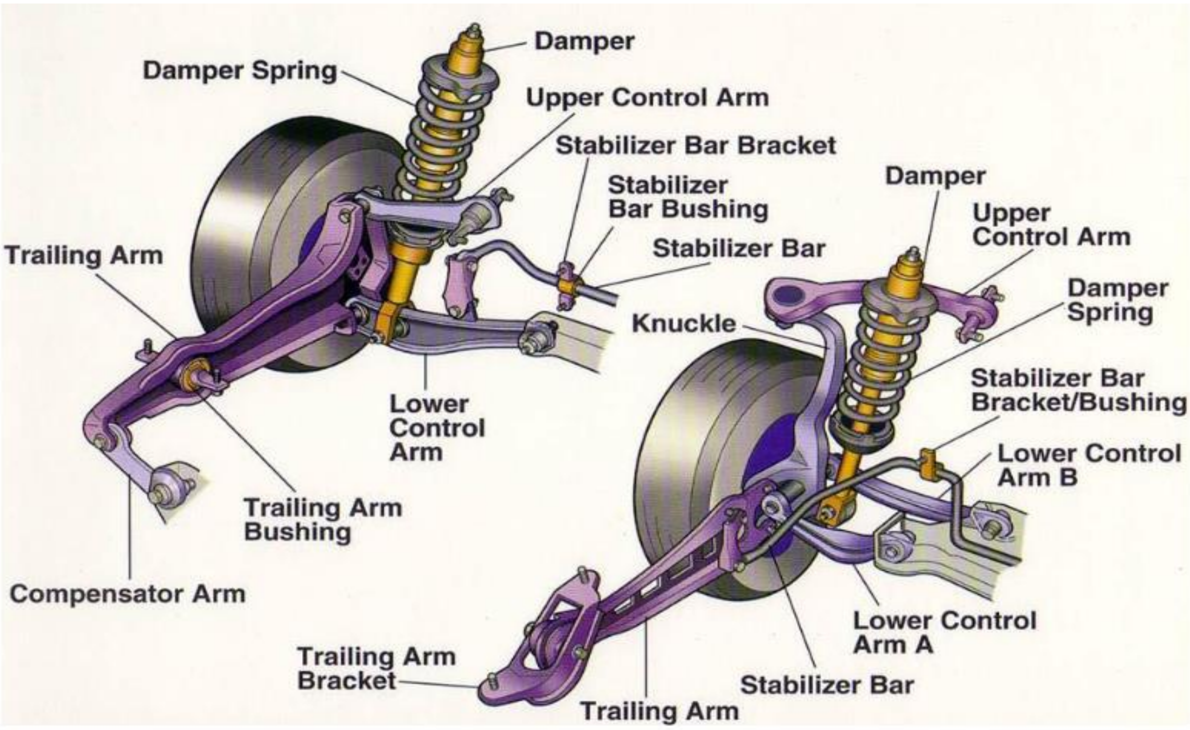



**Figure (II-1)**: Display components of suspension systems

For handling and reliable travel, suspension systems are crucial. The suspension mechanism is currently one of the most significant components of the auto system, which essentially determines the vehicle's safety and comfort while driving. The vehicle's suspension system transmits and regulates static and dynamic forces and reactions to the ground [76].

The suspension system connects the wheels of the vehicle to springs, safety nets, and other components. It is a procedure that physically separates the vehicle's wheel and body. A spring that converts kinematic energy into potential energy, or vice versa, and a shock absorber, a mechanical device made to disperse kinetic energy, are the three basic parts of the suspension system. The structure maintains the vehicle's weight and sets the suspension geometry [77].

Shock absorbers are hydraulic pump-like components that help control how the vehicle's springs and suspension move during impact and recoil. The major function of the shock absorber is to maintain constant tire contact with the road surface, which allows for the safest possible steering and braking responses from the vehicle. Figure (II-2) depicts an illustration of a shock absorber utilized in a car suspension system [78].

**Figure (II-2)**: The shock absorber utilized in a car suspension system

The primary function of the suspension system of a vehicle is to lessen the vertical acceleration that is conveyed to the vehicle body (indirectly to the occupants or loads), ensuring comfort in the lane. Hence, the suspension system's goals are to keep road shocks from transferring to the vehicle's components, protect the occupant from road shocks, and keep the vehicle stable when pitching or rolling while it is moving [79]. The deployment of various controlled suspension systems and their integration into the overall control of passenger vehicles may be influenced by the rising demands on the dynamics and stability of vehicles as a whole, as well as the quick development of hybrid and electric passenger cars [80]. The stability of suspension systems and how to enhance the necessary suspension performance, particularly driving comfort, road handling, and suspension deflection, have received a lot of attention from scientists [81]. Many car suspension models have been put forth up until this point. A thorough understanding of vehicle dynamics and automobile suspension networks is necessary to modify the suspension. Since the entire weight of the car rests on the contact surfaces of the tires, keeping the wheel in contact with the ground is crucial for the safety of the vehicle's movement [82]. In this chapter, we will explain in detail suspension systems, their types, and their analysis methods.

## II. 3. A Brief History of Suspension Systems

A smooth and safe ride for passengers is made possible by the suspension system, an essential part of modern cars. It is intended to stabilize the car and absorb stress and vibration from the road. Suspension has a lengthy and fascinating history that dates back to the oldest modes of mobility. The use of wooden blocks or springs to soften the ride in horse-drawn carriages was one of the earliest examples of suspension. They were normally constructed using several layers of curved timber pieces fastened together with bolts to form a flexible structure. Although this kind of suspension was good at absorbing stress and vibration, it was also cumbersome and prone to damage [83].

The first automobiles were created as self-propelled variations of ox-drawn carriages. Horse-drawn carriages, on the other hand, had been built for relatively slow speeds, and their suspension was not well suited to the faster speeds made possible by internal combustion engines [84].

Only with the onset of industrialization was the first practicable spring-suspension conceivable, which required highly developed metallurgical knowledge and abilities. The first patent for a spring-suspension vehicle was filed by Obadiah Elliott; the body of the carriage was fastened directly to the springs attached to the axles, and each wheel had two sturdy steel leaf springs on each side. Within a decade, the majority of British horse carriages had springs installed: steel springs for bigger carriages and hardwood springs for lighter, one-horse models to evade taxation. They were frequently fashioned from low iron and typically took the shape of many-layered leaf springs [85].

There have been leaf springs since the ancient Egyptians. With limited effectiveness at first, ancient military engineers powered their siege engines with leaf springs shaped like bows. Years later, the use of leaf springs in catapults was improved and made to function. A strong tree limb can also be utilized as a spring, as in the case of a bow. Springs are not just constructed of metal [86]. This system was used by horse-drawn carriages and the Ford Model T, and it is still employed in larger vehicles today, particularly in the rear suspension. This was the first modern suspension system, which, together with improvements in road building, signaled the single biggest advancement in road transportation prior to the invention of the automobile [87]. The Abbot Downing Company of Concord, New Hampshire, reintroduced leather strap suspension, which gave a swinging motion instead of the jolting up and down of a spring suspension, because the British steel springs were not well adapted for usage on America's poor roads at the time [88].

Shock absorbers were initially used in an automobile by the Mors of Paris in 1901. Henri Fournier won the important Paris-to-Berlin race on June 20, 1901, thanks to a damped suspension system on his "Mors Machine." The best rival was Léonce Girardot in a Panhard, who finished in 12 hours, 15 minutes, and 40 seconds, while Fournier's winning time was 11 hours, 46 minutes, and 10 seconds [89].

The first production vehicle with coil springs was the Brush Runabout, built by the Brush Motor Corporation, in 1906 [90]. In most cars today, coil springs are utilized. Torsion bars were first employed in a suspension system by Leyland Motors in 1920 [91].

The Lancia Lambda introduced independent front suspension in 1922, and mass market cars started to use it more frequently in 1932. All four Wheels of modern autos have independent suspension [92].

## II. 4. Classification of Vehicles Suspension Systems

Three different types of car suspension frameworks active, semi-active, and passive have been developed by diverse experts using varied techniques and algorithms. The passive suspension structure lacks vehicle stability in comparison to semi-active and active suspension systems [93]. The selection of the proper spring stiffness (due to the spring type and its characteristic) and shock absorbers characteristic reflected by the damping coefficient determines the dynamic behavior of passive automotive suspension systems [94].

### II. 4. 1. Passive Suspension Systems

The passive suspension system depicted in Figure (II-3), and Figure (II-4) show a general suspension that is made up of dampers and viscous linear springs with constant stiffness and damping coefficients. The car's passive suspension is dependable, easy to use, and reasonably priced [95]. This mechanism is used to hold the shock absorber and spring in place between the body frame and the wheel bracket, the rod pushes the piston from the outside. The piston can move via the apertures that permit liquid to enter through the cylinder parts. This liquid flow produces reaction forces that are proportional to the displacement of the unsprung and sprung masses with respect to the flow speed [96].

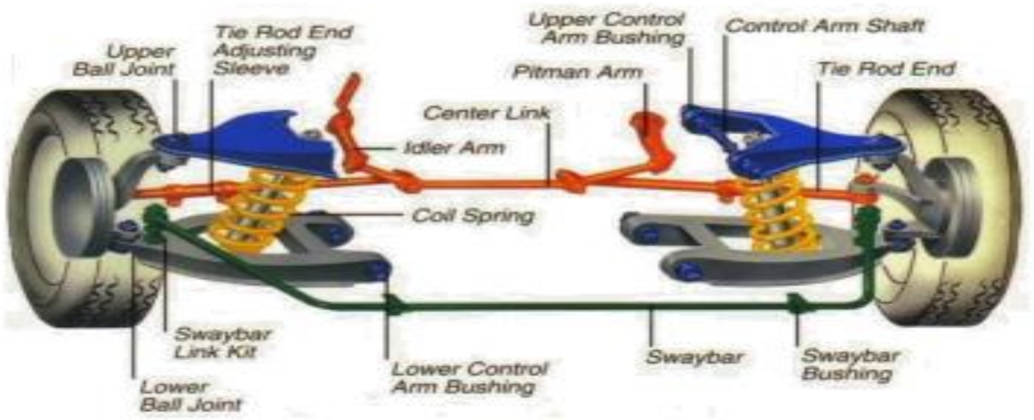



**Figure (II-3)**: Vehicle's general suspension components

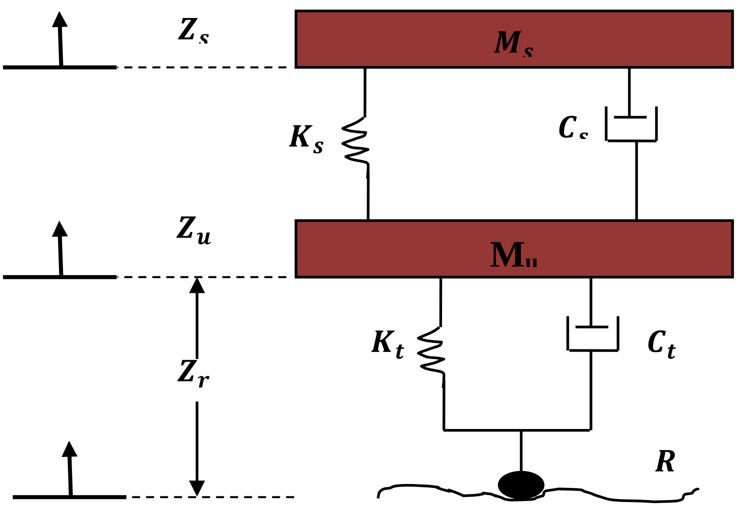



**Figure (II-4)**: A theoretical model for passive suspension of a quarter car

By turning the oscillations' energy into heat and dispersing it into the atmosphere, damping is accomplished. Since this system lacks external control and makes only minor adjustments to materials, valves, or even forms, it cannot produce acceptable results to address suspension issues [97]. The passive system's fixed damper and spring portion are insufficient to absorb energy to handle the load or road disturbance that has affected the vehicle system. Equations (II-1) serve as example of the mathematical formulation of the equation of motion for the passive suspension system.

$$\begin{cases} M_s\ddot{Z}_s = -K_s(Z_s - Z_u) - C_s(\dot{Z}_s - \dot{Z}_u) \\ M_u\ddot{Z}_u = K_s(Z_s - Z_u) + C_s(\dot{Z}_s - \dot{Z}_u) - K_t(Z_u - Z_r) - C_t(\dot{Z}_u - \dot{Z}_r) \end{cases} \quad \textbf{(II-1)}$$

where: $Z_r$ is an input for road unevenness $[m]$, $Z_s$ is a vertical displacement of the sprung mass $[m]$, $Z_u$ is a vertical displacement of the unsprung mass $[m]$, $M_s$ $(kg)$ is a sprung mass, $M_u$ $(kg)$ is a unsprung mass, $(C_s, C_t)$ is a damping coefficient $[kN.{}^{s}/_{m}]$, $K_t$ is a tire stiffness $[kN/m]$, and $K_s$ is a spring stiffness $[kN/m]$ [98].

**II. 4. 2. Semi-active suspension system**

The damping force of the shock absorber is controlled by a type of automotive suspension system called semi-active suspension in response to information from the constantly changing road surfaces. It aims to roughly implement the active suspension (to be discussed later) with a shock absorber that has a damping force adjustment [99].

A variable damper or other variable dispersing components are used in the semi-active suspension system. A viscous twin-tube damper is an illustration of a variable dissipater; the damping coefficient may be altered by altering the diameter of the piston hole. A semi-active dissipator that utilizes rheological magnet fluid is a rheological magnet (MR) damper. MR fluids are substances that change in rheological behavior in response to an applied magnetic field. This change is typically manifested as a rise in yield stress, which boosts the damper's dissipative power by regulating the electromagnetic field with the applied magnetic field [100]. To reduce actuation energy usage, semi-active suspension systems have been investigated in a number of different methods [101].

The semi-active suspension is similar to a passive suspension with simply a distinction in having a variable damping coefficient but furthermore a fixed spring constant and furthermore a lack of active power sources. This type of device permits seamless switching between passive dampers with a coefficient of semi-active damping, as shown in Figure (II-5) [102].

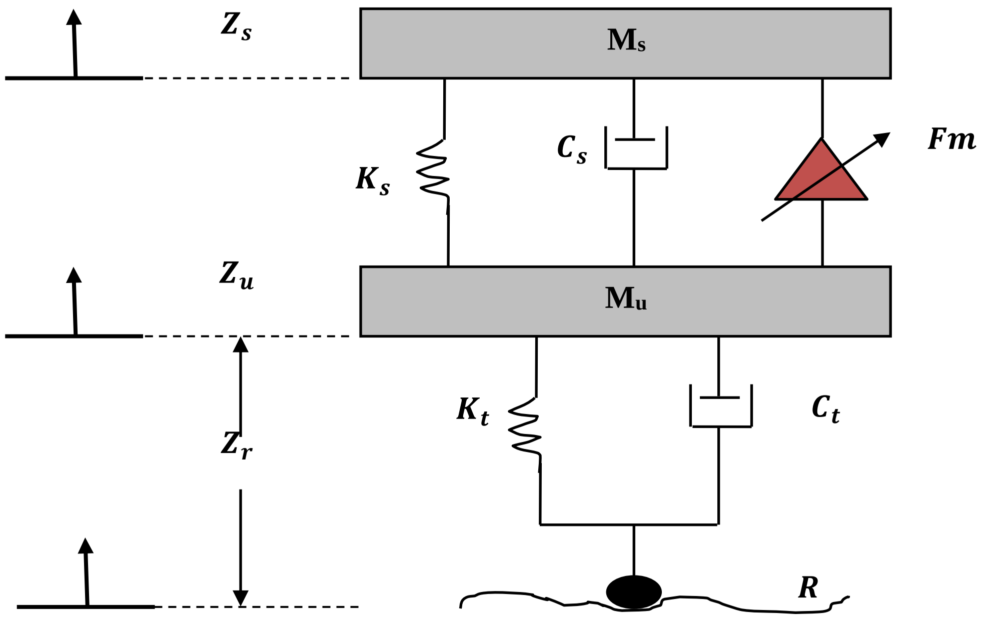



**Figure (II-5)**: A theoretical model for semi-active suspension of a quarter car

For the situation of a semi-active suspension system with two degrees of freedom, the equation of motion can be written as follows:

$$\begin{cases} M_s\ddot{Z}_s = -K_s(Z_s - Z_u) - C_s(\dot{Z}_s - \dot{Z}_u) + Fm \\ M_u\ddot{Z}_u = K_s(Z_s - Z_u) + C_s(\dot{Z}_s - \dot{Z}_u) - K_t(Z_u - Z_r) - C_t(\dot{Z}_u - \dot{Z}_r) - Fm \end{cases} \quad \textbf{(II-2)}$$

where: $Z_r$ is an input for road unevenness $[m]$, $Z_s$ is a vertical displacement of the sprung mass $[m]$, $Z_u$ is a vertical displacement of the unsprung mass $[m]$, $M_s$ $(kg)$ is a sprung mass, $M_u$ $(kg)$ is a unsprung mass, $(C_s, C_t)$ is a damping coefficient $[kN.{}^{s}/_{m}]$, $K_t$ is a tire stiffness $[kN/m]$, and $K_s$ is a spring stiffness $[kN/m]$, $Fm$ Damping coefficient of damper with controlling force.

### II. 4. 3. Active suspension system

The best suspension for a vehicle is an active suspension instead of the passive suspension given by huge springs, where the movement is entirely dependent on the road surface; it is used to regulate the vertical movement of the vehicle's wheels relative to the chassis or vehicle body [103], by maintaining the tires perpendicular to the road in corners, these technologies enable automakers to achieve a greater degree of ride quality and automobile handling, allowing for improved traction and control. Body rolls and pitch fluctuation are almost eliminated by the system in a variety of driving circumstances, including cornering, accelerating, and stopping [104].

Electronic control frameworks that regulate the activity of the suspension components are installed in active suspensions. The actuator, mechanical spring, and shock absorber are components of the active suspension, as shown in Figure (II-6) and Figure (II-7). They don't have the same performance limitations as passive suspensions, and they create a new advancement to remove the challenge of designing a compromise that is present in passive suspensions. In active suspension systems, the actuator enables the suspension to absorb wheel acceleration energy, minimizing the acceleration of the vehicle body [105].

As the actuator force controls both shock absorbers, such systems are much more sensitive to the induced vertical forces brought on by unforeseen changes in road access. Depending on the provided design, this actuator can be controlled by various types of controllers and operates by receiving or dispersing system power. Active suspension can lead to an enhanced suspension design with suitable control techniques, resulting in a better balance between vehicle handling comfort and driving stability [106].

