## People's Democratic Republic of Algeria

Ministry of Higher Education and Scientific Research
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Master in Mechanical Engineering

## Entitled

## MEMORY PRESENTED BY

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To obtain a Master's degree in

## Dynamic study of rotor mono-disc

Supported on 29/06/2020

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## dedicalion

First all I would like to thank Allah for guiding me to the right path and I would like to pay tribute to my parents for the unending support in this journey and words cannot express my full gratitude for their sacrifices I give thanks specially to my virtuous Belghalem Hadj and again thank you very much for all the members of my family and friends

Despite the time of Corona Virus Ahmad Allah, that I finished on time.

Islam

## Dévouement

Tout d'abord, je voudrais remercier Allah pour m'avoir guidé sur la bonne voie et je voudrais rendre hommage à mes parents pour leur soutien sans fin dans ce voyage et les adorations ne peuvent pas exprimer ma pleine gratitude pour leurs sacrifices Je rends grâce spécialement à mon vertueux BELGHALIM HADJ et encore merci beaucoup pour tous les membres de ma famille et mes amis

Malgré le temps de Corona Virus, Ahmad Allah, que j'ai terminé à temps

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## Introduction

The rotating machines are one of the most importing elements in the industry, they play a major role in the transportation section) turbo reactors of airplanes, turbo compressors of automobiles, and turbines for boats........ ect.........( modern the same thing goes for the energy industry ) hydroelectric turbines, turbo alternator for thermic centers, nuclear, solar .......(, the main component of rotating machines is the rotor which allows these machines to reach high speed for acquiring maximum results, nether less these results came with consequences, It creates forces of rotating unbalance, manufacturing defects can generate unacceptable stresses in the rotor and stator, or large differential displacement of the bearings. In the design of rotating machinery, it is necessary to predict the dynamic behavior of rotors in bending and in torsion. These predictions are as follows.

1. The static and dynamic behavior in torsion is performed. The natural frequencies which give the critical speeds must be determined. In addition, when electric motors or generators are present, the dynamic behavior during start-up or under short-circuit conditions must be predicted.
2. The dynamic behavior in bending is performed last. The natural frequencies as a function of the speed of rotation, which give the critical speeds, and possible instabilities must be determined. Then, the effect of forces of excitation is calculated, mass unbalance being the most important of these.
The essential studies in rotor dynamics concern the layout of the Campbell diagram, which represents the evolution of the natural frequencies according to the rotational speed, which makes it possible to determine the possible instabilities, and the calculation of imbalance responses, mainly when shifting critical gears, and possibly at an asynchronous force. To carry out such studies, we used the Rayleigh-Ritz method, which allows setting up a model allowing to treat simple cases and to put in evidence of the basic phenomena[1].
3. The work presented in this thesis concerns the modeling and simulation of the dynamic behavior of the simple rotor. In our first chapter we are going to talk about the basic knowledge of rotor dynamics and the characteristic or rotor elements. The second chapter will be about the maintenance of rotors with it many different types of maintenance. The third and last chapter we are going to represent the study of critical speeds of the rotor theoretically with using Cambell
diagram using MATLAB the analytic method and comparing between the two methods. we finish our work with a conclusion.

## Chapter1:

## Generalities of Rotor

## dynamics

## Chapter1: Generalities of Rotor dynamics

## 1.1: Introduction:

the rotating machinery have many components that faces many production problems to resolve this problems the industrial community developed over many years a mechanical branch that specializes in the study of rotating machinery under the name of rotor dynamics in our first chapter we will talk about rotor dynamics as a science the different parts of this branch and the characteristic of rotor components

## 1.2: Rotor Dynamics Definition:

Rotor dynamics, also known as rotor dynamics, is a specialized branch of applied mechanics concerned with the behavior and diagnosis of rotating structures. It is commonly used to analyze the behavior of structures ranging from jet engines and steam turbines to auto engines and computer disk storage. At its most basic level, rotor dynamics is concerned with one or more mechanical structures (rotors) supported by bearings and influenced by internal phenomena that rotate around a single axis. The supporting structure is called a stator. As the speed of rotation increases the amplitude of vibration often passes through a maximum that is called a critical speed. This amplitude is commonly excited by unbalance of the rotating structure; everyday examples include engine balance and tire balance. If the amplitude of vibration at these critical speeds is excessive, then catastrophic failure occurs. In addition to this, turbo machinery often develops instabilities which are related to the internal makeup of turbo machinery, and which must be corrected. This is the chief concern of engineers who design large rotors.

Rotating machinery produces vibrations depending upon the structure of the mechanism involved in the process. Any faults in the machine can increase or excite the vibration signatures. Vibration behavior of the machine due to imbalance is one of the main aspects of rotating machinery which must be studied in detail and considered while designing. All objects including rotating machinery exhibit natural frequency depending on the structure of the object. The critical speed of a rotating machine occurs when the rotational speed matches its natural frequency. The lowest speed at which the natural frequency is first encountered is called the first critical speed, but as the speed increases, additional critical speeds are seen. Hence, minimizing rotational unbalance and unnecessary external
forces are very important to reducing the overall forces which initiate resonance. When the vibration is in resonance, it creates a destructive energy which should be the main concern when designing a rotating machine. The objective here should be to avoid operations that are close to the critical and pass safely through them when in acceleration or deceleration. If this aspect is ignored it might result in loss of the equipment, excessive wear and tear on the machinery, catastrophic breakage beyond repair or even human injury and loss of lives.

The real dynamics of the machine is difficult to model theoretically. The calculations are based on simplified models which resemble various structural components (lumped parameters models), equations obtained from solving models numerically (Rayleigh-Ritz method) and finally from the finite element method (FEM), which is another approach for modeling and analysis of the machine for natural frequencies. There are also some analytical methods, such as the distributed transfer function method, which can generate analytical and closed-form natural frequencies, critical speeds and unbalanced mass response. On any machine prototype it is tested to confirm the precise frequencies of resonance and then redesigned to assure that resonance does not occur, [2]

### 1.3 Basic Principles of Rotor Dynamics:

The equation of motion, in generalized matrix form, for an axially symmetric rotor rotating at a constant spin speed $\Omega$ is where:

M is the symmetric Mass matrix
C is the symmetric damping matrix
G is the skew-symmetric gyroscopic matrix
K is the symmetric bearing or seal stiffness matrix
N is the gyroscopic matrix of deflection for inclusion of e.g., centrifugal elements.
In which $q$ is the generalized coordinates of the rotor in inertial coordinates and $f$ is a forcing function, usually including the unbalance.

The gyroscopic matrix $G$ is proportional to spin speed $\Omega$. The general solution to the above equation involves complex eigenvectors which are spin speed dependent. Engineering specialists in this field rely on the Campbell Diagram to explore these solutions.

An interesting feature of the rotor dynamic system of equations is the off-diagonal terms of stiffness, damping, and mass. These terms are called cross-coupled stiffness, cross-coupled damping, and cross-coupled mass. When there is a positive cross-coupled stiffness, a deflection will cause a
reaction force opposite the direction of deflection to react the load, and also a reaction force in the direction of positive whirl. If this force is large enough compared with the available direct damping and stiffness, the rotor will be unstable. When a rotor is unstable, it will typically require immediate shutdown of the machine to avoid catastrophic failure.[2]

### 1.3.1Campbell diagram



Fig. 1.1: Campbell diagram
The Campbell diagram, also known as "Whirl Speed Map" or a "Frequency Interference Diagram", of a simple rotor system is shown on the right. The pink and blue curves show the backward whirl (BW) and forward whirl (FW) modes, respectively, which diverge as the spin speed increases. When the BW frequency or the FW frequency equal the spin speed $\Omega$, indicated by the intersections A and B with the synchronous spin speed line, the response of the rotor may show a peak. This is called a critical speed.[2]

### 1.4 History of Rotor Dynamics

Research on rotor dynamics spans at least a 140-year history, starting with Rankin's paper on whirling motions of a rotor in 1869;

Rankine discussed the relationship between centrifugal and restoring forces and concluded that operation above a certain rotational speed is impossible;

Although this conclusion was wrong, his paper is important as the first publication on rotor dynamics.


Fig. 1.2: History of Rotor Dynamics[2]
De Laval, an engineer in Sweden, invented a one-stage steam turbine and succeeded in its operation.

He showed that it was possible to operate above the critical speed by operating at a rotational speed about seven times the critical speed. In the early days, the major concern for researchers and designers was to predict the critical speed, because the first thing that had to be done in designing rotating machinery was to avoid resonance.

Drunkenly (1894) derived an empirical formula that gave the lowest critical speed for a multicolor system.He was the first to use the term "critical speed'" for the resonance rotational speed.

Holzer (1921) proposed an approximate method to calculate the natural frequencies and mode shapes of tensional vibrations. The first recorded fundamental theory of rotor dynamics can be found in a paper written by Jeffcott (1919).

A shaft with a disk at the midspan is called the Jeffcott rotor. It is also called the Laval rotor, named after de Laval.

The developments made in rotor dynamics in the masterpiece written by Stodola (1924) This superb book explains nearly the entire field related to steam turbines. The dynamics of elastic shafts with disks, The dynamics of continuous rotors without considering the gyroscopic moment, The balancing of rigid rotors, and methods for determining approximate values of critical speeds of rotors with variable cross sections. Thereafter, the center of research shifted from Europe to the United States.

Campbell (1924) at General Electric investigated vibrations of steam turbines in detail. His diagram, representing critical speed in relation to the cross points of natural frequency curves and the straight lines proportional to the rotational speed, is now widely used and referred to as the Campbell diagram. As the rotational speed increased above the first critical speed, the occurrence of selfexcited vibrations became a serious problem.

In the 1920s, Newkirk (1924) and Kimball (1924) first recognized that internal friction of shaft materials could cause an unstable whirling motion. These phenomena, in which friction that ordinarily dampens vibration causes self- excited vibration, attracted the attention of many researchers.

Newkirk and Taylor (1925) investigated an unstable vibration called oil whip, which was due to an oil film in the journal bearings. Newkirk (1926) showed a forward whirl induced by a hot spot on the rotor surface, which was generated by the contact of the rotor and the surroundings. These hot spot instabilities called the Newkirkeffect. About a decade later, the study of asymmetrical shaft systems and asymmetrical rotor systems began. As these directional differences rotate with the shaft, terms with time-varying coefficients appear in the governing equations. These systems therefore fall into the category of parametrically excited systems. The most characteristic property of asymmetrical systems is the appearance of unstable vibrations in some rotational speed ranges.

Smith (1933)'s report is a pioneering work on this topic. Various phenomena related to the asymmetries of rotors were investigated actively in the middle of the twentieth century by Taylor (1940) and Foote, Poritsky, and Slade (1943), Brosen and Crandall (1961), and Yamamoto and Ota (1963a, 1963b, 1964).

Non stationary phenomena in passage through critical speeds have been studied since Lewis reported his investigation on the Jeffcott rotor in 1932. reports on this topic are classified into two groups.with a constant acceleration and with a limited driving torque. Due to complexity
of the problem numerical integrations was used. The asymptotic method developed by the Russian school of Krylov and Bogoliubov (1947) and Bogoliubov and Mitropol'skii (1958) considerably boosted the research on this subject.

The vibrations of rotors with continuously distributed mass were also studied. The simplest continuous rotor model corresponding to the Euler beam was first studied in the book by Stodola (1924).In the 1950s and 1960s, Bishop (1959), Bishop and Gladwell (1959), and Bishop and Parkinson (1965) reported a series of papers on the unbalance response and the balancing of a continuous rotor. Eshleman and Eubanks (1969) derived more general equations of motion considering the effects of rotary inertia, shear deformation, and gyroscopic moment.

The most important and fundamental procedure to reduce unfavorable vibrations is to eliminate geometric imbalance in the rotor. The balancing technique for a rigid rotor was established relatively early. A practical balancing machine based on this technique was invented by Lawaczeck in 1907.In 1925; Suehiro invented a balancing machine that conducts balancing at a speed in the postcritical speed range. In 1934, Thearle developed the two-plane balancing. The arrival of high-speed rotating machines made it necessary to develop a balancing technique for flexible rotors. Two representative theories were proposed. One was the modal balancing method proposed in the 1950s by Federn (1957) and Bishop and Gladwell (1959).The other was the influence coefficient method proposed in the early 1960s and developed mainly in the United States alone with the progress of computers. Goodman (1964) improved this method by taking into the least square methods.

In the latter half of the twentieth century, various vibrations due to fluid were studied. Hori (1959) succeeded in explaining various fundamental characteristics of oil whip by investigating the stability of shaft motion and considering pressure forces due to oil films. In 1964, Alford reported accidents due to labyrinth seals. Another one was a self-excited vibration called the steam whirl. The mechanism of this vibration in turbines was explained by Thomas (1958) and that in compressors was explained by Alford (1965).

As rotors became lighter and their operational speeds higher, the occurrence of nonlinear resonances such as subharmonic resonances became a serious problem. Yamamoto (1955, 1957a) studied various kinds of nonlinear resonances after he reported on subharmonic resonances due to ball bearings, in 1955.In the 1960s, Tondl (1965) studied nonlinear resonances due to oil films in journal bearings Ehrich (1966) reported subharmonic resonances observed in an aircraft gas turbine with squeeze-film damper bearings. The cause of strong nonlinearity in aircraft gas turbines is the radial clearance of squeeze-film damper bearings.

Later, Ehrich $(1988,1991)$ reported the occurrence of various types of subharmonic resonances up to a very high order and also chaotic vibrations in practical engines.

In the practical design of rotating machinery, it is necessary to know accurately the natural frequencies, modes, and forced responses to unbalances in complex-shaped rotor systems. Prohl (1945) used the transfer matrix method in the analysis of a rotor system by expanding the method originally developed by Myklestad (1944). This analytical method is particularly useful for multi rotor-bearing systems and has develop rapidly since the 1960s by the contribution of many researchers such as Lund and Orcutt (1967) and Lund (1974).

The finite-element method was first developed in structural dynamics and then used in various technological fields. The first application of the finite-element method to a rotor system w shafts began. In the 1970s, Gasch (1976) and Henry and Okah- Avae (1976) investigated vibrations, giving consideration to nonlinearity in stiffness due to open-close mechanisms. They showed that an unstable region appeared or disappeared at the major critical speed, depending on the direction of the unbalance. The research is still being developed and various monitoring systems have been proposed.[2]

### 1.5 Rotor Definition:

The rotor of turbo machinery consists of the shaft, the disc, the shaft, the bearings and the blade.

These elements are visible at the see figure( 1.3)[3]


Fig. 1.3: Description of Rotor.[3]

### 1.5.1 The Disc

The wheel can be modelled by a disc, it can be deformable or rigid. A wheel can begeometrically represented in cylindrical form; but in dynamic analysis, this form is limited[3]

### 1.5.2 The Shaft

It is an organ that carries the wheels. It can be rigid or flexible, these properties depend on its material. The shaft is considered deformable in dynamic analysis, this allows to take into account deformation effects. Geometrically, the cross-section of the shaft can be constant or variable. The bending study becomes complex for the variable section shaft because the movement isdescribed by linear differential equations with variable coefficients whose solutions are the functions of Bessel.[3]

### 1.5.3 The Bearings

The bearing is also called support; it can be flexible (isotropic or anisotropic) or rigid. The bearingis a dissipation system or not when flexibility is anisotropic.[3]

### 1.5.4 Unbalanc:

The baler is any eccentric mass of a rotor. It is located at a distance $d$ from the center ofgeometry of the tree as shown in Figure 1.4.[3]


Fig.1.4: Element of unbalance[3]

### 1.6 Classification of Rotors:

The huge domain of industry contains many different rotating machineries which uses different types of rotors.[4]

### 1.6.1 Jeffcott rotor:

The Jeffcott rotor (named after Henry Homan Jeffcott), also known as the de Laval rotor in Europe, is a simplified lumped parameter model used to solve these equations. The Jeffcott rotor is a mathematical idealization that may not reflect actual rotor mechanics.[4]

.1.5: Simple design of Jeffcott rotor [4].

### 1.6.2 Rigid Rotors

A Rigid Rotoris that which, when balanced in any two arbitrarily selected planes, will remain within the specified balance tolerance at any speed up to and including its maximum service speed.


Fig.1.6: Typical variation of unbalance amount of Rigid Rotors [4]

Balancing rigid rotors in a low speed, transportable balancing machine is a straight forward exercise when applying recognised balancing procedures, providing the specified permissible residual unbalance, calculated to the rotor's maximum service speed, can be achieved with repeatable results.

There are occasions when it may be difficult to ascertain whether or not a rotor meets the true definition of a rigid rotor. The best advice in this case is: if in doubt, treat it as "Flexible".[4]

### 1.6.3 Flexible Rotors:

A Flexible Rotoris that which does not satisfy the definition of a rigid rotor and which has a tendency to bend or distort due to centrifugal and unbalance forces, the effect of which can beinduced or aggravated by changes in operating loads and temperatures.For practical purposes, flexible rotors can be considered one of, or a combination of, three types:

Type 1 Those which lack inherent rigidity to an extent that causes them to whip, even at low speed.


Fig. 1.7:Typical variation of unbalance amount of Flexible rotors of the first type[4]

Type 2 Those which are subject to plastic deformation, induced by centrifugal forces, and changes in operating load and/or temperature.


Fig. 1.8:Typical variation of unbalance amount of Flexible rotors of the second type[4]

Type 3 Those which are subject to elastic deformation as operating speeds approach and coincide with rotor natural frequencies or "critical speeds".[4]


Fig. 1.9: Typical variation of unbalance amount of Flexible rotors of the third type.[4]

## Chapter2:

## Rotors Maintenance

## and surveillance

## Chapter 2: Rotors Maintenance and Surveillance.

## 2.1: Introduction

Throughout the more enterprising countries of the world, the benefits to industry of undertaking on site, the complete servicing and repair of strategic rotating machines have long been recognised and applied with considerable reward. The higher the capacity of the machines and the more strategic their nature of operation to production demands, the greater are the benefits to be realised in terms of reduced overall costs and machine downtime.

The most difficult maintenance procedures to undertake on site are always seen as the final machining and balancing of rotor assemblies, particularly the higher capacity and more precise "Flexible" rotors from machines such as turbines, compressors, electrical motors and generators.[4]

### 2.2 Maintenance

### 2.2.1: Maintenance Definition

British Standard Glossary of terms (3811:1993) defined maintenance as:

The combination of all technical and administrative actions, including supervision actions, intended to retain an item in, or restore it to, a state in which it can perform a required function.

Maintenance is a set of organised activities that are carried out in order to keep an item in its best operational condition with minimum cost acquired.