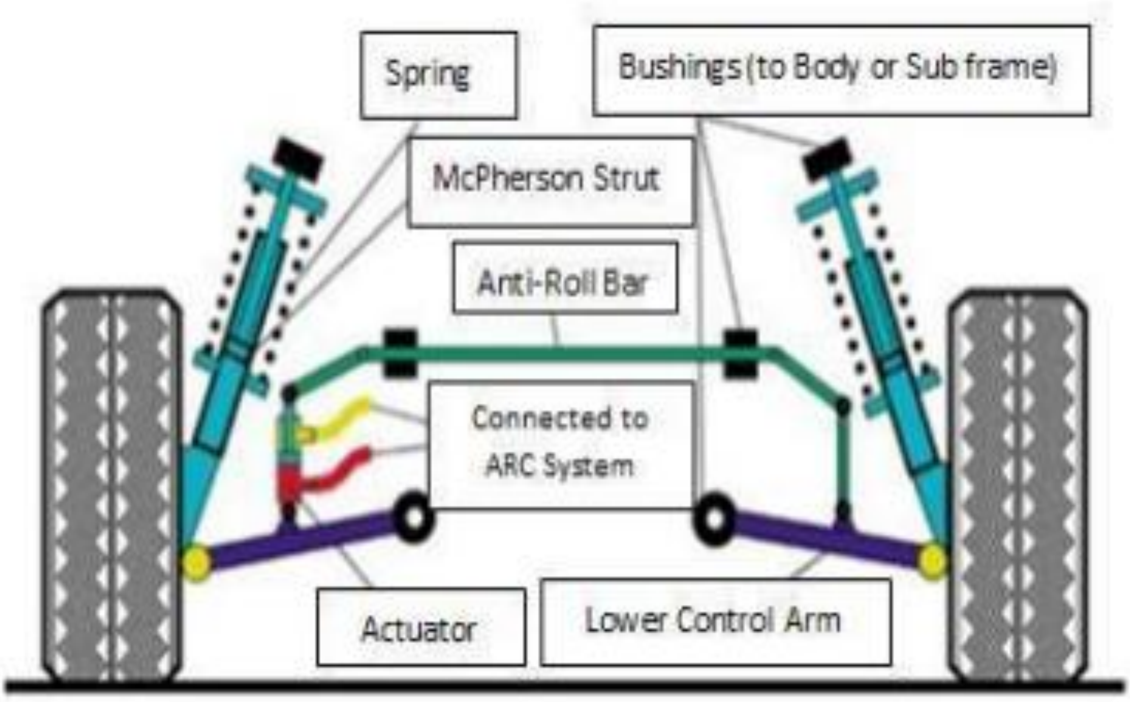



**Figure (II-6)**: Active Suspension System of a Car [107]

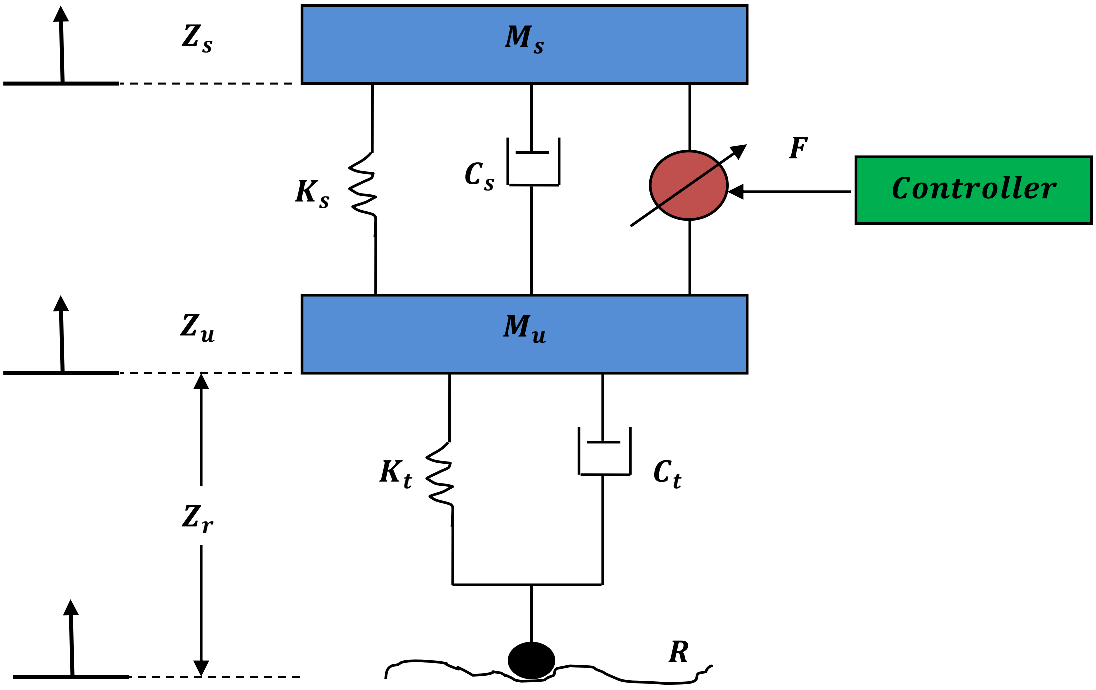



**Figure (II-7)**: A theoretical model for Active suspension of a quarter car

Newton's law of motion, as illustrated in Equations (II-3) can be used to derive the mathematical description of the equation of motion for the active suspension system in Figure (II-7). According to this law, the sum of the forces $(F)$ acting on the sprung and unsprung masses is equal to the product of the two forces' respective masses and velocities. Thus, the following is a mathematical statement for a sprung and an unsprung active suspension system.

$$\begin{cases} M_s\ddot{Z}_s = -K_s(Z_s - Z_u) - C_s(\dot{Z}_s - \dot{Z}_u) + F \\ M_u\ddot{Z}_u = K_s(Z_s - Z_u) + C_s(\dot{Z}_s - \dot{Z}_u) - K_t(Z_u - Z_r) - C_t(\dot{Z}_u - \dot{Z}_r) - F \end{cases} \quad \textbf{(II-3)}$$

where: $Z_r$ is an input for road unevenness $[m]$, $Z_s$ is a vertical displacement of the sprung mass $[m]$, $Z_u$ is a vertical displacement of the unsprung mass $[m]$, $M_s$ $(kg)$ is a sprung mass, $M_u$ $(kg)$ is a unsprung mass, $(C_s, C_t)$ is a damping coefficient $[kN.{}^{s}/_{m}]$, $K_t$ is a tire stiffness $[kN/m]$, and $K_s$ is a spring stiffness $[kN/m]$, $F$ is Actuator Force [108].

## II. 5. Characteristic of Suspension Systems

The performance and comfort of the vehicle's occupants are significantly impacted by the suspension system. As we discussed, the suspension system is made up of three components: springs, dampers, and linkages [109]. Every single one is important to suspension performance. The spring softens the system and stores energy. The damper dissipates energy and calms the system. Based on the needed geometrical constraints and characteristics for the vehicle, the connections constrain system motion. The ride comfort and handling of a suspension system are its primary features [110].

### II. 5.1. Ride Comfort

There are four ways to gauge how comfortable a ride is (also known as the human response to vibration [111]). The ISO 2631 standard [112] is one approach that is frequently utilized, although there are also additional standards as BS 6841 [113], VDI 2057 [114], and AAP [115]. The most significant measured factor for ride comfort appears to be vertical displacement and acceleration. To assess how comfortable a vehicle's ride is, consider its body acceleration vs frequency and RMS of acceleration. In other words, the absolute acceleration of the car body determines how comfortable the ride is. Similar to this, the vehicle body's settling period affects ride comfort [116]. Ride comfort for large trucks is determined by pitch plane motion as opposed to lateral and roll vibration [117].

### II. 5. 2. Handling

The highest feasible lateral acceleration or the percentage of the tire's available friction is referred to as "handling." The vehicle behaves linearly at levels below the linearity limit. The vehicle is physically difficult to handle at values higher than the friction limit of the tires, and even the most seasoned driver in a well-handling vehicle will lose control. The vehicle's designer must accomplish two goals: boost the tires' absolute friction limit and raise the linearity limit [118].

Dynamic handling tests and steady state handling tests are the two primary divisions of handling testing (also called transient response tests). The constant radius test, in which the car is driven in a circle, is the steady state handling test. Steering wheel angle and lateral acceleration are the most crucial variables that must be assessed. The test begins moving slowly [119]. The vehicle can no longer move in a consistent radius, so the speed is steadily increased. To determine if the car is oversteering, understeering, or acting neutrally, a graph of lateral acceleration against speed is employed [120]. There are two types of dynamic handling tests: closed-loop (where the driver attempts to maneuver the car along a predetermined course) and open-loop (where steering angle versus time is determined). The double-lane change test and the obstacle avoidance test are examples of closed-loop testing. An expert driver or a robot can do open-loop tests. The step-over and pulse-over tests are two examples of these [121]. The normal force of tires is one of the most significant measurable characteristics for handling; minimal values play a significant part in the assessment of vehicle handling [122].

## II. 6. Simulation Modelling for Passive Semi-Active and Active Suspension

Obtaining a mathematical model for the passive, semi-active, and active suspension systems for the quarter-car model is the goal of this chapter. Modern automotive suspension systems exclusively use fixed-rate springs and damping coefficients as passive components. Automobile suspension systems are frequently judged on their capacity to enhance passenger comfort and offer good road handling. Only passive suspensions provide a solution to these two competing demands. By directly manipulating the suspension force actuators, active and semi-active suspensions have the potential to minimize the traditional design as a compromise between handling and comfort [123].

### II. 6. 1. Simulation Modeling for Passive Suspension

The creation of a car's automatic suspension system turns out to be an intriguing control challenge. When designing the suspension system, one of the four wheels from a 1/4 vehicle model is used to simplify the issue to a one-dimensional spring-damper system [124]. The following figure (Figure (II-7)) shows the simulation model of a passive suspension system:

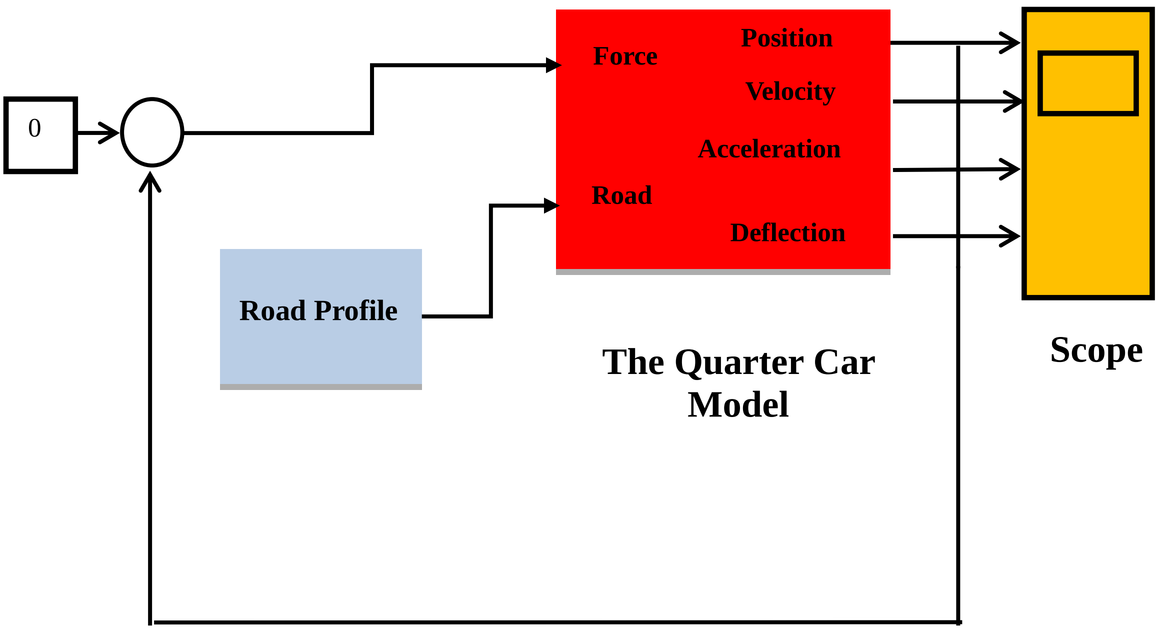



**Figure (II-8)**: Simulation model of passive suspension system using MATLAB/SIMULINK

## II. 6. 2. Simulation Modeling for Semi-Active and Active Suspension Systems

To keep the tires perpendicular to the road, active suspension technology allows automakers to achieve higher levels of ride quality and vehicle performance. Active suspension systems use an actuator to independently raise and lower the chassis at each wheel. In contrast, semi-adaptive suspension systems adjust shock absorber stiffness to account for changing road conditions or dynamic conditions [125]. The figure (Fig. (II-8)) shows the simulation modeling of active or semi-active suspensions:

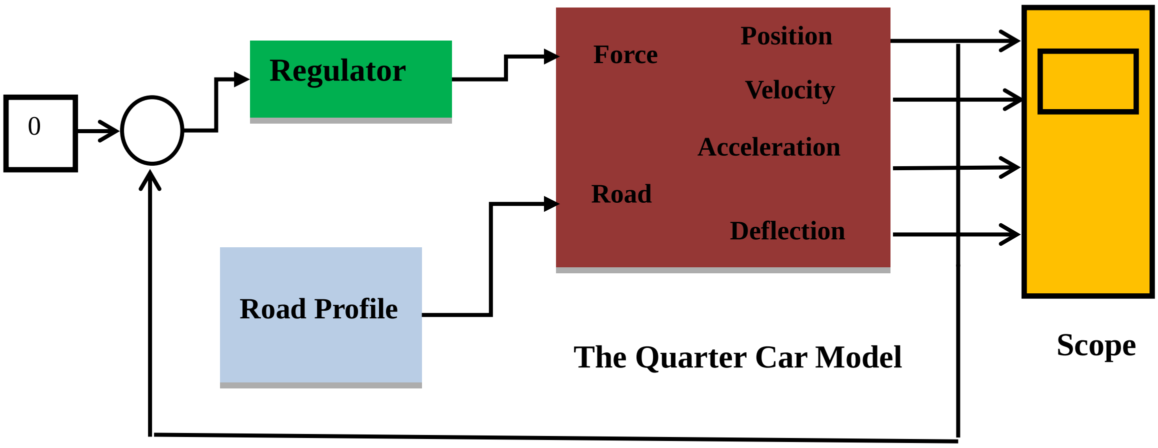



**Figure (II-9)**: Simulation model of semi-active and active suspension suspensions systems using MATLAB/SIMULINK

Adaptive or semi-active systems can only change the viscous damping coefficient of the shock absorber; they cannot contribute more energy to the suspension system. In contrast to adaptative suspensions, which typically have a slow time response and a limited range of damping coefficient values, semi-active suspensions have time responses that are close to a few milliseconds and can offer a wide range of damping values [126]. Due to this, adaptive suspensions frequently only provide a variety of riding modes (comfort, normal, sport, etc.) that correlate to various damping coefficients, as opposed to semi-active suspensions that alter the damping in real time depending on the driving conditions and the dynamics of the vehicle [127].

Despite having restricted intervention (for example, the control force can never have a different direction than the current vector of the suspension's velocity), semi-active suspensions are less expensive to build and consume very little energy. Research on semi-active suspension has recently made strides that have narrowed the capability gap between semi-active and fully active suspension systems [128]. The disadvantages of active suspension are its higher cost, increased complexity, and mass of the device, as well as its frequent maintenance needs in specific applications. Maintenance may need specialized equipment, and some problems may be difficult to diagnose [129].

## II. 7. Conclusion

The set of mechanical parts, springs, and dampers known as a suspension system attach the wheels to the chassis. It has traditionally served two purposes: controlling the car's steering and braking for safety and ensuring the comfort of the passengers against shocks, vibrations, and other variables. The wheels and axles of a wheeled vehicle are mechanically connected to the structure of the car by a system of springs or shock absorbers. It also helps to maintain the correct vehicle alignment and height. To maintain the steering wheel perpendicular to the ground and maximize traction, it also regulates the vehicle's orientation. The suspension also aids in protecting the vehicle and its contents from harm and deterioration. A car's front and rear suspensions might be made differently.

car's suspension system is in charge of ensuring a comfortable ride and maintaining control of the vehicle. The suspension system increases the friction between the tires and the road to provide steering stability and good handling.

This chapter contains an overview of suspension systems. We also studied the basic types of suspension systems and their role in car stability. Our focus will be on the active suspension system, which is the first focus of research because it provides better stability for a car than passive or semi-active suspension. The next chapter will study control systems that can be adopted in the active suspension.

# Chapter III

## Controller Design and Metaheuristic Optimization

## III. 1. Introduction

Control theory is the foundation for controlling nonlinear systems, providing methods for designing controllers which can be used in various applications such as robotics and autonomous vehicles [130]. In control theory, we analyze system behavior by studying its dynamics and stability properties under different input conditions. This analysis helps us identify possible areas where we may need additional control or intervention in order to maintain desired performance levels over time or when subjected to external disturbances or changes in environment/conditions etc.… [131].

For controlling these systems, scientists and researchers used many control systems that developed over time, leading to artificial intelligence, which gave great effectiveness to controllers of all kinds [132].

Artificial Intelligence (AI) is rapidly transforming the way we live and work. It’s changing how businesses operate, improving customer service, and creating new opportunities for growth. AI has become an integral part of our lives, from autonomous vehicles to virtual assistants like Alexa and Siri. As AI technology advances at a rapid pace, it's becoming increasingly important for businesses to understand its capabilities in order to stay competitive in today's market. With this understanding comes the potential for companies to create innovative products that can revolutionize their industry or even change the world as we know it [133].

The Artificial Intelligence is proving invaluable when it comes to tackling complex problems such as natural language processing (NLP), image recognition and more. This allows us to make sense of massive amounts of data faster than ever before– enabling us to provide more accurate predictions for any given situation. As we continue exploring the possibilities offered by this revolutionary technology, there’s no doubt that Artificial Intelligence will play a major role in shaping our future [134].

## III. 2. Control Techniques

James Clerk Maxwell initially outlined the theoretical underpinnings of governor operation in the 19th century, which is when control theory first emerged [135]. The development of PID control theory by Nicolas Minorsky from 1922 onwards followed the advancement of control theory by Edward Routh in 1874, Charles Sturm, and Adolf Hurwitz in 1895, who all contributed to the construction of control stability criteria [136].

There are a variety of other uses of mathematical control theory that go far beyond building process control systems for the industrial sector. Control theory has applications in the biological sciences, computer engineering, sociology, and operations research as well as wherever else that feedback occurs because it is the general theory of feedback systems [137]. In this chapter, we will focus on the classical control systems and the smart control systems used in our study.