Activities of maintenance function could be either repair or replacement activities, which are necessary for an item to reach its acceptable productivity condition or these activities, should be carried out with a minimum possible cost. [5]

### 2.2.2 Maintenance History

1. In the period of pre-World War II, people thought of maintenance as an added cost to the plant which did not increase the value of finished product. Therefore, the maintenance at that era was restricted to fixing the unit when it breaks because It was the cheapest alternative
2. During and after World War II at the time when the advances of engineering and scientific technology developed, people developed other types of maintenance, which were much cheaper such as preventive maintenance. In addition, people in this era classified maintenance as a function of the production system.
3. Nowadays, increased awareness of such issues as environment safety, quality of product and services makes maintenance one of the most important functions that contribute to the success of the industry. World-class companies are in continuous need of a very well organised maintenance program to compete world-wide. [5]


Fig.2.1Maintenance History (Adapted from Shenoy Bhadury, 1998) [5]

### 2.2.3 Maintenance Objectives

Maintenance objectives should be consistent with and subordinate to production goals. The relation between maintenance objectives and production goals is reflected in the action of keeping production machines and facilities in the best possible condition.

- Maximising production or increasing facilities availability at the lowest cost and at the highest quality and safety standards.
- Reducing breakdowns and emergency shutdowns.
- Optimising resources utilisation.
- Reducing downtime.
- Improving spares stock control.
- Improving equipment efficiency and reducing scrap rate.
- Minimising energy usage.
- Optimising the useful life of equipment.
- Providing reliable cost and budgetary control.
- Identifying and implementing cost reductions. [5]


Fig.2.2: Maintenance objective [5]

### 2.2.4 Types of Maintenance:

### 2.2.4.1 Run to Failure Maintenance (RTF)

The required repair, replacement, or restore action performed on a machine or a facility after the occurrence of a failure in order to bring this machine or facility to at least its minimum acceptable condition. It is the oldest type of maintenance. It is subdivided into two types:

- Emergency maintenance: it is carried out as fast as possible in order to bring a failed machine or facility to a safe and operationally efficient condition.
- Breakdown maintenance: it is performed after the occurrence of an advanced considered failure for which advanced provision has been made in the form of repair method, spares, materials, labour and equipment. [5]


### 2.2.4.2 Preventive Maintenance (PM)

It is a set of activities that are performed on plant equipment, machinery, and systems before the occurrence of a failure in order to protect them and to prevent or eliminate any degradation in their operating conditions.

British Standard 3811:1993 Glossary of terms defined preventive maintenance as:

The maintenance carried out at predetermined intervals or according to prescribed criteria and intended to reduce the probability of failure or the degradation of the functioning and the effects limited. [5]

### 2.2.4.3 Corrective Maintenance (CM)

In this type, actions such as repair, replacement, or restore will be carried out after the occurrence of a failure in order to eliminate the source of this failure or reduce the frequency of its occurrence.

In the British Standard 3811:1993 Glossary of terms, corrective maintenance is defined as:
The maintenance carried out after recognition and intended to put an item into a state in which it can perform a required function.

This type of maintenance is subdivided into three types:

- Remedial maintenance: which is a set of activities that are performed to eliminate the source of failure without interrupting the continuity of the production process. The way to carry out this type of corrective maintenance is by taking the item to be corrected out of the production line and replacing it with reconditioned item or transferring its workload to its redundancy.
- Deferred maintenance: which is a set of corrective maintenance activities that are not immediately initiated after the occurrence of a failure but are delayed in such a way that will not affect the production process.
- Shutdown corrective maintenance: which is a set of corrective maintenance activities that are performed when the production line is in total stoppage situation. [5]


### 2.2.4.4 Improvement Maintenance (IM):

It aims at reducing or eliminating entirely the need for maintenance. This type of maintenance is subdivided into three types as follows:

- Design-out maintenance: which is a set of activities that are used to eliminate the cause of maintenance, simplify maintenance tasks, or raise machine performance from the maintenance point of view by redesigning those machines and facilities which are vulnerable to frequent occurrence of failure and their long term repair or replacement cost is very expensive.
- Engineering services: This includes construction and construction modification, removal and installation, and rearrangement of facilities.
- Shutdown improvement maintenance: which is a set of improvement maintenance activities that are performed while the production line is in a complete stoppage situation. [5]


### 2.2.4.5 Predictive Maintenance (PDM)

Predictive maintenance is a set of activities that detect changes in the physical condition of equipment (signs of failure) in order to carry out the appropriate maintenance work for maximising the service life of equipment without increasing the risk of failure.

It is classified into two kinds according to the methods of detecting the signs of failure:

- Condition-based predictive maintenance
- Statistical-based predictive maintenance

Condition-based predictive maintenance depends on continuous or periodic condition monitoring equipment to detect the signs of failure.

Statistical-based predictive maintenance depends on statistical data from the meticulous recording of the stoppages of the in-plant items and components in order to develop models for predicting failures. [5]

### 2.3 Maintenance of Rotors:

### 2.3.1 Vibration: a good condition indicator

The operation of the machines entails efforts which will beoften the cause of subsequent failures (rotating efforts, turbulence, shocks, instability). The forces are in turn causing vibrations that will damage machinery structures and components.

The analysis of these vibrations will make it possible to identify the forces immediatelybefore they have caused irreversible damage.It will also, after analysis, allow us to deduce the origin and estimate the risk of failure.


Fig.2.3 Optimization of the maintenance policy[6]

$$
\text { Page } 30 \mid 97
$$

It is on these concepts that predictive maintenance is based.In order to implement it, we will have tobe able to determine the most frequent causes of failure, to evaluate theircosts, their likelihood of occurrence, and putting in place a policy to detect symptoms as soon as possible. [6]

### 2.3.2 the Recognize the defects

There is no predictive maintenance without minimal diagnosis of defects and their severity. This is why the first step of a monitoring action is to ask what defects are likely to occur on the machine to be monitored.

The second concerns the manifestations of these defects. What information, what parameters descriptions of the defect need to be developed and measured to have the right information; those that will allow to say whether the situation is normal or not (detection of abnormalities), but also those that will make it possible to find the origin later (diagnosis of the origin and severity of the abnormalities). Defects and manifestations [6]

### 2.3.3 Rotors and Rotating Parts

### 2.3.3.1 Mass imbalance of rotors.

Whatever the care given to the construction of the machines; itis not possible to match the axis of rotation with the centerthe gravity of each of the main rotorthe unbalance. As a result, the rotating shaft is subjected tocentrifugal forces that distort it. These forces result invibrations related to the rotation frequency.

The imbalances are usually due to machining defects,assembly of rotors or mounting. In operation, the rotors can then also deform under the effect of heating dissymmetric. Some examples of causes of imbalance are shown in figure 2.4


Fig.2.4 Some causes of unbalance [6]

On this figure, the defects seem exaggerated, especially if one is refers to the actual deformations of the rotors. But if we consider these defects can be amplified by resonance phenomena, the figures become realistic. Indeed, a decentralization of the $10 \mu \mathrm{~m}$ rotor relative to its axis of rotation may result in high vibration if the internal damping of the rotor is low.


Fig. 2.5Sudden evolution of vibration due to rupture or a slip [6]

With $1 \%$ damping, that is to say with a coefficient of amplification of 50, the vibration at critical speeds will be reach amplitude of $500 \mu \mathrm{~m}$. However, machining to within $10 \mu \mathrm{~m}$ is already a good machining, difficult to achieve. [6]

### 2.3.3.2 Unbalance of mechanical origin

### 2.3.3.2.1 Loss of fin, breakage of a blade

when a part of the rotor breaks and starts, an instantaneous evolution is generally observedof vibrations. This evolution is better perceived by monitoringsimultaneously the amplitude and phase of the vibrations ina vector representation (Figure 2.3).Loss of blades also result inflow (presence of repeated pressure pulses) thatwill see by specific analyses of type spectrum.

### 2.3.3.2.2 Modification of the installation

Sliding of the coupling plates translates asin the previous example by a sudden evolutionsynchronous vibration of the rotation. This kind of incident isto be correlated with changes in torsion forces (networks, evolution of the torque transmitted during a charge). This is especially true during the first charge after disassembly of the coupling which is then put back into placesliding when torsion forces become sufficient.

### 2.3.3.2.3 Erosion. Material deposition

The erosion of the blades can create an imbalance if the distributions not symmetrical (which is rare).The deposit of material meets on fans that workin very dirty atmospheres, such as for example fume draws. There is a slow evolution of the vibration's frequency of rotation, sometimes with discontinuities when a part of this deposit is detached by centrifugal forces. [6]

### 2.3.3.2.4 Creep, Cornering Defect

When starting a machine after a prolonged downtime, vibration may be observed under certain conditionscreated by permanent rotor deformation due to:

- a creep of hot rotors even when stopped for a short time;
- a creep of cold rotors if they are very flexible and stopped for a very long time; - a splash of water (local quenching due to the presence of cold fluid in a hot vapor flow). This phenomenon can occur when hot steam is sent to a steel pipe [6]


### 2.3.3.3 Thermal Unbalance

### 2.3.3.3.1Turbine Rotor Deformation

When the rotors are not homogeneous, or when the temperatureis not evenly distributed, the rotors deform under the effect of thermal stresses. If they deform dissymmetrical,the center of gravity moves and the forces vary.

The diagnostic criterion is then based on the correlation between temperature variations and vibration evolution. Speed the change will indicate the origin of the defect.

### 2.3.3.3.2 Distortion of alternator or rotors

Electric motors: thermal unbalanceas before, non-homogeneity of the rotor caninduce distortions.