**III. 2. 1. Proportional- Integral- Derivative (PID) Controller**

A closed-loop control system called a PID controller measures the process output variable and modifies the input in accordance with the error value. The discrepancy between the set point and the measured process variable is the error value [138]. PID controllers operate processes in a manner similar to how people do. to ride a bike at a set speed, as an illustration. The bike's throttle needs to be adjusted based on the error value (required value minus actual value) [139]. PID controllers compare measured process variables with set-point variables. Predict potential errors, modify the input, and eliminate all errors. PID controllers are used for a variety of things, including temperature control, liquid flow control, quad-copter flight control, and vehicle cruise control [140].

Proportional, integral, and derivative gains are used in PID control to regulate a process variable or system output. To calculate the error signal, they use the process output as input and compare it to the set point value. The proportional, derivative, and integration controllers are used to process the error value. In order to control the process variable, the PID controller sends input to the system in accordance with the error value [141].

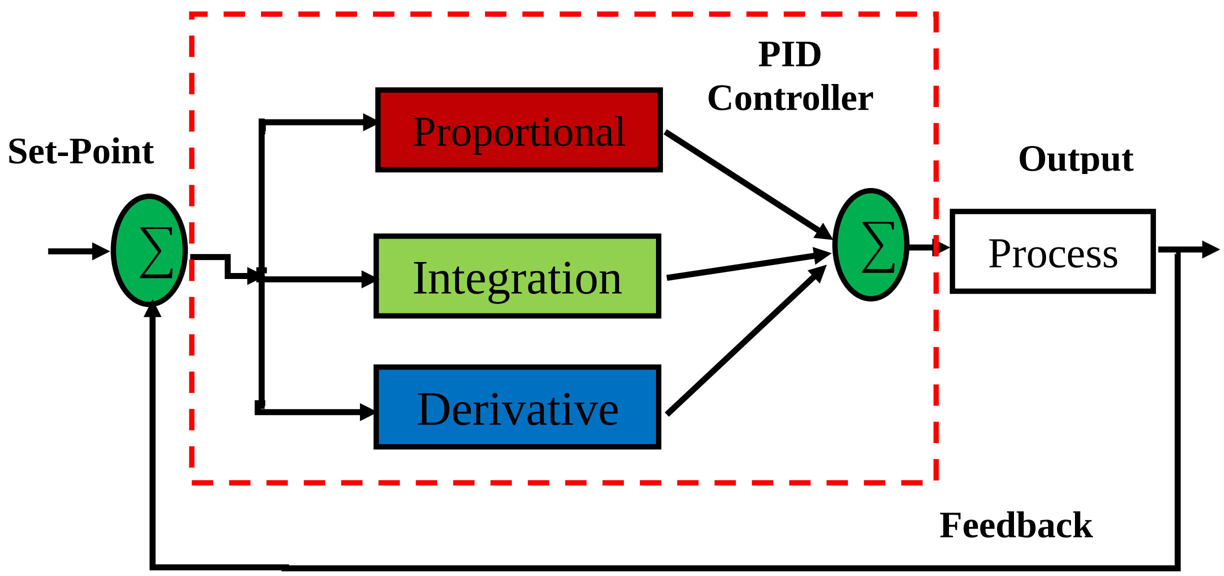



**Figure (III-1)**: Design of PID controller

The mathematical control function of PID control is:

$$U(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{de(t)}{dt} \quad \textbf{(III-1)}$$

Where $K_p$, $K_i$, $K_d$, the coefficients for the proportional, integral, and derivative terms, which are frequently abbreviated as P, I, and D, respectively, are all non-negative. In the usual form of the formula (III-1)$K_p$, and $K_i$ are replaced by $K_p/T_i$ and $K_p T_d$, when the $T_i$ is a reset time, and $T_d$ Derivative time. So, $K_p/T_i$ measures the length of time the controller will tolerate the output being continuously above or below the set point, and $K_p T_d$ is the time constant the controller will use to try to get close to the set point [142].

$$U(t) = K_p(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d \frac{de(t)}{dt}) \quad \textbf{(III-2)}$$

In spite of the fact that a PID controller has three control terms, certain applications only require one or two terms to provide effective control. In the absence of further control actions, this is accomplished by setting the unused parameters to zero and is referred to as a PI, PD, P, or I controller. Although integral terms are frequently required for the system to reach its target value, PI controllers are frequently used in applications where derivative action would be sensitive to measurement noise [143].

### III. 2. 2. Fractional Order PID (FOPID) Controller

Mathematics dealing with derivatives and integrals from non-integer orders is known as fractional-order. Fractional calculus was rediscovered by scientists and engineers two decades ago, and it is now being used in a growing variety of domains, particularly control theory. The development of efficient methods for the differentiation and integration of non-integer order equations has significantly contributed to the success of fractional-order controllers, which is beyond dispute. Recent years have seen a lot of interest in fractional-order proportional integral derivative (FOPID) controllers from both an academic and an industrial perspective. Because they have five parameters to choose from, they offer more versatility in controller design than ordinary PID controllers [144].

Podlubny put forth the idea of FOPID controllers in 1997 [145]. He also showed how this form of controller, when employed to handle fractional order systems, responded better than the traditional PID controller. The combined differentiation-integration operator known as the difference integral operator, denoted by $\alpha D_t^a$, is frequently employed in fractional calculus. The definition of this operator, which can be used to take both the fractional derivative and the fractional integral in a single equation, is [146]:

$$\alpha D_t^a = \begin{cases} \frac{d^\alpha}{dt^\alpha} & \alpha > 0 \\ 1 & \alpha = 0 \\ \int_0^t (d\tau)^{-\alpha} & \alpha < 0 \end{cases} \quad \text{(III-3)}$$

Where α is the Fractional order, a and t are limits. It is primarily recognized the FOPID there are Three types of fractional calculus: Fractional order calculus based on Riemann-Liouville and Grünwald-Letnikov, and Caputo [147, 148]:

- Riemann-Liouville Fractional Order Calculus is defined as follow:

$$aD_t^q f(t) = \frac{d^q f(t)}{d(t-a)^q} = \frac{1}{\Gamma(n-q)} \frac{d^n}{dt^n} \int_0^t (t-\tau)^{n-q-1} f(\tau) d\tau \quad \text{(III-4)}$$

Where $n-1 \leq q \leq n$, $\Gamma()$ is Gamma function, when $\Gamma(z) = \int_0^\infty t^{z-1-t} e^{-t}\, dt$. Under the Zero initial condition, the transform of Laplace's is obtained:

$$\int_0^\infty e^{st} D_t^q f(t) dt = s^p F(s) \quad \text{(III-5)}$$

- Grünwald-Letnikov Fractional Order Calculus is defined as follow:

$$aD_t^q f(t) = \frac{d^q f(t)}{d(t-a)^q} = \lim_{N\to\infty} \left[\frac{t-a}{N}\right]^{-q} \sum_{j=0}^{N-1} (-1)^j \binom{q}{j} f\left(t - j\left[\frac{t-a}{N}\right]\right) \quad \text{(III-6)}$$

- Caputo fractional derivative calculus is defined as follow:

$$D^\alpha f(t) = \frac{d^\alpha f(t)}{dt^\alpha} = \frac{1}{\Gamma(\alpha-1)} \int_0^t \frac{f^n(\tau)}{(t-\tau)^{\alpha+1-n}}\, d\tau, (n-1) < \alpha < n \quad \text{(III-7)}$$

Where $\Gamma(n) = (n-1)(n-2) \ldots \ldots (2)(1)!$.

The definition of the Grünwald-Letnikov and Riemann-Liouville are identical when the function $f(t)$ has $(n-1)$ order continuous derivative and these definitions are the most widely used. The integral and derivative orders are fractional in the fractional-order PID framework, unlike conventional PID. Thanks to the introduction of order flexibility and the use of fractional calculus, the FOPID controller can perform better than the traditional one.

The transfer function of $PI^\lambda D^\mu$ control has the following form [149]:

$$G_{Controller}(t) = K_p + K_i \frac{1}{s^\lambda} e(t) + K_d s^\mu e(t) \quad \text{(III-8)}$$

Where $G_{Controller}(t)$ is controller transfer function, $\frac{1}{s^\lambda}$ is an integrator term, and $S^\mu$ is a derivation term. $\lambda$ and $\mu \in R^+$are the non-integer order of the integral and derivative terms.

Instead of the classical methods of calculating the variables of the controllers, the digital realization algorithms of PID and fractional order PID (FOPID) calculus have been proposed recently for their application. The following figure (Figure (III-2)) show the structure of the FOPID controller [150]:

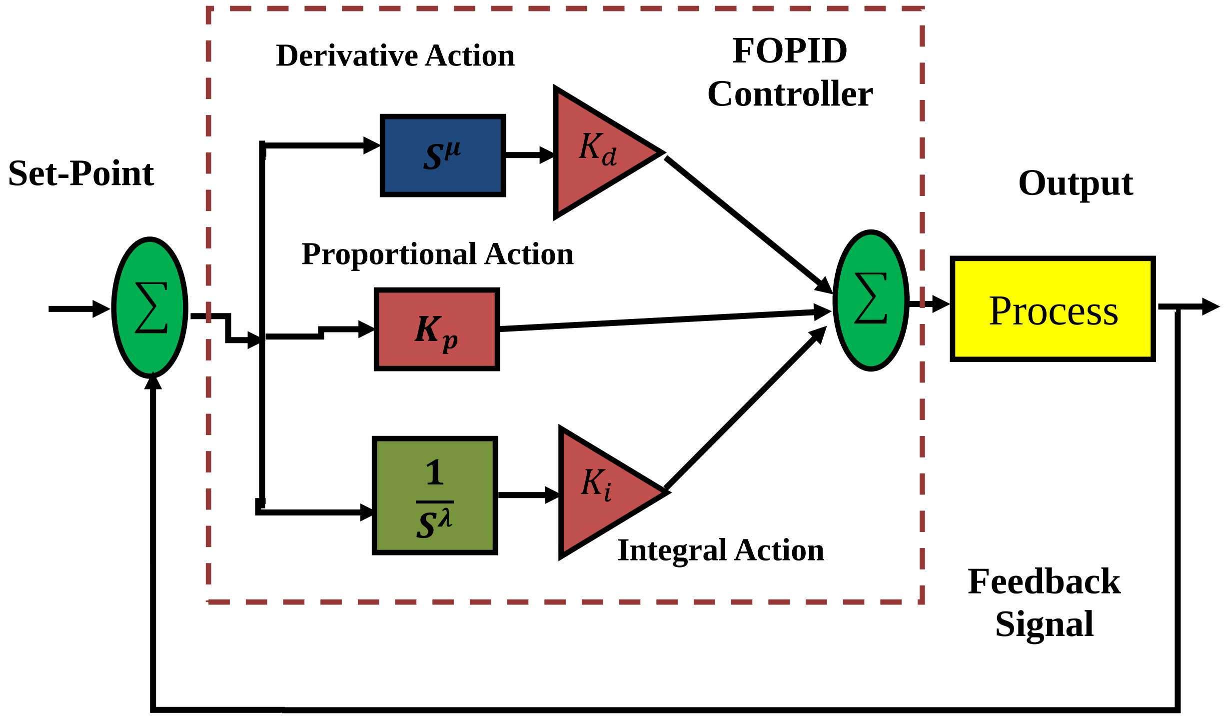



**Figure (III- 2):** Design of FOPID controller

### III. 2. 3. Linear Quadratic Regulator (LQR) Controller

While it is generally very difficult to solve the dynamic programming problem for continuous systems, there are a few key special cases where the solutions are very straightforward. In the cases of linear dynamics and quadratic cost, the majority of them involve variations. The linear quadratic regulator (LQR), which is the simplest instance, is defined as stabilizing a time-invariant linear system to the origin. Probably the most significant and influential finding in optimal control theory to date is the linear quadratic regulator. We will study the LQR control in this section [151].

#### III.2.3.1. Primary Derivation

The LQR state derivative feedback formulation take into account a state-space-based linear time-invariant system with the infinite-horizon cost function $J$ provided by [152]:

$$\dot{X} = AX + BU \quad \textbf{(III-9)}$$

$$J = \int_0^\infty [X^T QX + U^T RU]dt \quad Q = Q^T \geq 0, and\ R = R^T > 0 \quad \textbf{(III-10)}$$

The optimal $K$ is:

$$K = -R^{-1}B^T A^{-T}P \quad \textbf{(III-11)}$$

Where matrix $P$ calculated by the equation of algebraic Riccati (ARE):

$$PA^{-1} + A^{-T}P - PA^{-1}BR^{-1}B^{T}A^{-T}P + Q = 0 \qquad \textbf{(III-12)}$$

Finally, the optimal stabilizing control law is given by:

$$U(t) = -K\dot{X}(t) \rightrightarrows \quad U(t) = R^{-1}B^{T}A^{-T}P\dot{X}(t) \qquad \textbf{(III-13)}$$

The figure (Figure (III-3)) illustrate structure of the LQR control system [153]:

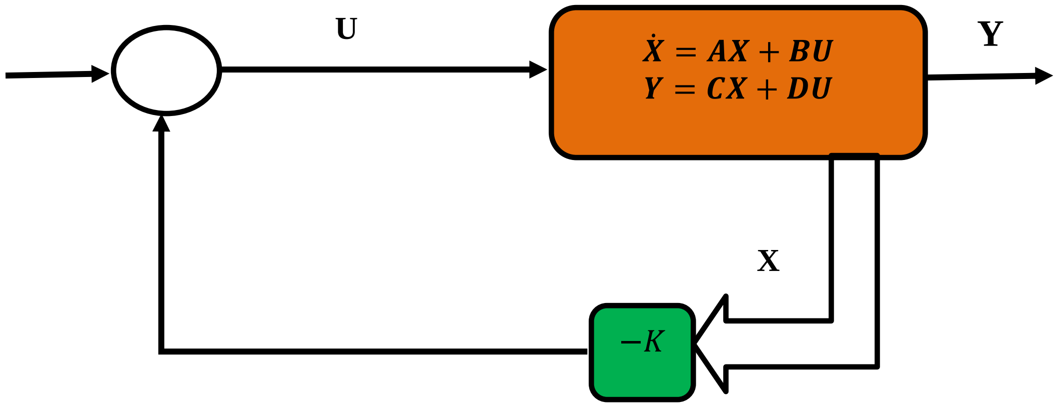



**Figure (III-3)**: The structure of the LQR control system

**III. 3. Optimization**

A key technique in engineering is optimization, which is the process of getting the best outcome possible under specific conditions, such as those of design, construction, or maintenance. The methodical search for such operating modes and designs is called optimization. It chooses the combination of steps or components that must be used to achieve optimum systems [154]. Optimization, in its most basic form, seeks the greatest or least value of an objective function corresponding to variables defined in a practical range or space. In a broader sense, optimization is the process of identifying the collection of variables that, under a variety of constraints, yields the best results for one or more objective functions [155]. An objective function and constraints are two examples of mathematical expressions that make up a single-objective optimization model [156].

$$Optimize\ f(x), x = (x_1, x_2, x_3, \ldots\ldots, x_i, x_N) \qquad \textbf{(III-14)}$$

$$Subject\ g_i(x) < b_j\,, \qquad j = 1, 2, 3, \ldots.., m \qquad \textbf{(III-15)}$$

$$x_i^{(L)} \le x_i \le x_i^{(U)}, i = 1, 2, 3, \ldots.., N$$

Where $f(x)$ is the objective function, $x$ is a set of decision variables $x_i$ that constitutes a possible solution to the optimization problem; $x_i$ is the $i^{th}$ decision variables; $N$ is the number of decision variables that determines the dimension of the optimization problem $g_i$ is the $j^{th}$ constraint; $b_j$ is a constant of the $j^{th}$ constraint; m is the total number of constraints; $x_i^{(L)}$ is the lower bound of the $i^{th}$ decision variables and $x_i^{(U)}$ is the upper bound of the $i^{th}$ decision variable [157]. Tasks involving minimization and maximization are referred to as optimization. The phrases minimization, maximization, and optimization are used interchangeably because a task requiring the maximization of the function $f$ is similar to a task involving the minimization of $-f$ (see Figure (III-4)) [158]:

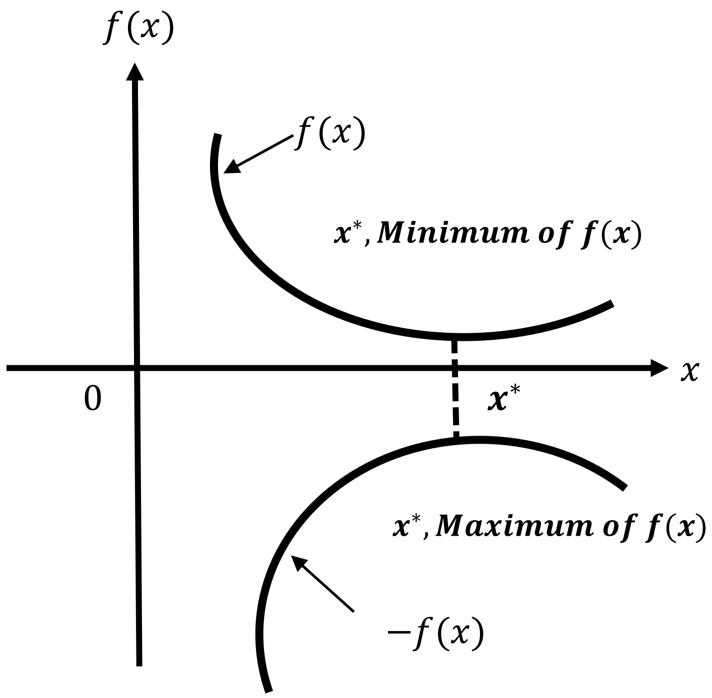



**Figure (III-4)**: Maximum of $-f(x)$ is the same as minimum of $f(x)$

### III. 3. 1. Stochastic Optimization or Improvement

The use of meta-heuristics in stochastic optimization is particularly suited for solving issues where appropriate global or local optima are challenging to locate using conventional techniques. Three features of this sort of optimization are frequently deciding factors in global optimization [159]:

- ❖ The only need for optimization using metaheuristics is that we must be able to evaluate the latter, which can therefore take any form, and we are not required to know the gradient of the function to be reduced;
- ❖ It is not necessary to choose a "good" beginning point because the initialization of the search space is done at random;

- Finally, this kind of optimization is stochastic, which limits trapping in local optima and allows for circumvention of the combinatorial explosion of possibilities.

**III. 3. 2. Objective Function**

The goal of an optimization problem is the objective function. By selecting variables or decision variables for the set of parameters that satisfy all restrictions, that aim could be maximized or decreased [160]. This is known as a feasible solution; optimal solutions are those that are possible and have objective function values that are superior to those of any other feasible solutions. For the purpose of representing and resolving the linear programming optimization issues, objective functions are frequently used. The choice variables x and y are used in the objective function, which has the form $Z = ax + by$. The best answer can be obtained by maximizing or minimizing the function $Z = ax + by$. Here, the restrictions $x > 0$ and $y > 0$ control the objective function. An objective function is used in optimization issues when the goal is to maximize profit, reduce expense, or use resources as little as possible [161].