Due to the significant energy dissipated by Joule effect or byhysteresis, it is necessary to cool the rotors. Any dissymmetry flow rate (blocked ventilation channels or different load losses). will

$$
\text { Page } 34 \text { | } 97
$$

result in a variation in power of vibration. The vibrations are then a function of the heating which dependsthe current in the rotor, but also the temperaturethe coolant, or its pressure.

A similar effect can be obtained in case of short-circuiting betweencoils, causing a heating dissymmetry. It is necessary; to find the origin of this deformation, complete the informationby electrical measures (insulation, resistance internal or current). [6]

### 2.3.3.3.3 Twisting and Twisting of Coils

If an obstacle prevents the expansion of a bar, or if the forcesfriction becomes high, the expansion of the winding cannotmore freely and the rotor twists. We observe in this a change in the vibratory level.

### 2.3.3.4 Friction. Evolutionary Unbalance

If passing through an orifice (bearing, seal for example)the tree warms up dissymmetrical, either because itfriction, either because the oil stirring causes a heatingmore intense on one side of the tree than on the other, if moreover, the deformationthe shaft that results from this heating moves in turnthe hot point (maximum vibration offset from the pointwhich gives birth to it) (cf. Figure 2.6), then, all the conditionsare brought together to initiate cyclical change of imbalance.

The equation of motion shows that the trajectory of the vector representing vibration at the rotation frequency is a dampened or diverging spiral depending on the position of the critical speeds, or more accurately depending on the angle value the vibration (displacement) with the force that gives birth to it (unbalance created by the warm-up).

The shift of the warm-up point is explained because there is phase shift between deformation and the force that creates it. This phase shift is the one that exists for example between a unbalance and the vibration that it induces:

- It is zero at low speed;
- It is equal to $\pi / 2$ at critical speed;
- It is equal to $\pi$ at high speeds

Figure 7 shows some examples of vibration observed in the case of friction on joints lubricated seal.

The phenomena of spiral evolution (also called unbalance thermal rotating) described above are mainly observed if the warm-up is not too strong and if the old contact point can be cool. [6]


Fig 2.6 Friction-heating phenomena(rotor / fixed parts) [6]

(a) représentation polaire (plan de Nyquist)

(b) amplitude de vibration en fonction du temps

Fig..2.7:"Soft" friction on lubricated joints seen by monitoring systems[6]

Example: on large machines ( 80 cm diameter rotor for example), the observed spiral will go around in a few hours. On smaller machines, cyclical evolution can be a lot faster; cycle in 10 to 15 minutes (trees 20 cm in diameter for example), or even a few seconds on some tree's millimeters in diameter. In the case of large thermal exchanges and particularly in the case of metal-to-metal friction, the of phenomena has a different pace and the beginning of the spiral is alone observed because the vibratory level quickly reaches the values alarm or shutdown of the machine.


Fig2.8 -exponential evolutions of vibrations duehard friction (metal/metal) [6]


Fig 2.9-Asymmetrical shaft: angular variation of stiffness K[6]

Example: on the turbines, a friction at the level of the labyrinthswill result in an exponential change in vibration(often called vibratory crisis) illustratedfigure 2.8. The duration of the evolution $\tau$ will be in the order of 10 to 15 minutes before the machine is forced to stop. If friction persists, it breaks the machine which then tends to slow down faster.

### 2.3.3.5 Asymmetrical rotors. Cracks

### 2.3.3.5.1 Dissymmetric trees

The behaviour of a tree with a stiffness dissymmetry for example, due to the presence of notches or winding (alternators, motors, and keypad) is particular.

When the rotor turns, the own weight forces are taken over by the stiffness of the tree, but the position of the center of the tree will be the same higher than stiffness will be important.

However, the stiffness varies over time. Figure 7 show that when the tree makes a turn, the stiffness varies twice per turn. The dissymmetric rotors create forces (thus vibrations) at twice the frequency of rotation ( 2 fr ) most often (blades, cordons, keys, etc.).

### 2.3.3.5.2 Transverse Shaft Cracking

Although this defect is infrequent, its consequences maybe important for safety. In this case, it is important to detectas soon as possible.A cracked tree has a stiffness that varies with directionof strength, essentially own weight, and supportive reactions (cf. Figure 2.10). There is therefore a certain analogy with behaviourof the dissymmetric rotor. But this time, the shaft arrow will be different depending on whether the crack is in the high position (compressed fibre,closed crack) or in the low position (tensioned fibre, crack open). The same movement reproduces at each turn of the tree, creating a periodic movement.


Fig.2.10.-Cracked shaft: angular variation of stiffness [6]
It is this property that is used to detect the appearance of a crack. Stiffness is a periodic function of time and the observed vibrations contain harmonics of rotation speed.

It is difficult, in normal operation, to separate what, in the vibration at frequencies $2 f \mathrm{r}$ and 3 fr , is due to a crack or a normal dissymmetry, from what comes from a machining fault of the tourbillions, or the non-linearity of the oil film.

To separate in the response of the tree what comes from the dissymmetry of the tree, we can look at what happens during the transients of speed. When the machine rotates at half the critical speed ( $\omega \mathrm{c}$ / 2)., the shaft is then excited at its critical speed by the effect of variable stiffness at twice the rotation frequency $(2 \omega c / 2=\omega c)$.

Note: A critical rotor speed is a rotational speed at which rotor vibrations pass through a maximum. It most often corresponds to a specific frequency of the tree.

The same applies when the machine is running at, or. Monitoring of a functioning rotor cracking will therefore include two flaps:

- Monitoring in operation. Ensure that the vibration measured at a point does not change too much over time, for example by following the deviation vector (current vibration less reference vibration), and by ensuring that its module remains below a limit value;
- speed transient monitoring. Ensure that no vibration peak appears at half (or one-third) of the critical speed.


### 2.3.3.6 Couplings

Couplings are components intended to connect two or more rotors. They must transmit torque. They may also allow for axial expansion of the machine, or radial movement. Only a few defects likely to disrupt their functioning will be mentioned.

### 2.3.3.6.1 Flatbed couplings

The defects of this type of coupling are mainly:

- Poor centring of the trays;
- A parallelism defect (not vertical of one of the trays to the rotation axis).

These two defects create a bump and therefore vibrations at the rotation frequency fr .

We can also have slides of the trays during operation. Their effect will be a brutal evolution of vibrations to fr (cf. § 2.1.3.3).

### 2.3.3.6.2 Cardin or double carding type couplings

A carding (Figure 2.11) is a joint designed to handle movement important relative axes of machine rotation trained and driving. It behaves like a dissymmetric tree, and as such, it will generate efforts at frequency $2 f$ r.

### 2.3.3.6.3 Finger Couplings

Finger couplings are organs that allow a axial and radial deformation due to the flexibility of the fingers. Rather, they are aimed at low efforts and the relative centring of shaft can evolve with the transmitted torque. So we can observe the evolution of the unbalance (vibration at frequency $f \mathrm{r}$ ) according to the transmitted couple.

### 2.3.3.6.4 Gear couplings

This type of coupling (cf. Figure 2.12) is often used to allow large axial movements between the driving machine and the driven machine or a large axial expansion of the rotors or long shafts with significant temperature variations.

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$$

The first defect of these couplings comes from a bad slip that prohibits dilatation. The tree upset in its expansion will flex (Figure 2.13) and its unbalance will evolve with the expansion of the tree. This type of defect can sometimes be highlighted by removing the engine torque to allow the toothing to slide. Vibrations will disappear by stopping the engine for a few seconds and restarting it.Other more complex phenomena can be observed:

- Toothing defects (as on the gears, with the same symptoms);
- Instabilities of centrifuged lubricant blades if thick.


### 2.3.3.7 Reduction gears and multipliers

### 2.3.3.7.1 Toothing noise

Typical gearing incidents (Figure 2.14) are related to tooth degradation (broken or damaged toothing, uniform or non-uniform wear, pitting (pitting, peeling) localized or distributed, poor centring). Fretting (corrosion under friction) can also be observed, which results in metal removal when the gear is poorly lubricated or when the forces are high.

The vibrations of the gears are dominated by stress at each contact. It is therefore in the frequencies $n f \mathrm{r}$ ( n whole) that the information is contained, especially if there is too much play, or on the contrary a too tight assembly.


Fig. 2.11-Cardan type coupling, or double carding [6]


Fig. 2.12- gear coupling [6]

Localized defects (defect of one tooth) also result in an impulse each time the damaged tooth is in contact with another. There is thus an appearance of a line at the frequency of contacts feaccording to the wheel concerned.


Fig 2.13-Tree sagging due to poor slip of teeth [6]


Fig. 2.14-Vibration of a gear [6]

### 2.3.3.7.2 Incorrect Centring

One can observe a modulation of this effort if the wheels are not well centred. This modulation results in the appearance in the vibratory spectrum of parallel lines around the toothing frequency.

The vibrations of a gearbox or a multiplier are a function of torque and radial forces.

### 2.3.4.1 Bearing Defects

Bearings (Figure 2.15) are among the most frequently used machine components and are a frequent source of failure.

The defects that can be encountered are the following:Slipping, gripping, corrosion (which causes slipping), false effect Brunel, and so on.

All these defects have one thing in common: they result sooner or later in the loss of metal fragments. This precursor to destruction is flaking. It results in repeated impacts of the balls on the bearing cage.