**III. 3. 3. Decision Variables and Decision Space**

The value of the objective function is determined by the choice factors. We look for the choice variables in each optimization issue that produce the best value for the objective function or optimal. Another hand the viable decision space is the collection of decision variables that conforms to the restrictions of an optimization problem. Each potential solution to an N-dimensional problem is an N-vector variable with N entries. This vector's elements are all choice variables. The objective function is optimized by finding a point (i.e., a vector of decision variables) or points (i.e., more than one vector of decision variables) in the decision space [162].

**III. 3. 4. Local and Global Optimum**

A well-defined optimization problem is known to have a well-defined decision space. The objective function's value is defined at each location in the decision space. A solution that has the best objective function in its immediate area is referred to as a local optimal. If the following stipulation is true, a viable decision variable $x^*$ is a local optimum of a maximizing issue in a one-dimensional optimization problem [163]:

$$f(x^*) \geq f(x), x^* - \varepsilon \leq x \leq x^* + \varepsilon \qquad \textbf{(III-16)}$$

The local optimal condition changes when there is a minimization problem:

$$f(x^*) \leq f(x), x^* - \varepsilon \leq x \leq x^* + \varepsilon \qquad \textbf{(III-17)}$$

Where $x^*$ is a local optimum; $\varepsilon$ is the limited length in the neighborhood about the local optimum $x^*$.

The figure (Figure (III-5)) illustrates global optimum and local optima for a one-dimensional maximization problem, with $L_1, L_2, L_3$ defined a local optimum and $G$ denoted the global optimum with the largest value of the objective function [164, 165].

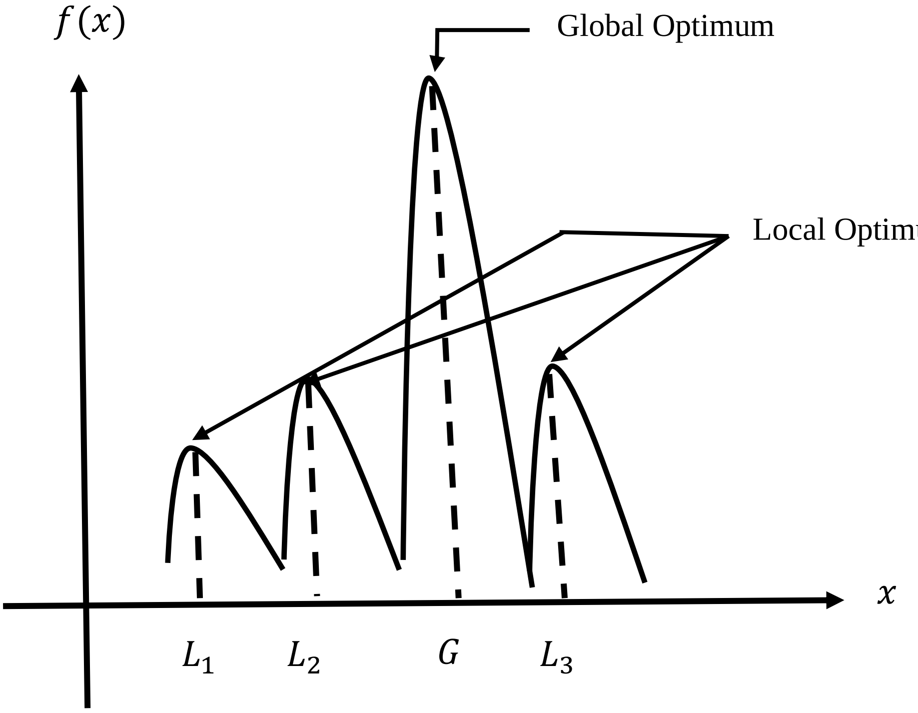



**Figure (III-5)**: Schematics of the one-dimensional maximization optimizing problem highlights global and local optimums

## III.4. Metaheuristic Algorithms

### III.4.1. Definition of Algorithms and Metaheuristic

Both the terms "meta" and "heuristic," which mean "upper level" and "the art of discovering new strategies," have their roots in ancient Greek [166]. Glover first used the term metaheuristic in 1986 [167] to describe a group of techniques that are conceptually superior to heuristics in that they serve as a blueprint for the creation of heuristics [168]. A metaheuristic is a higher-level method or heuristic that is used to locate, produce, or select a partial search algorithm, a lower-level procedure or heuristic that may be able to solve an optimization problem adequately. Metaheuristics can frequently identify good solutions with less processing work than calculus-based approaches or simple heuristics by searching over a vast set of plausible solutions [169].

A set of procedures used to solve a problem is known as an algorithm. Algorithms are sequences of iterative operations or stages that end when a predetermined convergence criterion is satisfied. Each step can be refined to a higher level of precision for simple tasks. Figure (III-6) displays a general algorithmic technique [170].

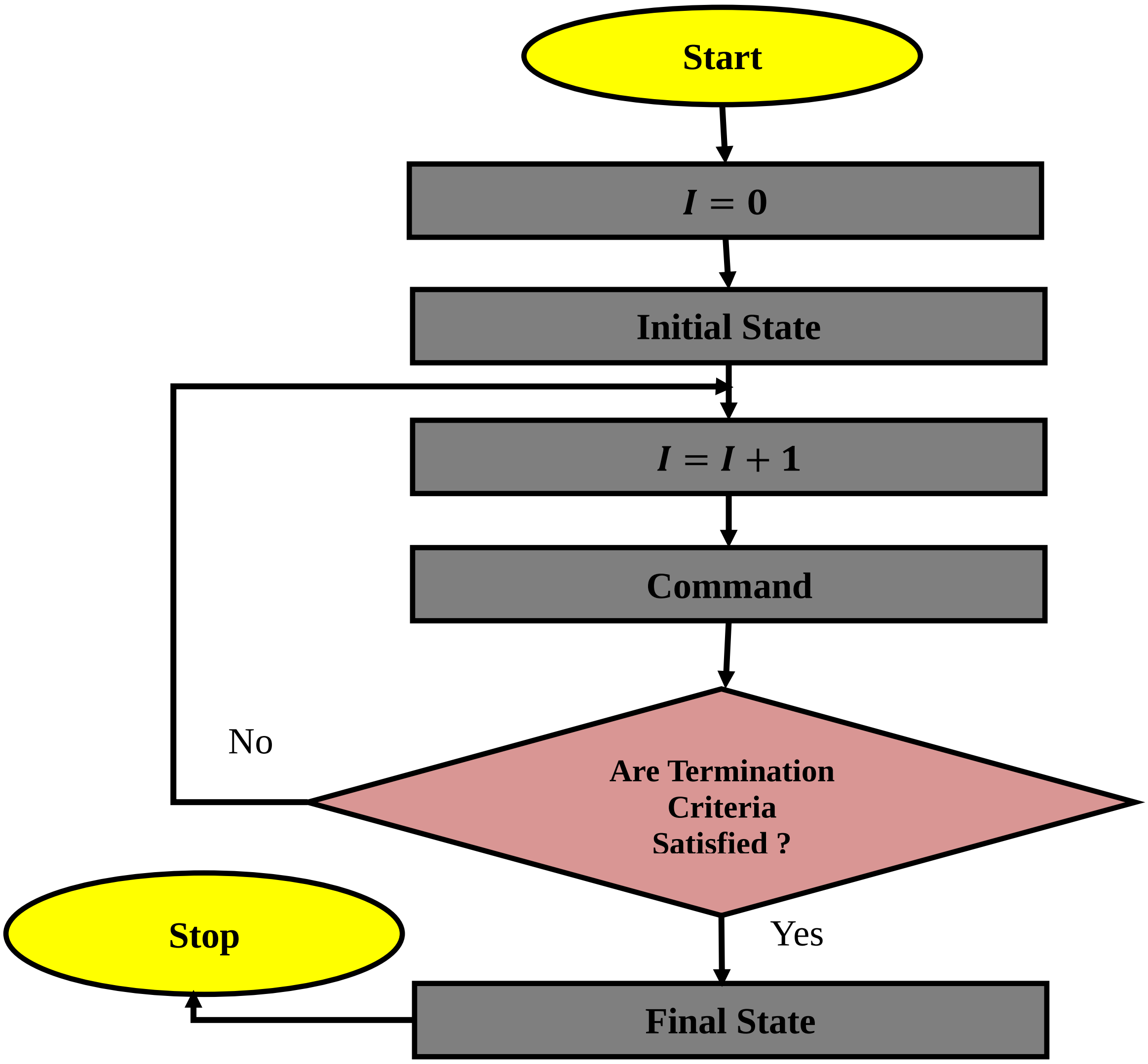



**Figure (III-6)**: General schematic of a simple algorithm; $I$ denote the counter of iteration

### III.4.2. Metaheuristics for Optimization

Some optimization techniques that are conceptually distinct from conventional mathematical programming methods have been developed in recent years. These techniques are classified as modern or unconventional optimization techniques. The majority of these techniques are based on certain biological traits and behaviors. In this section, the following metaheuristics are described for the best methods used for optimization [171]:

- Genetic Algorithms
- Particle Swarm Optimization

### III. 4.2.1. Genetic Algorithms

J. Holland created the genetic algorithm (GA) and expanded on this concept in his 1975, the name of a book is “Adaptation in Natural and Artificial Systems" [172]. John Holland also coined the phrase "genetic algorithm." Thus, a genetic algorithm is a method for simulating the biological systems-specific natural process of microscopic evolution and adaptation [173]. He developed the first genetic algorithms and explained how to apply the concepts of natural evolution to optimization issues. Holland's idea has been expanded upon, and as a result, Genetic Algorithms are now recognized as an effective technique for tackling search and optimization issues. The principles of evolution and genetics provide the foundation of genetic algorithms [174].

Each individual in the population serves as a potential solution to the optimization problem when GA acts on a population of individuals, a grouping of chromosomes. Typically, random initialization of the population is used. The population grows more and more suitable answers as the search progresses, and finally, it converges, becoming dominated by one answer. Holland also provided evidence of convergence to the situation in which chromosomes function as binary vectors. In the most general scenario, an individual's fitness influences the likelihood of its survival for the following generation [175].

Crossover, mutation, and inversion are the three (genetic) processes that GA employs to create new solutions from preexisting ones:

- ❖ Crossover: is a genetic procedure that involves joining two chromosomes, referred to as the parents, to create new chromosomes.
- ❖ Mutation: The chromosomes' properties undergo a random change known as Mutation. Usually, it operates at the gene level.
- ❖ Inversions: is a genetic process that alters how the genes are concatenated in a specific region of the chromosome, causing the new gene sequence (series) to be inverted in comparison to the original sequence [176].

The population evolves by iterative processes such as the genetic algorithm loops. Each involves the following actions [177]:

- ❖ Selection: Selecting individuals for reproduction is the initial stage. The probability of this selection is based on the relative fitness of the individuals, meaning that the fittest individuals are more likely to be selected for reproduction than the less fit ones.

- Reproduction: The chosen individuals breed offspring in the second step. The method can use both recombination and mutations to create new chromosomes.
- Evaluation: Where the new chromosomes' fitness is next assessed.
- Replacement: In the final step, members of the old population are eliminated and replaced by members of the new population.

The streamlined iterative procedure of the evolutionary algorithm, which employs a simple cycle of stages, is shown in Figure (III-7) [178]:

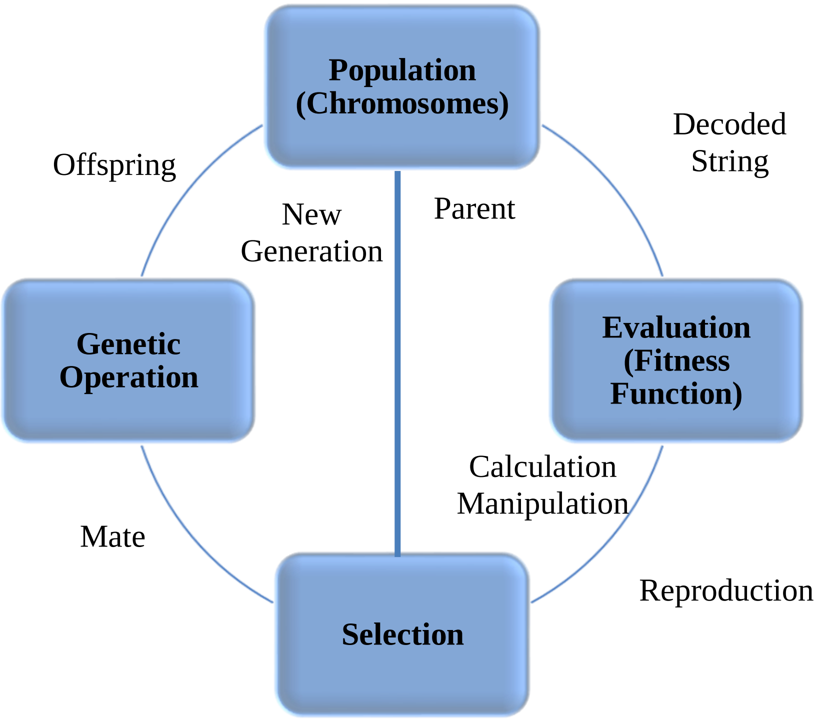



**Figure (III-7)**: General Operation of a GA

### III. 4.2.2. Particle Swarm Optimization Algorithm

One of the bio-inspired algorithms, particle swarm optimization (PSO), is straightforward in its search for the best solution in the problem area. It differs from other optimization techniques in that it does not depend on the gradient or any differential form of the objective and simply requires the objective function. There are also not many hyperparameters [179].

The social behavior of groups, such as that of flocking birds, schooling fish (see Figure (III-8)), or insect colonies (see Figure (III-9)), served as the inspiration for the evolutionary computation method known as particle swarm optimization (PSO). It is well known that a group can successfully accomplish an objective by using the common information of every element. Eberhart and Kennedy [180] first developed the PSO algorithm in 1995 as a substitute for population-based search techniques (such as genetic algorithms) to address optimization issues.

**Figure (III-8)**: A Flock of birds and A School of Fish

**Figure (III-9)**: The Path of the Ants

Each particle (a fish or a bird) in this procedure, which refers to the population's components, is a potential candidate for the answer. Each particle is viewed as a moving point with a specific velocity in the N-dimensional search space. Each particle continuously modifies its speed based on its own and its partners' experiences in order to travel towards a better solution area. Three factors can affect a particle's movement (Figure (III-10)) [181]:

- ❖ Physically, a particle tends to move in the direction that it is currently moving;
- ❖ cognitively, a particle tends to move toward the best location that it has already passed;
- ❖ socially, a particle tends to rely on the experience of its congeners and, as a result, to move toward the best location that its neighbors have already arrived at.

Each particle state in PSO exhibits a position and velocity, which are initialized by a random process population formation. Noting that there are three characteristics that each particle has [182, 183]:

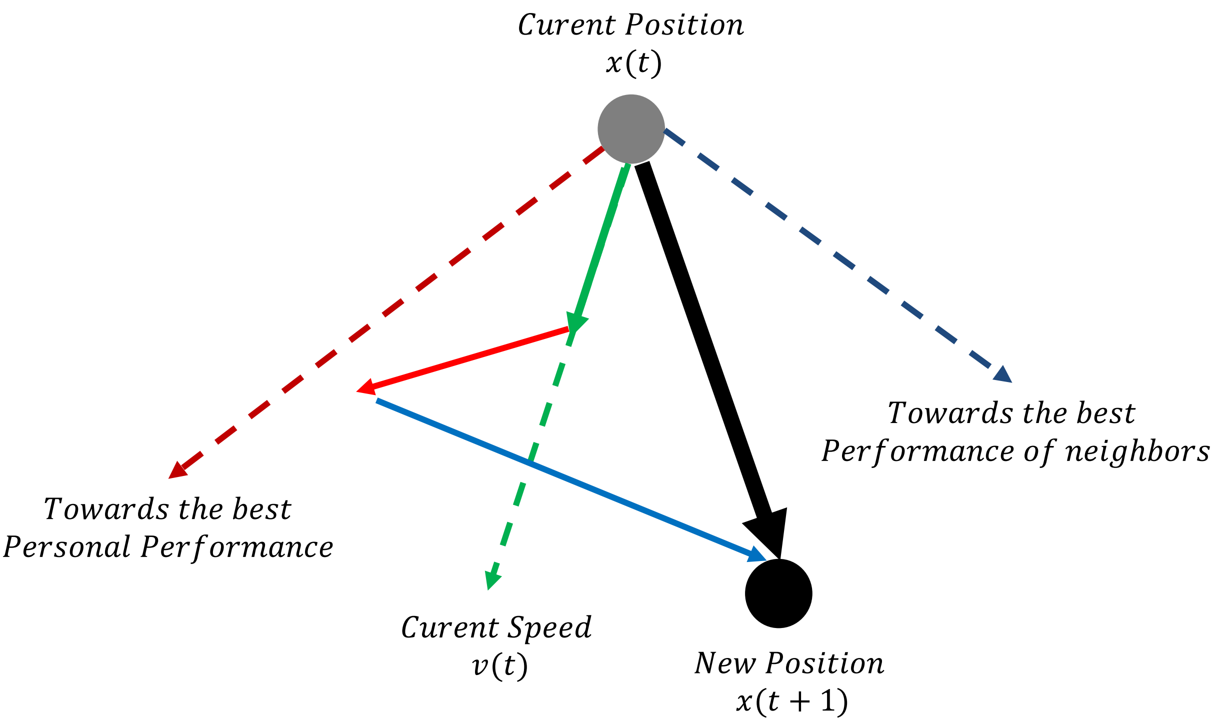



**Figure (III-10)**: The Principe of displacement of a particle

$x_k^i$: $i^{th}$ Particle Vector Position at time $k$; $v_k^i$: $i^{th}$ Particle Velocity at time $k$, which represents the search direction and used to update the position vector; $f(x_k^i)$ is a fitness or objective, determines the best position of each particle over time.

The following equations govern how the particle velocities and position are updated mathematically [184]:

$$v_{k+1}^i = wv_k^i + C_1r_1\left(Pbest_{ij} - x_k^i\right) + C_2r_2\left(Gbest - x_k^i\right) \quad \textbf{(III-18)}$$

With

$$w = w_{max}\left(\frac{iter}{maxiter}\right)\left(w_{max} - w_{min}\right) \quad \textbf{(III-19)}$$

Where $i = 1,2, \dots, n$ is the number of particles in the swarm, $r_1$ and $r_2$ are random numbers between (0,1). $C_1$ and $C_2$ are correction factors ($C_1$ pulls each particle toward a local best position, it is called cognitive parameter, $C_2$ is a social parameter, it pulls the particle toward a global best position, $w_{max}$ is a final weight and $w_{min}$ is an initial weight, *maxiter* is the maximum iteration number and *iteration* is the current iteration number [185].