Many devices allow a good detection of bearing anomalies. Their goal is to detect repeated shocks as soon as possible. However, at the beginning of the flaking, the shock, of short duration, does not change not the average energy of the system. So it doesn't show up if you study the level of
vibration. To improve the detection, one realizes filtering by the vibration sensor. An accelerometer is used for this purpose broadband bandwidth, with low resonance, which, excited by the shocks, will respond to its resonance and play the part selective filter (see Figure 2.15). It lets in shocks, not noise background. This filtering ensures better fault discrimination. It just measures the level of the output signal, which, in the absence of shock, is low and increases very quickly, in case of defect. This type of defects is therefore characterized by an increase in the value effective signal and peak factor.


Fig. 2.15-bearing [6]

The basis for detecting the bearing defect is therefore to detect the increase in signal energy, or rather, if we want to have an early detection, of what in the energy indicates the presence of small repeated shocks.

The techniques are many, and consist in analyzing the signal provided by an accelerometer. A number of descriptors are available:

- Effective value of acceleration: $\gamma$ eff.
- peak value: $\gamma_{c}$; or peak to peak $\gamma_{c c}$
- peak factor: . : $\gamma_{\text {eff }} \gamma_{\mathrm{cc}}$
- Or mixing of these values: $\frac{y c c}{\text { yeff }+\varepsilon \gamma \mathrm{eff}}$


Fig.2.16-Detection and location of a bearing defect [6]


Fig. 2.17- Example of envelope spectrum [6]

Other quantities may be added to these basic quantities more complex, considered by some authors as more representative of this type of defect. One of them is flattening (Kurtosis): this size is close to a factor of form.

We can also look at the spectrum of the signal envelope if the defect is to be located (see Figure 2.17). It is calculated from the demodulated and filtered signal and contains the frequencies defect characteristics. Indeed, the characteristic frequencies of the defects localized on the parts of a bearing are as follows ( Figure 2.17):

- For the cage: $\quad f_{d c a}=f_{a}(1-(d / D) \cos \phi)$
- The outer ring: $F_{d b e}=N f_{d c a}$
- The inner ring: $F_{d b}=N_{i}\left(f_{a}-f_{d c a}\right)$


#### Abstract

$f_{\mathrm{a}}$ shaft rotation frequency, N number of balls, d diameter of a ball, D average bearing diameter, $\varphi$ Contact angle.


This different bearing technology characteristic information can be provided by the manufacturer.

### 2.3.4.2 Oil Film Bearing Lubrication Defects-Instability

Defects encountered on fluid bearings are primarily due to Tampering with pads, bumpers or bristles. Bad lineage, high vibrations, and especially an interruption of lubrication or a lack of lifting are some possible causes of destruction of the regulation (antifriction alloy based on lead or tin). A potential difference between the rotor and bearing can induce pitting or pitting (especially on alternators).
The presence of particles in the oil can cause scratches in the regulation.
Another lubrication problem is that of self-excited vibrations that appear in a tier when certain conditions are filled. A bearing that is too lightly loaded, or has radial play too strong, may become unstable. Indeed, it is interesting to note the behaviour of a bearing is highly non-linear. The results of calculation of stiffness and damping coefficients of a bearing, depending on the position of the shaft in the bearing show that, if the tree is insufficiently loaded, that is to say if the point of operation of the bearing imposes too weak eccentricity, stiffness can even become negative. In this case, the bearing is unstable and generates self-arousing vibrations.

Violent vibrations then occur at a similar frequency half of that of the rotation. The evolution is then often fast and the level is not stable.

The dominant frequency of vibration is at the half-frequency of rotation. It can be close to $1 / 3$ or $1 / 4$ of the rotation if a critical rotor speed synchronizes the phenomenon.
It is assumed that a critical speed between 0.3 and 0.7 can impose its frequency on oil film instability.

For example, in Figure 2.18a, the vibration spectrum is recognized of a tree supported by a bearing at the limit of stability. There is energy in an area close to half the rotation speed but it is not organized; on figure 2.18 b , all the energy is contained in a single frequency line. Amplitude increases.

The organization can come from the rotation of the machine, and in this case the frequency will be about half that of the rotation.

It may also be due to the presence of a specific rotor (or bearing) frequency in the vicinity of $f_{\mathrm{r} 2}$ $l f_{\mathrm{r} 3} / f_{\mathrm{r} 4} \ldots . . .$. and in this case, the observed frequency is that specific frequency.

The instabilities are generally corrected by a resumption of lineage, a reduction of backlashes (for example, the «localization» or «lemonade» of the circular bearing) or geometric modifications of the pad.

The behaviour of the machine depends to a large extent on the type of bearing:

- circular bearing: good load capacity, low stability;
- oval or lemon bearing: good vertical load capacity, lowhorizontal stiffness, stability better than the
circular bearingbut not very good;
- lobe bearing: good stability, more limited to overload resistanceexceptional;
- Skid bearing: good stability, significant warm-up, pivotsflexible.


### 2.3.4.3 Degradation of Lineage

Although referred to by the same word, the effects of a loosening will be very different depending on whether it is the assembly of the rotors or the position of the supports. In fact, aligning the rotors before assembling them consists of setting the coupling plates parallel and concentric (Figure 2.19), thus aligning the rotation axes. These criteria must be met in both the horizontal and vertical planes.

Two conditions are met:

- The rotation axes are confused, so no unbalance (cf. Figure 2.6);


Fig.2.18- Vibration spectrum of a bearing [6]


Fig. 2.19- Rotor Alignment Defects [6]

- The reactions of the supporters are those that correspond to the own weight; each bearing the intended portion of the weight of rotors.

If the first condition is not met, we create a lump.

If the positions of the supporters are not what they should be, then we have a designate. It can have a double effect:
loosening introduces alternate forces into the tree because the shaft rotates;

- It changes the stiffness of supports and therefore the responses machine dynamics at these forces, when they are not linear, or when the rotors are not symmetrical.

Another consequence of the release, linked to the non-linear behaviour of the bearings, is to modify the distribution of the harmonic vibrations of the rotation frequency. This effect of evolution of the harmonic 2 can also be observed in the case where one has a dissymmetric rotor or a coupling that does not present rotation symmetry (blades, cardoons), because internal forces will be created by the rotation of the machine at twice its rotation frequency. Finally, there is often a link between designate, instability and friction of the rotating parts on the fixed parts.

The breakdown of the lineage results from deformations of the parts fixed, body, solid, beam or bearing support due to:

- Thermal effects, for example: beam deformations during start-up on certain machines, unplanned plugging ventilation orifices, dilated stators;
- Mechanical effects. Although mechanical effects are less frequent than thermal causes, it was possible to highlight the influence of static forces on structures as per example of vacuum or water level in turbine condensers steam-driven, compaction of beams or cracking of beams or slabs;
- Of external effects. Indeed, the vibrations of some machines could be correlated with the external climatic conditions, and in particular with the sun, the water table level, or even with the tides. [6]


### 2.3.4.4 Slack, loosening, incorrect fastening

This class of defects concerns the attachments of their supports or massifs. If the installation is defective, we can observe different phenomena. If there is play, the operation will not be linear, and the movement sinusoidal vibratory will transform into a movement even richer in harmonics as the signal will be distorted.

If the fixation has insufficient stiffness it is by measurements of distorted that it can be highlighted. The weak points of a structure appear as singularities of form (Excessive local deformation,

$$
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$$

behavioral dissymmetry vibratory). In the same way we will be able to highlight resonances of structures, or changes in time of anchors by an evolution of deformed. [6]

## Chapter3: dynamic equationsrotor elements characteristic

## Chapter 03 Characteristic of Rotor elements

### 3.1 Introduction

The rotating machinery are a very sensitive part in the industry which needs full time observation study and maintenance, to do this and study the behaviour of the rotor one must know all of the rotor proprieties and its elements characteristic to define the needed equations and determine the final rotor behaviour .

In this chapter we are going to study the characteristic of all rotor components and its relation with the dynamics of the rotor and prove step by step the method to study its dynamics and plot the Campbell diagram ,define the critical speed frequencies.

### 3.2 Determination the Characteristic of Rotor elements

The basic elements of a rotor are the disk, the shaft, the bearings and the seals .The mass unbalances which cannot be completely avoided must also be considered. Kinetic energy expressions are necessary to characterize the disk, shaft and mass unbalances. Strain energy is necessary to characterize the shaft. The forces due to bearings or seals are used to calculate their virtual work, and then the corresponding forces acting on the shaft are obtained. The general rotor equations are provided by means of the following steps:

- The kinetic energy $T$, the strain energy $U$ and the virtual work $8 W$ of external forces are calculated for the elements of the system.
- A numerical method is chosen: The Rayleigh-Ritz method for a very small number of degrees of freedom, and the finite element method for engineering applications.
- Lagrange's equations are applied in the following form:

$$
\begin{equation*}
\frac{d y}{d t}\left(\frac{\partial T}{\partial q_{i}}\right)-\left(\frac{\partial T}{\partial q}\right)+\left(\frac{\partial U}{\partial q_{i}}\right)=F q_{i} \tag{3.1}
\end{equation*}
$$

where $N(1>i>N)$ is the number of degrees of freedom, $q$; are generalized independent coordinates, $F q$; are generalized forces, and denotes differentiation with respect to time $t$.[1]
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### 3.2.1 Disc