The particle position is the update by velocity (see equation (III-18)) as:

$$x_{k+1}^i = v_{k+1}^i + x_k^i \quad \textbf{(III-20)}$$

According to equations (III-18) and (III-20), the PSO principle involves controlling each particle's velocity and location toward its pi and p g locations at each time step until a maximum change in the fitness function is smaller than a predetermined tolerance, which gives us the following stopping criteria (III-21) [186]:

$$|f(Gbest_{k+1}) - f(Gbest_k)| \leq \varepsilon \tag{III-21}$$

The figure (Figure (III-11)) shows the PSO algorithm flew chart:

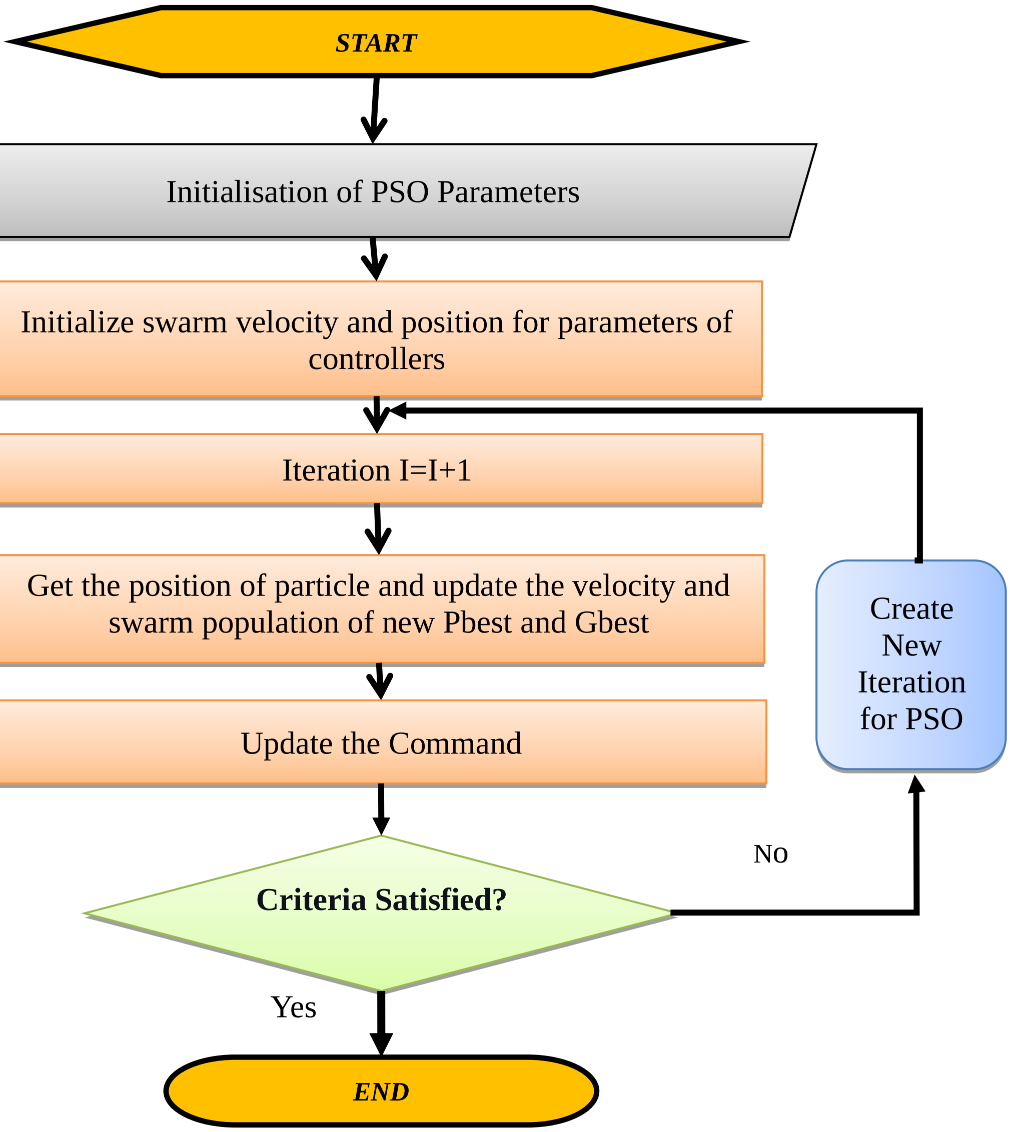



**Figure (III-11)**: PSO Algorithm flew chert

### III. 5. Conclusion

The goal of this chapter is to give an overview of the controller techniques used in this study, metaheuristic algorithms, and the theory behind the later-used optimization techniques, such as genetic algorithms, and particle swarm optimization.

# Chapter IV

## Control Active Suspension System of Quarter Car Model

## IV.1. Introduction

Active suspension systems are the new frontier in automotive technology, offering drivers unprecedented control over their vehicle's handling and performance. By using a combination of sensors, actuators, and software algorithms to adjust the stiffness of each wheel independently, active suspensions can provide superior stability and comfort on rough roads while also improving cornering ability. This makes them an attractive option for both commuters who want a smoother ride as well as performance enthusiasts looking to maximize their car’s potential [187]. The most common type of active suspension system is called “electronically controlled dampers,” or ECDs for short. These work by constantly monitoring data from various sources, such as speed sensors, accelerometers, and gyroscopes in order to detect when changes need to be made in order to maintain optimal levels of grip on different surfaces or during specific maneuvers like cornering or braking hard into turns. When this happens, they will automatically adjust the dampening force applied at each wheel accordingly, which helps keep all four tires firmly planted on the ground regardless of what driving conditions you may encounter along your journey [188].

In addition, active suspension systems offer many advantages over traditional passive systems, including improved road-holding capabilities, greater agility around corners, better steering response time due to increased feedback from wheels due to more accurate sensing data being sent back directly from the wheels themselves instead of relying solely upon driver input via the steering column. Finally, these types of suspensions can also reduce body roll significantly, giving cars that sporty feel without compromising the overall safety level, so it's a win-win situation for everyone involved [189]. The other major benefit associated with an active suspension system is its potential for improving fuel economy through more efficient energy management techniques that minimize drag forces acting against a moving car’s momentum when faced with bumps in roadways which would otherwise cause air resistance losses if not properly managed via shock absorbers tuned according to strategic parameters set forth within each individual setup configuration. As such, this technology has become increasingly attractive among automakers looking towards making their vehicles lighter yet still able to maintain optimal performance characteristics across varying environmental conditions [190].

Numerous studies addressing the modern control of suspension systems can be found in the literature. Yu Wei, and Yang Jing, and all Sam improved the management of a quarter-car model by applying the Linear Quadratic Regulator (LQR) based a genetic algorithm in reference [191]. The weakness of the suggested strategy is that it was unable to guarantee satisfactory results on noisy roads. A useful terminal sliding mode control for the quarter vehicle model was employed by G. Wang and all in [192]. Many researchers studied this model and applied a different controller for improve car performance.

This chapter aims to focus on active suspension system for quarter car model with two degrees of freedom. We applied three types of controllers the PID, FOPID, and LQR control. For the PID, and FOPID controllers we adjust them using intelligent techniques, and we chose Genetic Algorithms (GA) and Particle Swarm Optimization (PSO).

## IV. 2. Mathematical Model of Quarter Car Dynamic

The quarter car system has two degrees of freedom and consists of two bodies: the sprung mass of the car and the unsprung mass of the suspension, wheels, and tires. Even though it is a simplified model, the systems' stiffness and damping are used to avoid considering everything rigid. As a result, the stiffness and damping of the tire and wheel between the road and the suspension, as well as the stiffness and damping of the spring and damper between the suspension and the vehicle, are taken into account [193].

A dynamic model can be used to depict the active car suspension system, as shown in Figure (IV-1) [194].

A damper is positioned in between the sprung and unsprung masses in the model. Instead of using a passive suspension system, active suspensions require the addition of a force element $F$ (Force Actuator) (see Figure (IV-1) [195]) whose job it is to apply the desired force between the vehicle and its wheel. This essay's goal is to offer a suggestion for creating a controller for this force [196].

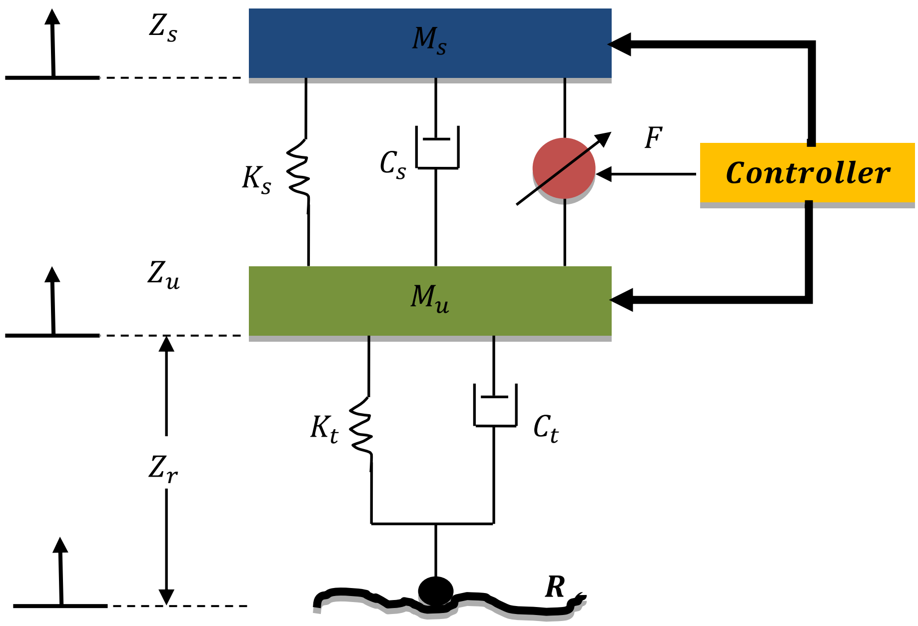



**Figure (IV-1)**: The Dynamic Model of Active Suspension for Quarter Car Model

The corresponding parameters and variable of figure (Figure (IV-1)) are listed in Table (IV-I):

**Table (IV-1)** : Description of quarter car properties

| Parameters | Description |
|---|---|
| $M_s$ | Body Mass (Sprung Mass) |
| $M_u$ | Suspension Mass (Unspring Mass) |
| $K_s$ | Spring of suspension system |
| $K_t$ | Spring of wheel and tire |
| $C_s$ | Damping of suspension system |
| $C_t$ | Damping of wheel and tire |
| $F$ | Actuator Force |
| $Z_s$ | Body Displacement |
| $Z_u$ | Wheel Displacement |
| $Z_r$ | Vertical position of the road profile |
| $R$ | Road Profil Input |

The following are the dynamic equations for the quarter-car model:

$$\begin{cases} M_s\ddot{Z}_s = -K_s(Z_s - Z_u) - C_s(\dot{Z}_s - \dot{Z}_u) + F \\ M_u\ddot{Z}_u = K_s(Z_s - Z_u) + C_s(\dot{Z}_s - \dot{Z}_u) - K_t(Z_u - Z_r) - C_t(\dot{Z}_u - \dot{Z}_r) - F \end{cases} \quad \textbf{(IV-1)}$$

Dynamic equation (IV-1) of the quarter-car model's state space representation is described as follows [197]:

$$\begin{cases} \dot{Z} = AZ + BF + GZ_r \\ y = CZ + DF + EZ_r \end{cases} \quad \textbf{(IV-2)}$$

which results in the state space form that follows:

$$\begin{cases} Z_1 = \dot{Z}_s \\ Z_2 = Z_s \\ Z_3 = \dot{Z}_u \\ Z_4 = Z_u \end{cases} \Rightarrow \begin{cases} \dot{Z}_1 = -\frac{C_s}{M_s}Z_1 - \frac{K_s}{M_s}Z_2 + \frac{C_s}{M_s}Z_3 + \frac{K_s}{M_s}Z_4 + \frac{1}{M_s}F \\ \dot{Z}_2 = Z_1 \\ \dot{Z}_3 = \frac{C_s}{M_u}Z_1 + \frac{K_s}{M_u}Z_2 - \frac{(C_s+C_t)}{M_u}Z_3 - \frac{(K_s+K_t)}{M_u}Z_4 + \frac{K_t}{M_u}Z_r + \frac{C_t}{M_u}\dot{Z}_r - \frac{1}{M_u}F \\ \dot{Z}_4 = Z_3 \end{cases} \quad \textbf{(IV-3)}$$

The state equation for variable $\dot{Z}_3$ changes to: if we assume that the vertical velocity caused by the road profile $\dot{Z}_r$ is very tiny (neglected).

$$\dot{Z}_3 = \frac{C_s}{M_u}Z_1 + \frac{K_s}{M_u}Z_2 - \frac{(C_s+C_t)}{M_u}Z_3 - \frac{(K_s+K_t)}{M_u}Z_4 + \frac{K_t}{M_u}Z_r - \frac{1}{M_u}F \quad \textbf{(IV-4)}$$

As a result, the equation for the state and output is:

$$\underbrace{\begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \\ \dot{Z}_3 \\ \dot{Z}_4 \end{bmatrix}}_{\dot{Z}} = \underbrace{\begin{bmatrix} -\frac{C_s}{M_s} & -\frac{K_s}{M_s} & \frac{C_s}{M_s} & \frac{K_s}{M_s} \\ 1 & 0 & 0 & 0 \\ \frac{C_s}{M_u} & \frac{K_s}{M_u} & \frac{(C_s+C_t)}{M_u} & \frac{(K_s+K_t)}{M_u} \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{M_s} \\ 0 \\ -\frac{1}{M_u} \\ 0 \end{bmatrix}}_{B} F + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{K_t}{M_u} \\ 0 \end{bmatrix}}_{G} Z_r \quad \textbf{(IV-5)}$$

$$y = \begin{bmatrix} Z_s \\ \dot{Z}_s \\ \ddot{Z}_s \\ Z_s - Z_u \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -\frac{C_s}{M_s} & -\frac{K_s}{M_s} & \frac{C_s}{M_s} & \frac{K_s}{M_s} \\ 0 & 1 & 0 & -1 \end{bmatrix}}_{C} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{1}{M_s} \\ 0 \end{bmatrix}}_{D} F + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{E} Z_r \quad \textbf{(IV-6)}$$

The vehicle parameters values using in this study show in the Table (IV-2) [198]:

**Table (IV-2)**: The vehicle parameters values

| **Parameters** | ***Value*** | ***Unit*** |
|---|---|---|
| $M_s$ | 972.2 | Kg |
| $M_u$ | 113.6 | Kg |
| $K_s$ | 42,719.6 | N/m |
| $K_t$ | 101,115 | N/m |
| $C_s$ | 1,095 | N.s/m |
| $\boldsymbol{C_t}$ | 14.6 | N.s/m |

## IV.3. The Controllers Design

The goal of the control design in this study is to use fractional order PID (FOPID) and PID controllers employing metaheuristic techniques to provide the car with the necessary dynamic behavior under road fluctuations, and comparison with Linear Quadratic Regulator LQR control

### IV.3.1. Adjusted PID and FOPID Using GA and PSO Optimizations

Genetic algorithms are based on archiving data and converting it to a binary system, and this is what happens to the parameters of the proportional integral derivative (PID) controller and the fractional order PID (FOPID) controller (see Figure (IV-2)) [199].

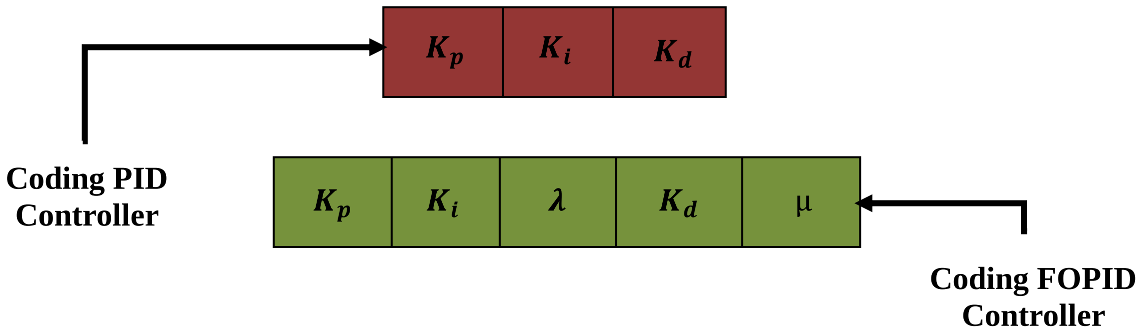



**Figure (IV-2)**: PID and FOPID chromosomes after coding

The choice of the objective function (fitness) for the assessment of each chromosome's appropriateness is the most crucial assignment in GA. We employed the following objective function in this study [200].

$$J = \int_0^T t|e(t)|dt \quad \textbf{(IV-7)}$$

Where $e(t)$ is the error to be used for minimize the objective function $J$. So, for that we use the fitness value to guide simulation in the best solution of the problems as show in the equation (IV-8) [201]:

$$Fitness\,Value = \frac{1}{J} \quad \textbf{(IV-8)}$$

The PSO method works by creating an initial n-particle swarm that moves randomly in an i-dimensional search space. It utilizes optimization particles that show a potential resolution to a problem and can produce a high-quality solution in less time. All particles that are seeking a specific velocity can be altered [202].