The disk is assumed to be rigid and is then characterized solely by its kinetic energy. $R O$ (XYZ) is an inertial frame and $R(x, y, z)$ is fixed to the disk (Figure 1.1). The $x y z$ coordinate system is related to the $X Y Z$ coordinate system through a set of three angles $\Psi, \theta$ and $\Phi$. To achieve the orientation of the disk one first rotates it by an amount $\Psi$ around the Z axis; then by an amount $\theta$ around the new $x$ axis, denoted by $\mathrm{x}_{1}$; and lastly by an amount $\Phi$ around the final y axis. The instantaneous angular velocity vector of the $x y z$ frame is

$$
\begin{align*}
& \vec{\Omega}=\dot{\Psi} Z+\dot{\theta} x_{1}+\dot{\phi} Y  \tag{3.2}\\
& \left\{\begin{array}{c}
\vec{X}_{1}=\cos \Psi \vec{X}+\sin \Psi \vec{Y}+0 \vec{z} \\
\vec{y}_{1}=-\sin \Psi \vec{X} \cos \Psi \vec{Y}+0 \vec{z} \\
\vec{Z}_{1}=O \vec{x}+O \vec{Y}+\vec{Z}
\end{array}\right. \tag{3.3}
\end{align*}
$$



Fig 3.1 Axles rotation by $\psi$

$$
\left\{\begin{array}{c}
\vec{x}_{2}=\vec{x}_{1}  \tag{3.4}\\
\vec{y}_{2}=0 \vec{x}_{1}+\cos \theta \vec{y}_{1}+\sin \theta \vec{z}_{1} \\
\vec{z}_{2}=0 \vec{x}_{1}-\sin \theta \vec{y}_{1}+\cos \theta \vec{z}_{1}
\end{array}\right.
$$



Fig 3.2 Axles rotation by $\theta$
$\left\{\begin{array}{c}\vec{x}=\cos \phi \vec{x}_{2}+0 \vec{y}_{2}+\sin \phi \vec{z}_{2} \\ \vec{y}=0 \vec{x}_{2}+\vec{y}_{2}+0 \vec{z}_{2} \\ \vec{z}=-\sin \psi \vec{x}_{2}+0 \vec{y}_{2}+\cos \phi \vec{z}_{2}\end{array}\right.$


Fig 3.2 Axles rotation by ${ }^{\phi}$
with
$\vec{\Omega}=\dot{\Psi} \vec{z}+\dot{\theta} \vec{x}_{1}+\dot{\phi} \vec{y}$
$\vec{\Omega}=\dot{\Psi}\left(\sin \theta \vec{y}_{2}+\cos \theta \vec{z}_{2}\right)+\dot{\theta} \vec{x}_{1}+\dot{\phi} \vec{y}$
$\vec{\Omega}=\dot{\Psi}(\sin \theta \vec{y}+\cos \theta(\sin \phi \vec{x}+\cos \phi \vec{y}))+\dot{\theta}(\cos \phi \vec{x}-\sin \phi \vec{z})+\dot{\phi} \vec{y}$
Where $Z, x_{1}$ and $y$ are unit vectors along the axes $Z, x_{1}$ and $y$. The kinetic energy of the disk about its center of mass $O$ is calculated using the frame $R$. In this system the angular velocity vector becomes

$$
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$$

$\vec{\Omega}_{\frac{R}{R_{0}}}=\left[\begin{array}{l}w_{x} \\ w_{y} \\ w_{z}\end{array}\right]=\left[\begin{array}{c}\dot{\Psi} \cos \theta \sin \dot{\phi}+\dot{\theta} \cos \phi \\ \dot{+} \dot{\Psi} \sin \theta \\ \dot{\Psi} \cos \theta \cos \phi+\dot{\theta} \sin \phi\end{array}\right]$


Fig 3.3. Reference frames for a disk on a rotating flexible shaft[1]

Let $m_{d}$ be the mass of the disc and

$$
I_{D_{0}}=\left[\begin{array}{clc}
I_{d x} & 0 & 0 \\
0 & I_{d y} & 0 \\
0 & 0 & I_{d z}
\end{array}\right]
$$

$$
I_{d x}=I_{d z}
$$

The energy of the disc is
$T_{d}=\frac{1}{2} M_{d}\left(u^{2}+w^{2}\right)+\frac{1}{2}\left(I_{d x} w^{2}{ }_{x}+I_{d y} w^{2}{ }_{y}+I_{d z} w^{2}{ }_{y}\right)$
$T_{d}=\frac{1}{2} M_{d}\left(\dot{u}^{2}+\dot{w}^{2}\right)+\frac{1}{2}\left(I_{d x} w^{2}{ }_{x}+I_{d y} w^{2}{ }_{y}+I_{d z} w^{2}{ }_{y}\right)$
$=\frac{1}{2} M_{d}\left(\dot{u}^{2}+\dot{w}^{2}\right)+$
$\frac{1}{2} I_{d x}\left(\Psi^{2} \cos ^{2} \theta \sin ^{2} \phi+\dot{\theta^{2}} \cos ^{2} \dot{\phi}-2 \dot{\theta} \dot{\Psi} \cos \theta \sin \phi \cos \phi+\dot{\Psi}^{2} \cos ^{2} \theta \cos ^{2} \phi+\theta^{2} \sin ^{2} \theta+\right.$ $2 \dot{\theta} \Psi \cos \theta \sin \phi \cos \phi)+I_{d y}\left(\dot{\phi}^{2}+\dot{\Psi}^{2} \sin ^{2} \theta+2 \dot{\theta} \dot{\Psi} \sin \theta\right)$

So :

$$
T_{d}=\frac{1}{2} M_{d}\left(\dot{u}^{2}+\dot{w}^{2}\right)+\frac{1}{2} I_{d x}\left(\dot{\Psi}+\dot{\theta}^{2}\right)+\frac{1}{2} I_{d y}\left(\dot{\phi}^{2}+2 \dot{\phi} \dot{\Psi} \theta\right)
$$

$\dot{\mathscr{I}}=\Omega$ is the rotation speed de rotation du rotor and it's constant so $\frac{1}{2} I_{d y} \phi^{2}$ Is a constantand
$\dot{\mathscr{I}}$ and does not relate to the Lagrange, then energy of disc is
$T_{d}=\frac{1}{2} M_{d}\left(\dot{u}^{2}+\dot{w}^{2}\right)+\frac{1}{2} I_{d x}\left(\dot{\Psi}+\dot{\theta}^{2}\right)+\frac{1}{2} I_{d y} 2 \dot{\Omega} \Psi \dot{t}$

### 3.2.2 The Shaft

The shaft is represented as a beam with a circular cross-section, and is characterized by strain and kinetic energies.

### 3.2.2.1 Kinetic energy

The general formulation of the kinetic energy of the shaft comes from an extension of the disk equation (1.6). For an element of length $L$, the expression for the kinetic energy is

We have
$\left\{\begin{array}{l}I_{d x}=\int\left(y^{2}+z^{2}\right) d m \\ I_{d y}=\int\left(x^{2}+z^{2}\right) d m \\ I_{d w}=\int\left(x^{2}+y^{2}\right) d m\end{array}\right.$
For the Disc

$$
I_{d x}=I_{d z}
$$

The mass of the Disc is

$$
m_{d}=\rho_{d} v_{d}
$$

The mass of the shaft is

$$
d m_{a}=\rho d v=p . s . d_{y}
$$

With $s$ is the air of the right section of the Shaft
$S=\pi \cdot r^{2}$
The general formulation of the kinetic energy of the shaft comes from an extension of the disk equation (1.6). For an element of length $L$, the expression for the kinetic energy is

We have

$$
\begin{align*}
& T_{s=} \frac{1}{2} \int d m_{a}\left(\dot{u}^{2}+\dot{w}^{2}\right)+\int\left(x^{2}+y^{2}\right) d m_{a}\left(\dot{\Psi}^{2}+\dot{\theta}^{2}\right)+\frac{1}{2} \int\left(x^{2}+z^{2}\right) d m_{a}^{2} \\
& \quad+\int\left(x^{2}+z^{2}\right) \Omega \Psi \theta d \dot{m}_{a} \\
& d m_{a}=\text { p.s. } d_{y} \\
& T_{s}=\frac{\rho . s}{2} \int_{0}^{l} d y\left(\dot{u}^{2}+\dot{w}^{2}\right)+\frac{\rho I}{2} \int_{0}^{l} d y\left(\dot{\Psi}^{2}+\dot{\theta}^{2}\right)+\rho I L \Omega^{2}+\rho I \Omega \int_{0}^{L} \Psi \dot{\theta} d y \tag{3.11}
\end{align*}
$$

where $p$ is the mass per unit volume, $S$ is the cross-sectional area of the beam, supposed to be constant, and $I$ is the area moment of inertia of the beam cross-section about the neutral axis, also supposed to be constant.

The first integral of (3.6) is the classical expression for the kinetic energy of a beam in bending; the second integral is the secondary effect of rotator inertia (Timoshenko beam); the term $\rho I L \Omega^{2}$ (3.9) is a constant and has no influence on the equations; and the last integral represents the gyroscopic effect.

### 3.2.2.1 The Shaft deformation energy

The following notation is used (refer to Figure 1.2):
C is the geometric centre of the beam, $\mathrm{B}(\mathrm{x}, z)$ is a typical point on the cross-section, E is the Young's modulus of the material, $\varepsilon$ and $\sigma$ are strains and stresses, and $u^{*}$ and $\mathrm{w}^{*}$ are displacements of the geometric centre with respect to the $\mathrm{x}, \mathrm{z}$ axis.

If second-order terms are included, the longitudinal strain of point B can be shown to be

$$
\begin{equation*}
\frac{\partial^{2} u^{*}}{\partial y^{2}}-z \frac{\partial^{2} w^{*}}{\partial y^{2}}+\frac{1}{2}\left(\frac{\partial u^{*}}{\partial y}\right)^{2}+\frac{1}{2}\left(\frac{\partial w^{*}}{\partial y}\right)^{2} \tag{3.12}
\end{equation*}
$$

With

$$
x \varepsilon_{L}=\quad \frac{\partial^{2} u^{*}}{\partial y^{2}}-z \frac{\partial^{2} w^{*}}{\partial y^{2}}
$$

And

$$
\varepsilon_{n l}=\frac{1}{2}\left(\frac{\partial u^{*}}{\partial y}\right)^{2}+\frac{1}{2}\left(\frac{\partial w^{*}}{\partial y}\right)^{2}
$$

$$
\varepsilon=\varepsilon_{L}+\varepsilon_{n L}
$$



Fig 3.4 Coordinates ot the geometric center $C$ and an arbitrary pclint $B$ t)n the shat $\cdot t[1]$
$\varepsilon_{\llcorner }$contains the linear terms and $\varepsilon_{\mathrm{nL}}$ the nonlinear terms.

The strain energy is
$U_{a}=\frac{1}{2} \int \varepsilon^{t} \sigma d v$
where " T " is the matrix transposition symbol. The relationship between stresses and strains is
$U_{a}=\frac{E}{2} \int \varepsilon^{t} \varepsilon d v=\frac{E}{2} \int\left(\varepsilon_{L}^{2}+\varepsilon_{n L}^{2}+2 \varepsilon_{L} \varepsilon_{n L}\right) d v$
The symmetry of the beam cross-section with respect to .t and $z$ results in $\varepsilon_{L} \varepsilon_{n L} d v=0$ In the case of deformation linear $\varepsilon_{n L} \cong 0$

$$
\begin{gathered}
U_{a}=\frac{E}{2} \int_{0}^{L} \int\left(\mathrm{x} \frac{\partial^{2} u^{*}}{\partial y^{2}}-z \frac{\partial^{2} w^{*}}{\partial y^{2}}\right)^{2} d s \cdot d y \\
U_{a}=\frac{E}{2} \int_{0}^{L} \int\left[\mathrm{x}^{2}\left(\frac{\partial^{2} u^{*}}{\partial y^{2}}\right)^{2}+z^{2}\left(\frac{\partial^{2} w^{*}}{\partial y^{2}}\right)^{2}-2 x z \cdot \frac{\partial^{2} u^{*}}{\partial y^{2}} \cdot \frac{\partial^{2} w^{*}}{\partial y^{2}}\right] \mathrm{ds} . \mathrm{dy} \\
2 x z \cdot \frac{\partial^{2} u^{*}}{\partial y^{2}} \cdot \frac{\partial^{2} w^{*}}{\partial y^{2}}=0
\end{gathered}
$$

due to rotor system
$. U_{a}=\frac{E}{2} \int_{0}^{L} \int\left[\mathrm{X}^{2}\left(\frac{\partial^{2} u^{*}}{\partial y^{2}}\right)^{2}+z^{2}\left(\frac{\partial^{2} w^{*}}{\partial y^{2}}\right)^{2}\right] \mathrm{ds} . \mathrm{dy}$
.$\left\{I_{X}=\int Z^{2} d s \quad I_{z}=\int x^{2} d s\right\}$
quadratic torque of the section of the shaft
$U_{a}=\frac{E}{2} \int_{0}^{L} \int\left[\mathrm{I}_{\mathrm{Z}}\left(\frac{\partial^{2} u^{*}}{\partial y^{2}}\right)^{2}+\mathrm{I}_{\mathrm{X}}\left(\frac{\partial^{2} w^{*}}{\partial y^{2}}\right)^{2}\right] \mathrm{dy}$
After the projection at $(0, x, y)$
$\left\{\begin{array}{c}u^{*}=u \cdot \cos \Omega t \\ w^{*}=u \cdot \sin \Omega t\end{array}\right.$
We replace $u^{*}$ and $w^{*}$ en fonction of u and w at $U_{a}$ we have
$-\frac{\partial^{2} u^{*}}{\partial y^{2}}=\frac{\partial^{2} u^{*}}{\partial y^{2}} \cdot \cos \Omega t-\frac{\partial^{2} w}{\partial y^{2}} \cdot \sin \Omega t$
$\frac{\partial^{2} w^{*}}{\partial y^{2}}=\frac{\partial^{2} u}{\partial y^{2}} \cdot \sin \Omega t+\frac{\partial^{2} w}{\partial y^{2}} \cdot \cos \Omega t$.
$U_{a}=\frac{E}{2} \int_{0}^{L}\left[I_{z}\left(\frac{\partial^{2} u}{\partial y^{2}} \cdot \cos \Omega t-\frac{\partial^{2} w}{\partial y^{2}} \cdot \sin \Omega t\right)^{2}+I_{x}\left(\frac{\partial^{2} u}{\partial y^{2}} \cdot \sin \Omega t-\frac{\partial^{2} w}{\partial y^{2}} \cdot \cos \quad \Omega t\right)^{2}\right] d y$

$$
\left\{\begin{array}{c}
I_{x}=I_{z}=I \\
\cos ^{2} \Omega t+\sin ^{2} \Omega t=1
\end{array}\right.
$$

(Symétric of the rotor), After simplifying the equation we get

$$
\begin{equation*}
U_{a}=\frac{E}{2} \int_{0}^{L}\left[\left(\frac{\partial^{2} u}{\partial y^{2}}\right)^{2}+\left(\frac{\partial^{2} u}{\partial y^{2}}\right)^{2}\right] d y \tag{3.13}
\end{equation*}
$$

### 3.2.3 The Bearings

### 3.2.3.1 Generalized forces

The stiffness and viscous damping terms are assumed to be known, and the bending influence can in general be neglected (Figure 1.3). The virtual work $8 W$ of the forces acting on the shaft can be written as

$$
\begin{aligned}
& \delta w=-k_{x x} u^{*} \delta u^{*}-k_{x z} w^{*} \delta u^{*}-k_{z z} w^{*} \delta u^{*}-k_{z x} w^{*} \delta u^{*}-C_{x x} \dot{u}^{*} \delta u^{*}-C_{x z} \dot{u}^{*} \delta u^{*}- \\
& C_{x x} \dot{w}^{*} \delta w^{*}-C_{z x} \dot{w}^{*} \delta w^{*} \\
& \quad=\quad F_{u^{*}} \delta u^{*}+F_{w^{*}} \delta u^{*} \\
& =\quad-\left(k_{x x} u^{*}+k_{x z} w^{*}+C_{x x} \dot{u}^{*}+C_{x z} \dot{u}^{*}\right) \delta u^{*}-\left(k_{z z} w^{*}+k_{z x} w^{*}+C_{x x} \dot{w}^{*}+C_{z x} \dot{w}^{*}\right) \delta w^{*} \quad \text { On }
\end{aligned}
$$

$$
\left\{\begin{array}{c}
F_{u^{*}} \\
F_{w^{*}}
\end{array}\right\}=-\left[\begin{array}{cc}
k_{x x} & k_{x z} \\
k_{z x} & k_{z z}
\end{array}\right]\left\{\begin{array}{c}
u^{*} \\
w^{*}
\end{array}\right\}-\left[\begin{array}{cc}
C_{x x} & C_{x z} \\
C_{z x} & C_{z z}
\end{array}\right]\left\{\begin{array}{c}
\dot{u}^{*} \\
\dot{w}^{*}
\end{array}\right\}
$$



Fig.3.5. Bearing stiffness and damping [1]
We suppose that $k_{x x} \pm 0$ and $k_{z z} \pm 0$

$$
\begin{aligned}
& \delta w=k_{x x} u^{*} \delta u^{*}-k_{z z} w^{*} \delta w^{*} \\
& \left\{\begin{array}{c}
u^{*}=u \cdot \cos \Omega t-w \cdot \sin \Omega t \\
w^{*}=u \cdot \sin \Omega t-w \cdot \cos \Omega t
\end{array}\right.
\end{aligned}
$$

We replace $u^{*}$ and $w^{*}$ at $\delta w$ then we ge

$$
\begin{aligned}
& \delta w=-k_{x x}(u \cdot \cos \Omega t-w \cdot \sin \Omega t)(\delta u \cdot \cos \Omega t-\delta w \cdot \sin \Omega t)- \\
& k_{z z}(u \cdot \sin \Omega t+w \cdot \cos \Omega t)(\delta u \cdot \sin \Omega t+\delta w \cdot \cos \Omega t) \\
& \quad \delta W=-\left(k_{x x} u \cdot \cos ^{2} \Omega t+k_{z z} u \cdot \sin ^{2} \Omega t-k_{x x} w \cdot \cos \Omega t \cdot \sin \Omega t+k_{z z} w \cdot \cos \Omega t \cdot \sin \Omega t\right) \delta u- \\
& \left(k_{x x} w \cdot \sin ^{2} \Omega t+k_{z z} w \cdot \cos ^{2} \Omega t+k_{x x} u \cdot \cos \Omega t \cdot \sin \Omega t+k_{z z} u \cdot \cos \Omega t \cdot \sin \Omega t\right) \delta w
\end{aligned}
$$

Then
$\delta w=-\left[\left(k_{x x} \cos ^{2} \Omega t+k_{z z} w \sin ^{2} \Omega t\right) u-\frac{\sin 2 \Omega t}{2}\left(k_{x x}-k_{z z}\right) w\right] \delta u-\left[\left(k_{x x} \sin ^{2} \Omega t+\right.\right.$ $\left.\left.k_{z z} \cos \Omega t^{2}\right) w-\frac{\sin 2 \Omega t}{2}\left(k_{x x}-k_{z z}\right) u\right] \delta w$