To optimize the parameters of PID and FOPID, we employed the fitness function specified in equations (IV-7) and (IV-8) for Particle Swarm Optimization (PSO), which, like the GA approach, needs an objective function to improve. The figure (Figure (IV-3)) shows the flowcharts of Gas and PSO for finding the optimal parameters of PID and FOPID controllers:

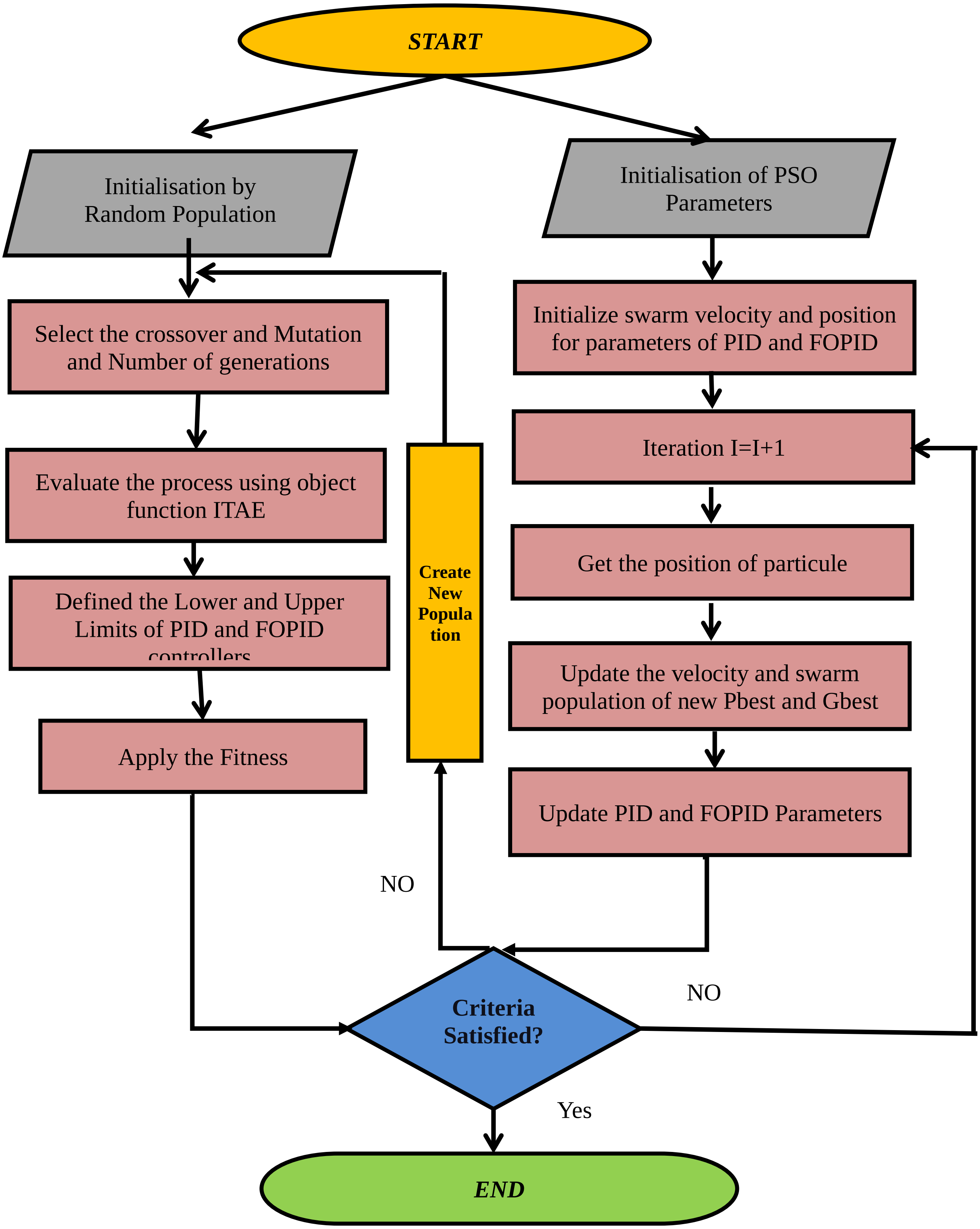



**Figure (IV-4)**: The flew charts of GAs and PSO for optimal parameters of PID and FOPID controllers

Table (IV-3) and Table (IV-4) illustrate the parameters of controllers using in the study (PID and FOPID) optimized by GA and PSO respectively:

**Table (IV-3)**: Parameters Tuned the suggested controllers by Genetic Algorithms (GAs)

| GA Features | Controller | |
|---|---|---|
| | ***PID*** | ***FOPID*** |
| ***Fitness Function*** | *ITAE* | *ITAE* |
| ***Population size*** | *50* | *50* |
| ***Maximum number of Generation*** | *100* | *100* |
| ***Selection*** | *Roullette* | *Roullette* |
| ***Crossover*** | *Two Point* | *Two Point* |
| ***Mutation*** | *Uniform with Rat=0.01* | *Uniform with Rat=0.01* |
| ***Interval for Controllers*** | *0<kp<10*<br>*0<ki<0.085*<br>*0<kd<1.3×10^4* | *0<kp<10*<br>*0<ki<0.085*<br>*0<kd<1.3×10^4,*<br>$0<\lambda<0.250,\ 1<\mu<1.1$ |

**Table (IV-4)**: Parameters Tuned the suggested controllers by Particle Swarm Optimization (PSO)

| PSO Features | Controller | |
|---|---|---|
| | ***PID*** | ***FOPID*** |
| ***Object Function*** | *ITAE* | *ITAE* |
| ***Function Tolerance*** | *1×10^-6* | *1×10^-6* |
| ***Inertial Range*** | *[0.1 1.1]* | *[0.1 1.1]* |
| ***Max Iteration*** | *6000* | *10000* |
| ***Swarm Size*** | *30* | *50* |
| ***Interval for Controllers*** | $0< k_p \leq 5$<br>$0< k_i <0.085$<br>$0 < k_d <1.3\times10^4$ | $0< k_p \leq 0.10$<br>$0< k_i <0.014$<br>$0< k_d <1.3\times10^4,\ 0< \lambda <0.140$<br>$1< \mu <1.1$ |

The PID and FOPID's optimal parameters, which were found using GA and PSO optimization techniques, are shown in Table (IV-5).

**Table (IV-5)**: Optimized Parameters of PID and FOPID Controller

| Tuned Methods | | $k_p$ | $k_i$ | $k_d$ | $\lambda$ | $\mu$ |
|---|---|---|---|---|---|---|
| **Metaheuristic Methods** | *GA_PID* | *6.314* | *0.087* | *10995.09* | – | – |
| | *GA_FOPID* | *2.09* | *0.011* | *11848.659* | *0.037* | *1* |
| | *Pso_PID* | *5* | *0.007* | *11778* | – | – |
| | *Pso_FOPID* | *0.061674* | *0.008789* | *1.2*10^4* | *0.06284* | *1* |

### IV.3.2. Linear Quadratic Regulator (LQR Control)

The linear quadratic regulator (LQR) is briefly defined in Chapter III, along with the creation of an LQR-based compensator. Also described is the application of integral feedback to get rid of steady-state error (see Chapter III equation (III-10) to (III-13)).

The matrix $Q$ and $R$ Taking by:

$$Q = \begin{bmatrix} 10^{-2} & 0 & 0 & 0 \\ 0 & 10^{-5} & 0 & 0 \\ 0 & 0 & 10^{-3} & 0 \\ 0 & 0 & 0 & 10^{-4} \end{bmatrix}, R = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.10 \end{bmatrix}$$

The values of obtained feedback gain matrix of LQR given by:

$$K = \begin{bmatrix} 0.0010 & -0.0002 & -0.0001 & 0.0010 \\ 0.2596 & -0.3767 & 0.0968 & 0.3772 \end{bmatrix}$$

The following figure (Figure (IV-4)) illustrate the active suspension for quarter car model designing by controllers suggested (PID and FOPID), and figure (Figure (IV-5) show the model of an active suspension system controller by LQR control:

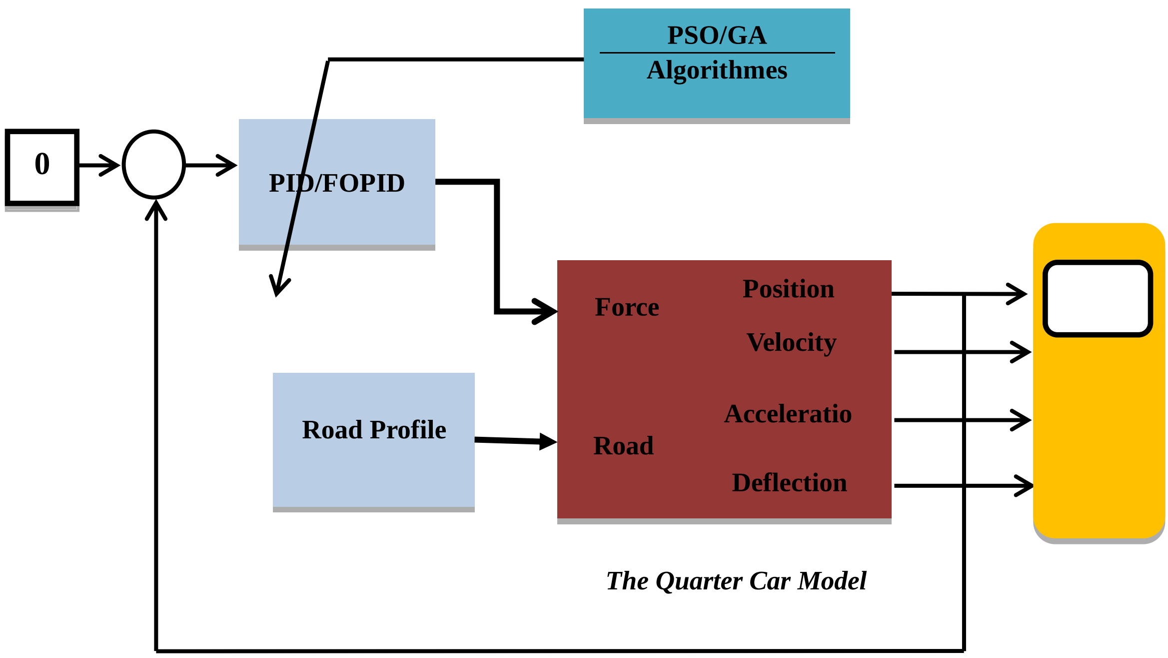



**Figure (IV-4)**: Block Diagram explain the Optimization Process by Metaheuristic Algorithms

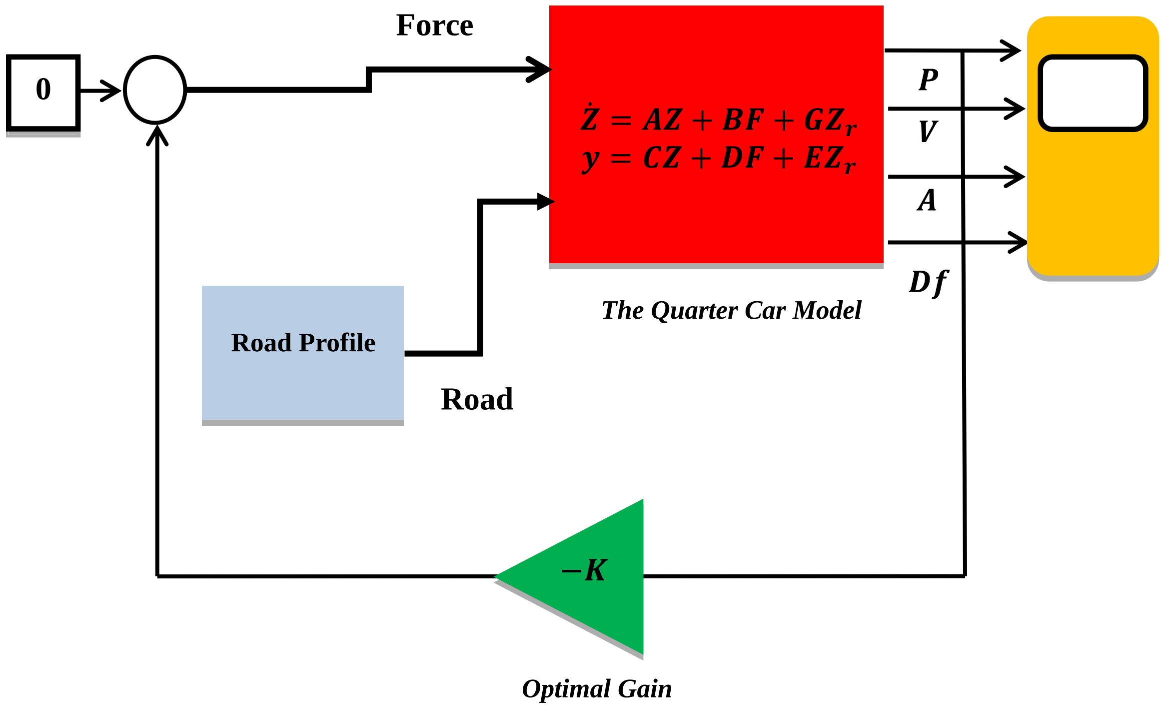



**Figure (IV-5)**: The optimal control for control an active suspension system model by LQR

### IV. 4. Simulation Results and Discussions

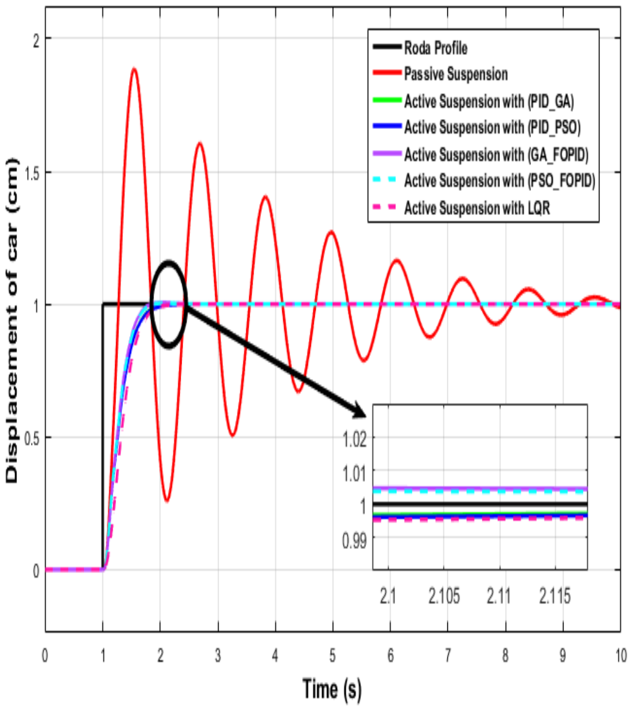
In summary, the results that we will discuss prove that the active suspensions offer numerous benefits including improved stability under harsh driving scenarios and greater efficiency gains achieved through effective energy management strategies implemented into each unique design layout. Consequently, it's no surprise why many modern cars now come equipped with advanced versions of this technology allowing drivers everywhere to access top-notch automotive experiences like never before [203].

Five distinct types of road disturbances are used in the simulation, and the results are based on a mathematical model of a quarter automobile. A step input signal of amplitude 1(cm), a sinusoidal input signal to represent a bumpy road of amplitude 1(cm), a variable-step input signal to represent an excavated road of amplitude 1(cm), and a noisy road for a road made of high-intensity vibration of 0.2 noise power and 0.1 sample time are the inputs for testing the performance of the closed-loop suspension system. The ramp road with an amplitude of 0.1(cm) is the final road in this study. The descriptions of a different road types are provided in the table below (Table (IV-6)). We display in these results the vehicle's speed (Velocity), body acceleration, quarter-car body displacement, and suspension deflection.

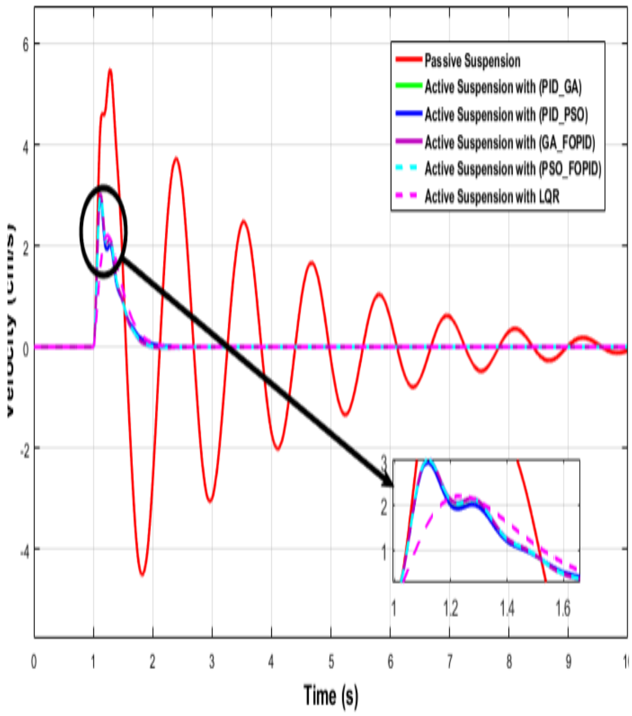
**Table (IV-6)**: Different roads profiles types for testing the active suspension system of quarter car model

| **Roads Profiles** | **Equation and Values** | |
|---|---|---|
| | ***Equation*** | ***Values*** |
| ***Step Road Profile*** | $R_p(t) = A \times \begin{cases} 1 \; for \; t \geq 0 \\ 0 \; for \; t < 0 \end{cases}$ | $A = 1cm$ |
| ***Sinusoidal Road Profile*** | $R_p(t) = A \times \sin(wt + \Phi)$ | $A = 1cm$ |
| ***Variable Step Road Profile*** | $R_p(t) = A \times [R_{p0}(t) - 2R_{p0}(t-3) + 2R_{p0}(t-6) - 2R_{p0}(t-9)]$ | $A = 1cm$ |
| ***Random Road Profile*** | $R_p(t) = Sigma + randn(N, 1) + M_u$ | $Sigma = 2$<br>$N = 0.2$<br>$M_u = 0.1$ |
| ***Ramp Road Profile*** | $R_p(t) = A \times \begin{cases} 1 \; for \; t \geq 0 \\ 0 \; for \; t < 0 \end{cases}$ | $A = 1cm$ |

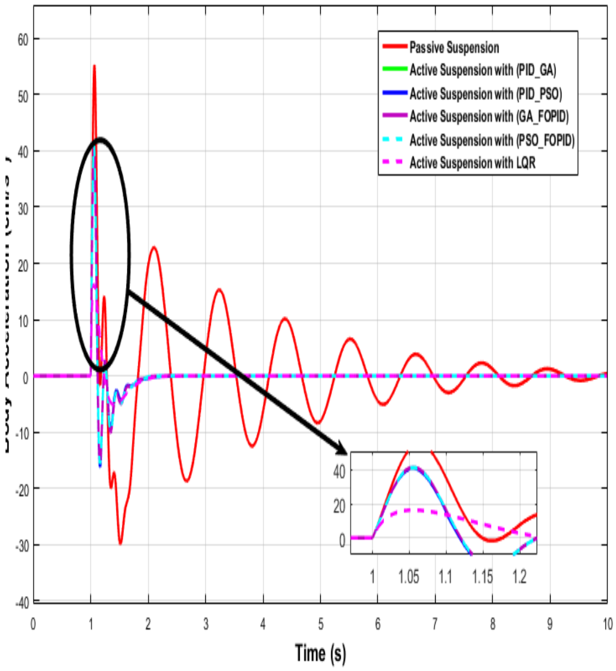
### IV.4.1. Step Road Profile (A Road with a hole)



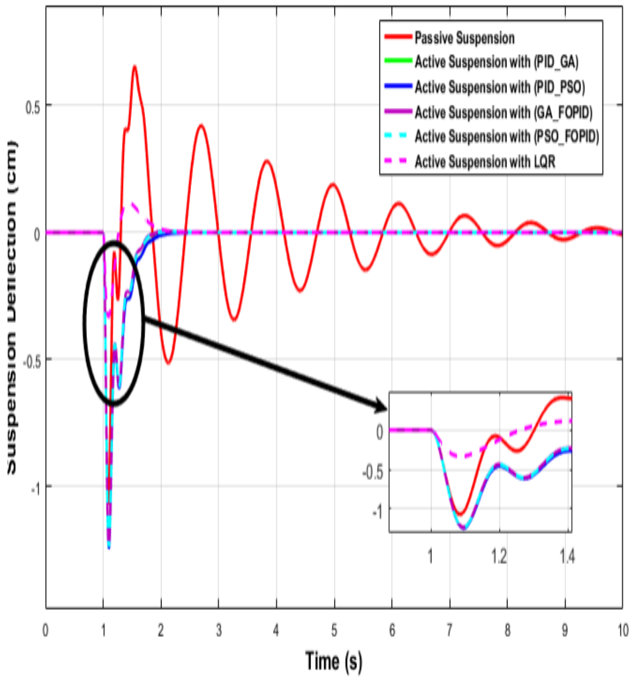

$(a)$




$(b)$




$(c)$




$(d)$

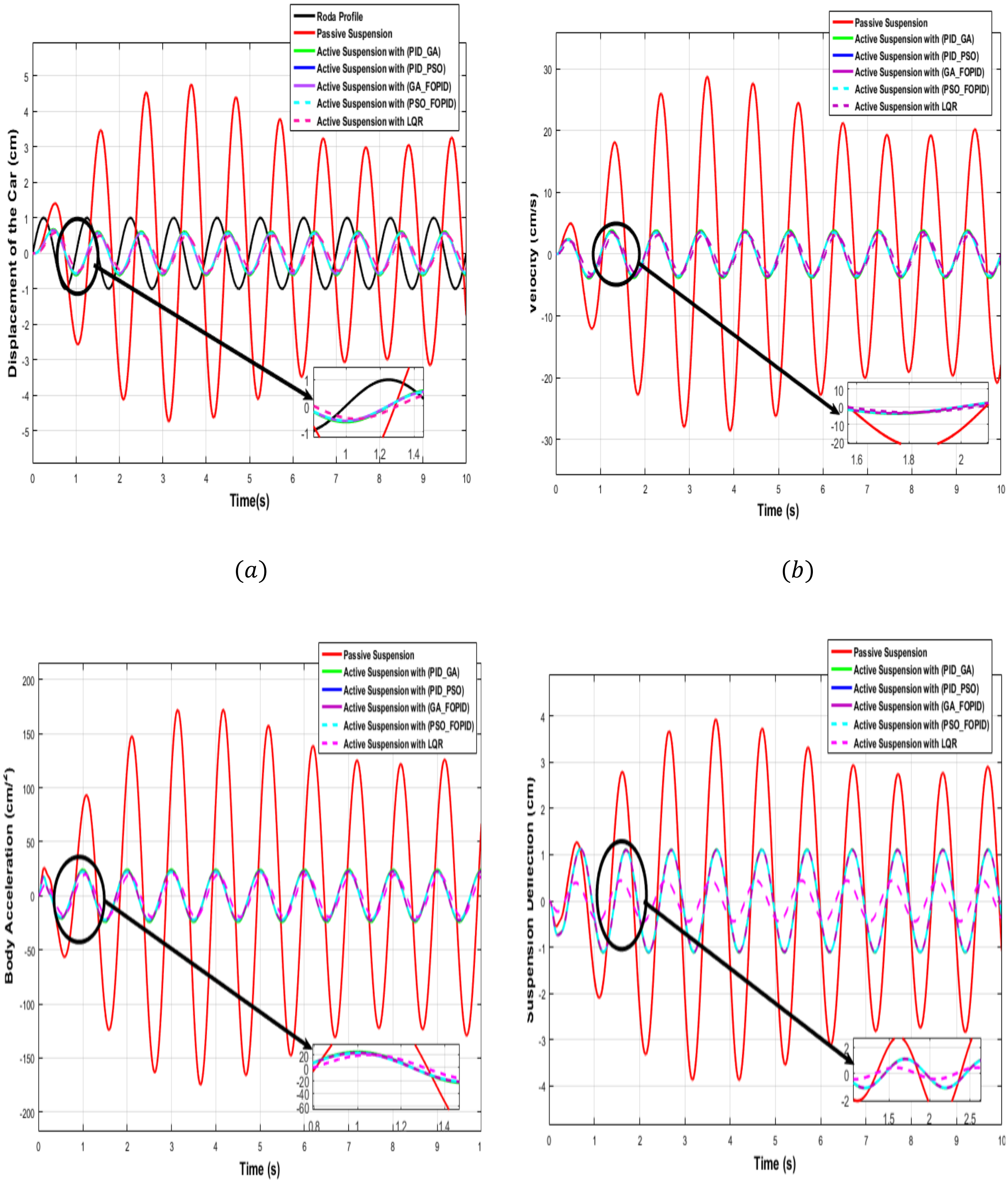
**Figure (IV-6)**: Step Responses of optimal PID and FOPID Comparison with LQR control:$(a)$ Body Displacement, $(b)$ Velocity, $(c)$ Body Acceleration, $(d)$ Suspension Deflection

Figure (IV-6) shows the body Displacement, body deflection, body velocity, and body acceleration for step responses for optimal PID and FOPID controllers, and LQR control. Results compare passive suspension, intelligent controllers, and optimal control.

Figures (b), (c), and (d) demonstrate the stability of the active suspension system with LQR better, where the road vibration minimizes. Figure (a) demonstrates the effectiveness of the active suspension Controller using LQR, which is exhibited in the zoom part. The behavior of the suspension in various stages (active and passive) is depicted in Figure (IV-6) (a) (Part Zoom), where we can observe that the RMS error for a step road input span from 68.49% to 75.77%.

**IV.4.2. Sinusoidal Road Profile (A Bumpy Road)**

A bumpy road can be defined as any uneven terrain that causes your car's suspension system to work harder than usual, resulting in an uncomfortable ride for passengers inside the vehicle. This could include roads with large potholes, gravel surfaces, unpaved roads, and cobblestone streets.




$(a)$ $(b)$

$(c)$ $(d)$

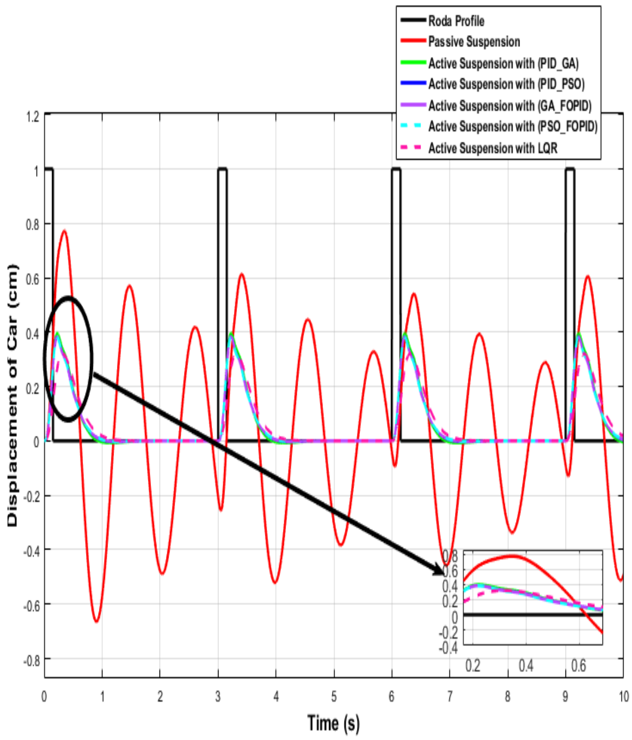
**Figure (IV- 7)**: Bumpy Road Responses of optimal PID and FOPID Comparison with LQR control:$(a)$ Body Displacement, $(b)$ Velocity, $(c)$ Body Acceleration, $(d)$ Suspension Deflection

The results compare passive and active suspension utilizing the optimal control (LQR) and intelligent controllers with different situations of the vehicle (Displacement, velocity, acceleration, and deflection). The bumpy road profile (Sinusoidal input) is shown in (Figure (IV-7)).

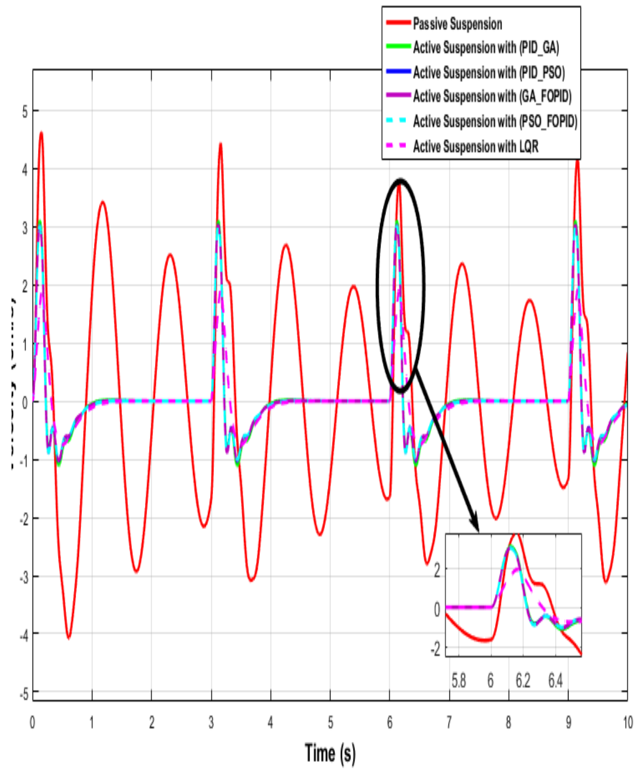
In comparison to other approaches, figures (a), (b), (c), and (d) demonstrate superior stability of an active suspension with a Linear Quadratic Regulator (LQR Control) and minimize the overshot of the road. RMS error values for all types of suspension systems examined in this section range from 83.77% to 86.67%. The outcomes show how well the suggested controller works.

### IV.4.2. Variable -step Road Profile and Random Road Profile (An Excavated Road and Noisy Road)

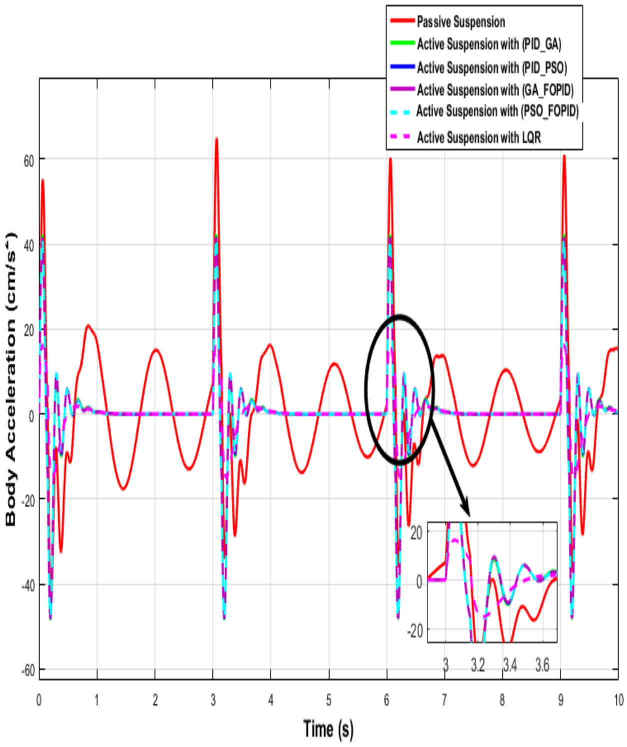
Excavated roads are a type of road that has been dug out from the ground and reinforced with stone, asphalt, or other materials. These types of roads can be used for transportation purposes such as highways and city streets. The excavation process is often done to create a more efficient route for vehicles to travel on or provide access to an area that was previously inaccessible due to terrain features like hillsides or ravines.



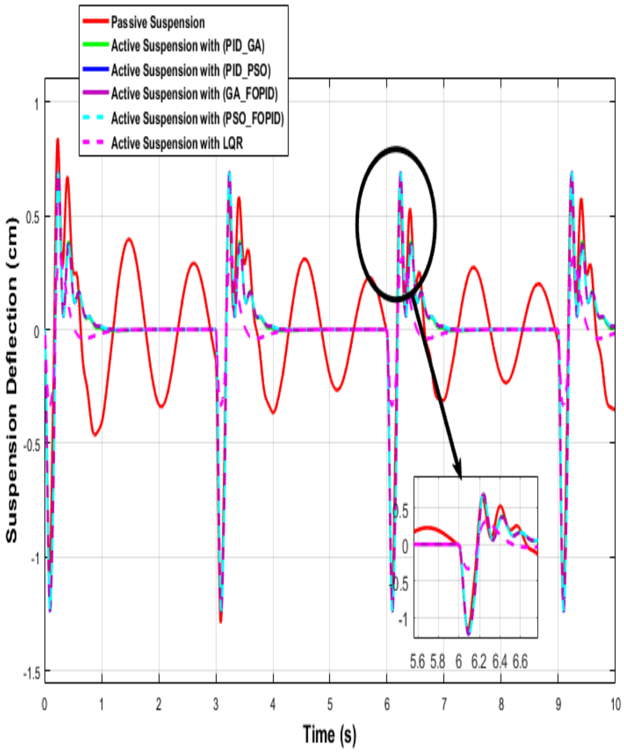

(a)




(b)




$(c)$




$(d)$

**Figure (IV-8)**: Excavated Road Responses of optimal PID and FOPID Comparison with LQR control:$(a)$ Body Displacement, $(b)$ Velocity, $(c)$ Body Acceleration, $(d)$ Suspension Deflection

The excavated road (Variable-Step) was used as an input for the car suspension (Figure (IV-8)), and a passive and active suspension with optimal and intelligent controllers were contrasted. The various scenarios demonstrate the effectiveness of LQR control, in reducing vibration of the dug road. The most recent route picked was noisy; this style of the road had intense vibrations. Figure (IV-8) shows the same comparison as Figures (IV-9), where show the noisy road or another word and we can denote vibration road it illustrates that the active suspension has a high ability to reduce the impact of the car's vibrations, particularly the active suspension controlled by Linear Quadratic Regulator, which produced better results in the stability of the car on the road and reduces the vibrations.

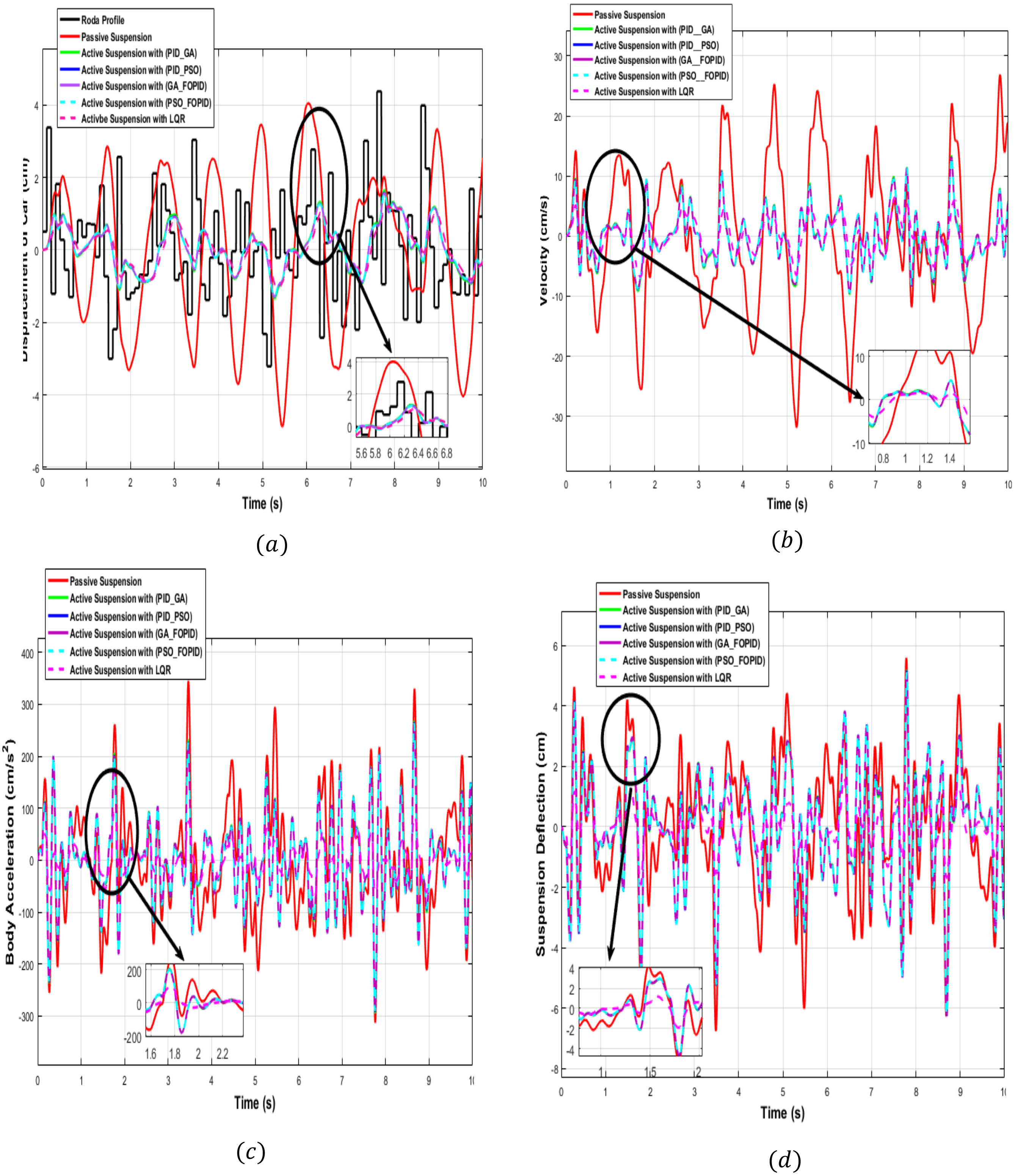
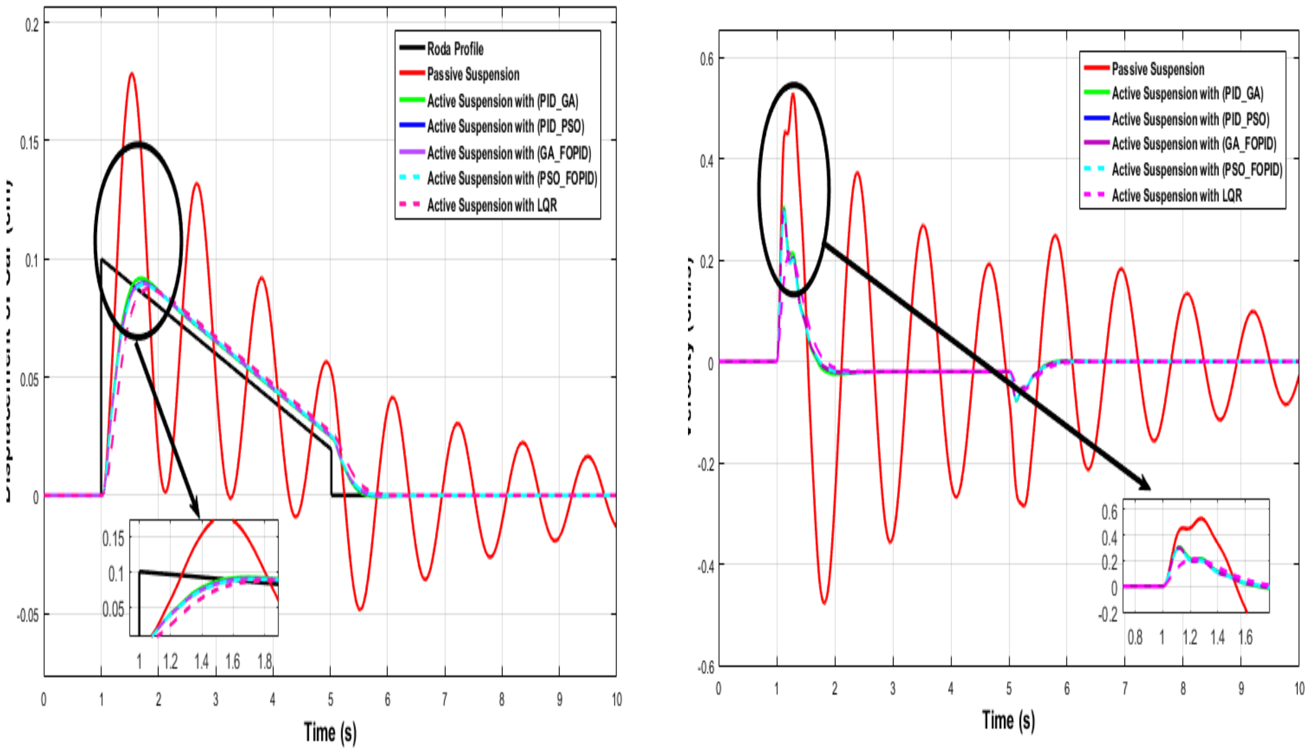



**Figure (IV-9):** Noisy Road Responses of optimal PID and FOPID Comparison with LQR control:$(a)$ Body Displacement, $(b)$ Velocity, $(c)$ Body Acceleration, $(d)$ Suspension Deflection.

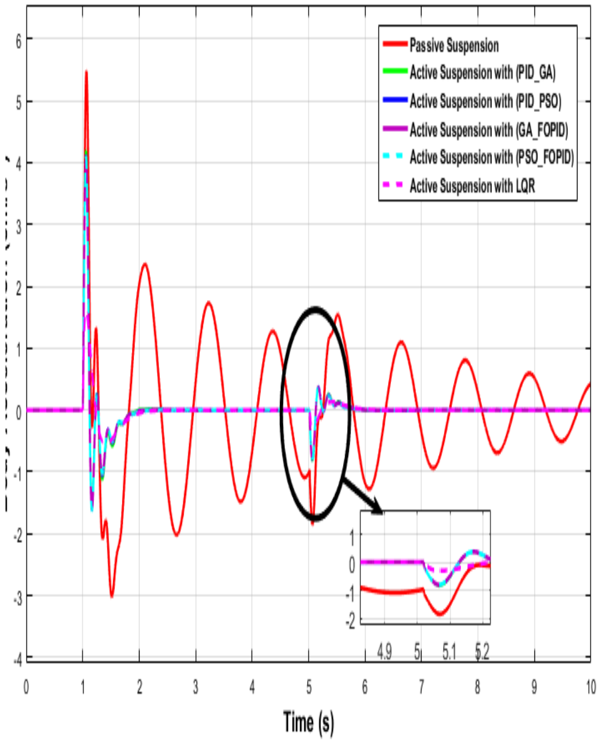
Figures (IV-8) and Figure (IV-9) depict the suspension reaction for dug and noisy roadways, respectively. RMS error rates range from 71.27% to 75.05% for a noisy road and from 62.41% to 66.46% for an excavated road. The simulation results, which are displayed in Figures (IV-6) through Figure (IV-9), indicate how well the active suspension controller with the suggested optimal control modified (LQR control) in all types of road profiles, particularly those with significant vibration.

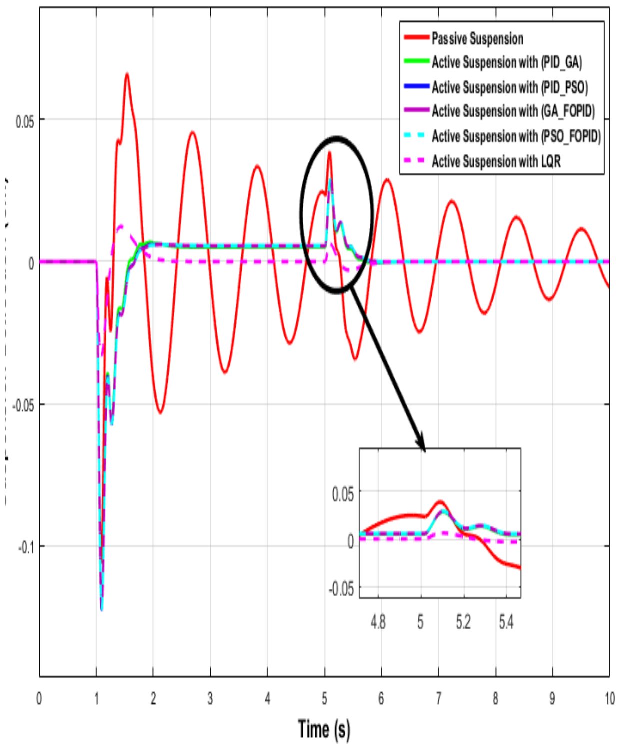
### IV.4.2. Ramp Road Profile (A Sloping Road)

A sloping (Ramp Road) road is defined as any roadway that has an incline or declines greater than 3%. This means that the surface of the roadway rises more quickly than it would on flat ground and may even require extra effort from your engine to reach higher speeds when going uphill. On downhill slopes, cars must also use brakes to slow down in order to prevent accidents due to excessive speed gain caused by gravity alone. Figure (IV-10) shows an example of a Sloping Road with the response of an active suspension for a quarter-car model.




$(a)$ $(b)$




$(c)$ $(d)$

**Figure (IV-10)**: Sloping Road Responses of optimal PID and FOPID Comparison with LQR control:$(a)$ Body Displacement, $(b)$ Velocity, $(c)$ Body Acceleration, $(d)$ Suspension Deflection.

The results show in Figure (IV-10) compare passive and active suspension utilizing the optimal control (LQR) and intelligent controllers with different situations of the vehicle (Displacement, velocity, acceleration, and deflection). The results testing by the sloping road profile (Ramp Road input).

In comparison to other approaches, figures (a), (b), (c), and (d) demonstrate best stability of an active suspension with a Linear Quadratic Regulator (LQR Control Significantly reduce road gradients. RMS error values for all types of suspension systems examined in this section range from 25.37% to 26.24%. The outcomes show how well the suggested controller works.

To assess the system performances (riding comforts and handling performance) in the time domain, the RMS error (Root Mean Square) (see the equation (IV-9)) criterion is applied. The RMS error values for all employed methods are shown in Table (IV-7).

$$RMSE = \frac{\sqrt{\sum_{i=1}^{N}\|y(i)-\hat{y}(i)\|^2}}{N} \quad \textbf{(IV-9)}$$

Where $N$ is the Number of data; $y(i)$ is the $i-th$ measurement; and $\hat{y}(i)$ is its corresponding prediction [204].

**Table (IV-7)**: RMS Error of the Active Suspension System for a Quarter Car Model

| Roads / Methods | ***Step Road Input*** | ***Bumpy Road Input*** | ***Excavated Road input*** | ***Noisy Road Input*** | ***Romp Road Input*** |
|---|---|---|---|---|---|
| **Passive Suspension** | $1.0044e^{+00}$ | 2.555 | $3.480e^{-01}$ | 2.164 | $5.251e^{-02}$ |
| **PID_GA** | $9.725e^{-01}$ | $4.146e^{-01}$ | $1.281e^{-01}$ | $6.217e^{-01}$ | $3.919e^{-02}$ |
| **FOPID_GA** | $9.707e^{-01}$ | $3.881e^{-01}$ | $1.237e^{-01}$ | $5.969e^{-01}$ | $3.900e^{-02}$ |
| **PID_PSO** | $9.708e^{-01}$ | $3.898e^{-01}$ | $1.240e^{-01}$ | $5.986e^{-01}$ | $3.902e^{-02}$ |
| **FOPID_PSO** | $9.704e^{-01}$ | $3.405e^{-01}$ | $1.230e^{-01}$ | $5.927e^{-01}$ | $3.897e^{-02}$ |
| **LQR** | $9.649e^{-01}$ | $9.725e^{-01}$ | $1.143e^{-01}$ | $5.400e^{-01}$ | $3.873e^{-02}$ |

The actual difference between the estimated and measured values is used by the RMS statistics to offer data regarding the system's short-term performance. The RMS data shown in Table VII demonstrate that LQR control yields the best controller performances. The findings demonstrate that, when the road profile is altered, the optimal control achieves a better balance between ride comfort and handling performance. It also has a higher resilient ability.

### IV. 5. Interpretations of Results

A graph of unsprung mass displacement is shown in Figure (IV-6)-(a). According to the graph, the black line denotes the road profile, the red line denotes the passive suspension system's unsprung mass displacement. The active suspension system's displacement was shown by the green, blue, purple, azure, and fuchsia lines, which were regulated by PID and FOPID and optimized by genetic algorithms and PSO algorithms, with last regulator is LQR control. The working distance between is the unsprung mass displacement. The condition of the road and the unsprung mass or tire tread area. The velocity, body acceleration, and body deflection were depicted in Figure (IV-6)-(b), (c), and (d). According to the simulation results depicted in Figure (IV-6) 6, the passive suspension system's unsprung mass is significantly higher than that of active suspension systems in terms of displacement, velocity, body acceleration, and suspension deflection. It was noted that the unsprung mass in passive suspension's positive displacement stepped up at 1.562 seconds and fell at the same instant at roughly 2.106 seconds. The active suspension system's unsprung mass displacement towards the positive direction is approximately 1.015 cm for active control by PID_GA, 0.9955 cm for active control by FOPID_GA, 0.9929 cm for active control by PID_PSO, 0.9893 cm for active control by FOPID_PSO, and the final value is 0.552 cm for an active suspension controlled by LQR contra under the same road conditions. Both passive suspension systems feature vibrational response characteristics with a time delay response of roughly 1.569 seconds. It was once more noted that the active suspension system's unsprung mass displacement characteristics. Under the identical driving conditions shown in Figure (IV-6), the active suspension system exhibits the least unsprung mass displacement and consistent vibrational characteristics. The graph in Figure (IV-6) truly depicts how a car tire reacts to an uneven road surface. Figure (IV-6) {(b), (c)} illustrate velocity, body acceleration, and these graphs show the stability of the car on the road surface. This gives passengers and the driver greater comfort by reducing road vibrations. The simulation findings as shown in Figure (IV-6) (d) reveal that, the deflection of the suspension of the passive suspension system is substantially higher than the active suspension systems. It was observed that, the deflection characteristics of the nature of the vibration is opposite to the vibrational characteristics of body acceleration of unsprung mass. The graph of passive, and active suspension systems under the same road condition. The passive suspension system deflected more than the active suspension systems; it was once more noted. Again, there is a time delay of about 1.094 seconds before the start of the vibrational deflection which lasted for about 1.542 seconds before the deflection begins to smoothen out to normality. Here, the graph in Figure (IV-6) {(d)} shows that, the deflection displacement first moved towards the negative direction to start the vibration which is opposite of the unsprung mass displacement. The drop value for the active suspension system, which is controlled by the proportional controller and the partial controller, ranges between 1,240 cm and 1,229 cm. The passive suspension system has the highest magnitude, with a suspension deflection of 0.6525 cm in the positive direction and 1.048 cm in the negative direction. The positive vibrations are non-existent, while the active suspension controlled using LQR has a suspension deflection magnitudes 0.1165 cm towards the positive direction and 0.3336 cm towards the negative direction under the same road condition. systems. The vibrational response characteristics of passive, and active suspension systems are the varied magnitudes of suspension deflections.

The same concerning the figures (Figures (IV-7) to Figure (IV-10)), also proves that the active suspension is controlled by using LQR, which gives better results and optimal fell for road vibrations, as we applied different types of roads to prove this theory. In this dissertation we choose LQR control because is a type of control algorithm and we called a Linear Quadratic Regulator (LQR) is used to optimize the behavior of a system with multiple inputs and outputs. It is a current control approach that is frequently utilized in robots, automobiles and control engineering. LQR is based on the linear quadratic (LQ) optimal control theory, a mathematical framework for maximizing system performance by minimizing a cost function that assesses the discrepancy between the system's desired and actual behavior. This cost function is used by the LQR algorithm to determine the best control inputs to steer the system toward the desired behavior. LQR's ability to handle systems with multiple inputs and outputs as well as systems with time-varying dynamics is one of its main advantages. This makes it an effective tool for managing complicated systems, such as robots. In compared with PID (Proportional-Integral-Derivative) control and FOPID (Fractional order PID) control, LQR is generally more precise and robust, but it also takes more computer capacity. Fuzzy logic, on the other hand, is a more advanced control method that can manage uncertainty and non-linear systems, but it necessitates more data for optimization. In conclusion, LQR is a strong control algorithm with advantages over other control methods like PID and fuzzy logic. It is widely used in robotics and control engineering.

## IV. 6. Conclusion

The suspension of a vehicle is a complex, nonlinear system. They can be divided into three groups: passive suspensions, which merely include a spring and a damper; semi-active suspensions; and active suspensions, which rely on a controller to enable the suspension to be adjusted for various road conditions.

We discussed an examination in this chapter that relates to enhancing the car's robustness and stability for an active quarter car suspension system. The optimal control approach, which is based on the linear quadratic regulator, is based on metaheuristic optimization of PID and FPID utilizing GA and PSO algorithms. Results were compared to those from intelligent approaches and the best way. Notably, the LQR control for the active suspension system produced better results after comparison: smaller RMS, smaller overshoot, shorter settling time, and shorter rising time, which can ensure ride comfort and road holding capabilities while exposed to road disturbances.

# General Conclusion

# General Conclusion

In the industrial field, Active suspension systems are advanced automotive technologies that provide superior ride quality and handling. They have become increasingly popular in recent years due to their many advantages over traditional passive suspension systems. Active suspensions offer improved performance, better control of body roll, increased stability during cornering, and a smoother overall ride experience. Additionally, they can be customized for specific driving conditions or preferences to further enhance the driving experience. The work presented in the dissertation studied the active suspension system of quarter car model, this model defined by 2-DOF. We controlling an active suspension system by three different controllers PID, FOPID, and LQR control.

PID controller is one type commonly used for controlling active suspensions system due to its robustness against external disturbances. It consists of three components: proportional gain which adjusts the output according to error magnitude; integral gain which eliminates steady-state errors; derivative gain which reduces overshoot or oscillations when step input is applied. The parameters must be tuned carefully so that they will not lead to instability or poor performance due to bad tuning values. On the other hand, the FOPID controller uses fractional derivatives instead integer derivatives like PID does allowing better accuracy at low-frequency signals compared with traditional PID but also making a more difficult parameter tuning process since there’s no general rule about how much fraction should use on each term in order obtain best results. In order to avoid these problems, we used optimization techniques. Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) are two of the most popular optimization techniques used in engineering applications. Both of these algorithms can be used to optimize PID controllers, FOPID controllers, or both.

The main advantage that PSO has over GA is its ability to quickly search through a large parameter space without getting stuck at local minimum points as GA does. This makes it ideal for finding optimal parameters with complex systems where multiple objectives must be optimized simultaneously such as when designing a controller with multiple inputs and outputs or when dealing with nonlinear systems which require more sophisticated control strategies than linear models allow for. Additionally, since PSO requires fewer parameters compared to genetic algorithms it is faster overall allowing engineers more time to experiment on their design instead of waiting around while an algorithm runs its course. On other hand, LQR provides good dynamic response while still maintaining stability by using a feedback mechanism based on state variables rather than just error signals as others do, and minimize the vibration of roads. This allows faster response times without compromising stability however this approach requires precise knowledge about plant model structure so it can become expensive if accurate information isn't available beforehand.

The thesis studies the comparison between passive and active suspension of quarter car model with all technics, and we will be testing these systems using five types of roads.

In the comparison between the types of controllers for control an active suspension system, and passive suspension system studied in this thesis, LQR offers better stability since it takes into account all possible inputs from both disturbance sources and actuators whereas Optimized PID/FOPID only considers certain input variables like error signal magnitude, etc. Also, LQR provides superior tracking accuracy compared with optimized FOPIDs because they have access to additional information about plant dynamics that cannot easily be obtained through other means. On the other hand, optimized FOPID offer more flexibility when designing complex systems due to their ability to adjust parameters quickly without having manually.

The perspective of this dissertation is to provide powerful force controllers for active vehicle suspension systems. Few active suspension controllers have been experimentally proven to operate correctly because most of them in the literature did not integrate actuator dynamics. Because they combine the benefits of deterministic robust control techniques and adaptive control techniques while avoiding many of their shortcomings, adaptive robust control techniques are used in this dissertation to provide precise force production. To further increase the usability and effectiveness of the GA and PSO algorithms techniques. The PID, FOPID, and LQR controller types are used in the initial design of a force controller. Second, in order to make the final force controller simpler to build, we used GA and PSO algorithms to optimize PID and FOPID. The performance and robustness of the controllers are then enhanced through the suggestion of the LQR adaptation mechanism. Finally, a modular approach is devised to enhance the controller's identification process. These improvements are significant in the following ways. The output feedback extension is crucial for cutting the price of sensors. The LQR control helps to stabilize the system under unmodeled uncertainty while also enhancing system performance during routine operations. According to simulation results from a quarter-car test rig, these force controllers perform satisfactorily when the LQR control laws are used.

We discussed nonlinear systems and outlined the fundamental ideas and theorems pertaining to system stability in **Chapter I**.

The principle of suspension systems, including all the different classifications, is discussed in **Chapter II**.

The principles of genetic algorithms, particle swarm optimization algorithms, and biogeography-based optimization algorithms are briefly discussed in **Chapter III**. These algorithms allow for the provision of a sufficiently good solution to an optimization problem and serve as a basis for PID and FOPID controller research.

In **Chapter IV**, we provide the results of controlling the active suspension system for a quarter-car model through phases of PID and FOPID controller parameter optimization and LQR controller parameter calculation. The findings compare the passive suspension to the active suspension.

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