### 3.2.4 Unbalance of mass

W have $C D=d$

The unbalance is due to a mass $m_{b}$
We named the distance d from the centre C


Fig 3.6 Mass unbalance[1]

$$
\begin{aligned}
\overrightarrow{O D} & =\left\{\begin{array}{c}
u+d \cdot \sin \Omega t \\
c t e \\
w+d \cdot \cos \Omega t
\end{array}\right. \\
\vec{v} & =\frac{\overrightarrow{d o D}}{d t}=\left\{\begin{array}{c}
u \dot{+} d \Omega \cos \Omega t \\
0 \\
\dot{w}-d \Omega \sin \Omega t
\end{array}\right.
\end{aligned}
$$

The kinetic energy of unbalance is note

$$
\begin{aligned}
T_{b} & =\frac{1}{2} m_{b} v^{2}=\frac{1}{2} m_{b}\left[(\dot{u} \cdot \mathrm{~d} \Omega \cos \Omega t)^{2}(\dot{w} \cdot \mathrm{~d} \Omega \sin \Omega t)^{2}\right] \\
& =T_{b}=\frac{1}{2} m_{b}\left[\dot{u}^{2}+\dot{w}^{2}+d^{2} \Omega^{2}+2 d \Omega(\dot{u} \cdot \cos \Omega t-\dot{w} \cdot \sin \Omega t)\right]
\end{aligned}
$$

${ }^{\frac{1}{2}} m_{b} d^{2} \Omega^{2}$ is a constant due time. so we don't consider it in the equation of Lagrange.
*we neglect $\frac{1}{2} m_{b}\left(\dot{u}^{2}+\dot{w}^{2}\right)$ because the mass of unbalance is small when compared with the mass of the rotor and $\dot{u}+\dot{w}$ is negligible compared to the rotation speed so
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$$
\begin{equation*}
T_{b}=m_{b} d \Omega(\dot{u} \cdot \cos \Omega t-w \cdot \sin \Omega t) \tag{3.15}
\end{equation*}
$$

### 3.3 Simple Model of mono-rotor



Fig 3.7. MI)del of the rotor[1]
This is the model depicts on the figure.
The rotor axis is along the $y$ axis and the rotation speed is constant
Only one degree of freedom is used for each movement in the $x$ and $z$ directions
The expression of displacement in the $x$ and $z$ direction is

$$
\left\{\begin{array}{l}
u(y \cdot t)=f(y) \cdot q_{1}(t) \\
w(y \cdot t)=f(y) \cdot q_{2}(t)
\end{array}\right.
$$

$q_{1}$ et $q_{2}$ cordonnées généralisés indépendants comme $\Psi$ et $\theta$ sont petits elles sont approximées par:


Fig. 3.8: Coordinates [1]
the rotation angle $c$ is derived from $u$ and $w$

$$
\begin{gather*}
\theta=\frac{\partial w}{\partial y}=\frac{d f(y)}{d y} q_{2}(t)  \tag{3.16}\\
\Psi=\frac{-\partial u}{\partial y}=\frac{-d f(y)}{d y} q_{2}(t)  \tag{3.17}\\
\\
\left\{\begin{array}{l}
\dot{u}(y \cdot t)=\frac{d u(y \cdot t)}{d t}=f(y) \cdot \dot{q}_{1} \\
\dot{w}(y \cdot t)=\frac{d w(y \cdot t)}{d t}=f(y) \cdot \dot{q}_{2}
\end{array}\right.
\end{gather*}
$$

## Determination $\theta$ of and $\Psi$

We have

$$
\begin{gathered}
\frac{d f}{d y}=g(y) \\
\dot{\theta}=\frac{d \theta}{d t}=\frac{d}{d t}\left(\frac{d f(y)}{d y} \cdot q_{2}(t)\right)=\frac{d f}{d y} \cdot \dot{q}_{2}=g(y) \cdot \dot{q}_{2}=g\left(l_{1}\right) \cdot \dot{q}_{2}
\end{gathered}
$$

$$
\dot{\Psi}=\frac{d \Psi}{d t}=-\frac{d}{d t}\left(\frac{d f(y)}{d y} \cdot q_{1}(t)\right)=\frac{-d f}{d y} \cdot \dot{q}_{1}=-g(y) \cdot \dot{q}_{1}=-g\left(l_{2}\right) \cdot \dot{q}_{1}
$$

replace these derivatives in the equation of kinetic energy of Disk we have
$T_{d}=\frac{1}{2} M_{d}\left(\dot{u}^{2}+\dot{w}^{2}\right)+\frac{1}{2} l_{d x}\left(\dot{\Psi}^{2}+\dot{\theta}^{2}\right)+l_{d y} \Omega \dot{\Psi} \theta+\frac{1}{2} l_{d y} \Omega^{2}$
The disc is at a distance

$$
\begin{gather*}
y=l_{1} \quad f(y)=f\left(l_{1}\right) \\
T_{d}=\frac{1}{2}\left[M_{d} f^{2}\left(l_{1}\right)+I_{d x} \cdot g^{2}\left(l_{1}\right)\right]\left(\dot{q}_{1}^{2}+\dot{q}_{2}^{2}\right)+I_{d y} \Omega g^{2}\left(l_{1}\right) \cdot \dot{q}_{1} \cdot q_{2} \tag{3.18}
\end{gather*}
$$

the kinetic energy of the tree is given by

$$
\begin{align*}
& T_{s}=\frac{\rho . s}{2} \int_{0}^{L} f^{2}(y) d y\left(\dot{q}_{1}^{2}+\dot{q}_{2}^{2}\right)+\frac{\rho \cdot L}{2} \int_{0}^{l} g^{2}(y) d y\left(\dot{q}_{1}^{2}+\dot{q}_{2}^{2}\right)+\rho I \Omega \int_{0}^{L} g^{2}(y) d y \cdot \dot{q}_{1} \cdot q_{2} \\
& T_{s}=\left[\frac{\rho . s}{2} \int_{0}^{L} f^{2}(y) d y+\frac{\rho . L}{2} \int_{0}^{l} g^{2}(y) d y\right]\left(\dot{q}_{1}^{2}+\dot{q}_{2}^{2}\right)+\left[\rho I \Omega \int_{0}^{L} g^{2}(y) d y\right] \dot{q}_{1} \cdot q_{2}  \tag{3.19}\\
& \quad T=T_{S}+T_{d=}\left[M_{d} f^{2}\left(l_{1}\right)+I_{d x} \cdot g^{2}\left(l_{1}\right)+\rho \cdot s \int_{0}^{L} g^{2}(y) d y\right]\left(\dot{q}_{1}^{2}+\dot{q}_{2}^{2}\right)- \\
& \Omega\left[I_{d y} \Omega g\left(l_{1}\right)+\rho I \int_{0}^{l} g^{2}(y) d y\right] \dot{q}_{1+} q_{2}
\end{align*}
$$

Can be written in compact form
$T=\frac{1}{2} m\left(\dot{q}_{1}^{2} \dot{q}_{2}^{2}\right)-\Omega a . \dot{q_{1}} \cdot \dot{q}_{2}$
Deformation energy of the Shaft:
We have

$$
U_{a}=\frac{E I}{2} \int_{0}^{L}\left[\left(\frac{\partial^{2} u}{\partial Y^{2}}\right)^{2}+\left(\frac{\partial^{2} u}{\partial Y^{2}}\right)^{2}\right] d y
$$

And

$$
\left\{\begin{array}{l}
u=f(y) \cdot q_{1}  \tag{3.21}\\
w=f(y) \cdot q_{2}
\end{array}\right.
$$

$$
\begin{equation*}
h(y)=\frac{\partial^{2} f(y)}{\partial y^{2}} \tag{3.22}
\end{equation*}
$$

in compact form

$$
\begin{equation*}
U_{a}=\frac{1}{2} k\left(q_{1}^{2}+q_{2}^{2}\right) \tag{3.23}
\end{equation*}
$$

with

$$
\begin{equation*}
k=E I \int_{0}^{L} h^{2}(y) d y \tag{3.24}
\end{equation*}
$$

## Generalized strength of bearings

The work of the external forces acting on the level is given by
$\delta w=-k_{x x} u\left(l_{2}\right) \delta u\left(l_{2}\right)-k_{x z} w\left(l_{2}\right) \delta u\left(l_{2}\right)-k_{z z} w\left(l_{2}\right) \delta u\left(l_{2}\right)-k_{z x} u\left(l_{2}\right) \delta w\left(l_{2}\right)-$ $C_{x x} \dot{u}\left(l_{2}\right) \delta u\left(l_{2}\right)-C_{x z} \dot{w}\left(l_{2}\right) \delta u\left(l_{2}\right)-C_{z z} \dot{w}\left(l_{2}\right) \delta w\left(l_{2}\right)-C_{z x} \dot{u}\left(l_{2}\right) \delta w\left(l_{2}\right)$
$u \cdot \delta u=f(y) \cdot q_{1} \cdot f(y) \cdot \delta q_{1}=f^{2}(y) \cdot q_{1} \cdot \delta q_{1}$
Donc
$\delta w=-k_{x x} f^{2}(y) \cdot q_{1} \cdot \delta q_{1}-k_{x z} f^{2}(y) \cdot q_{2} \cdot \delta q_{1}-k_{z z} f^{2}(y) \cdot q_{2} \cdot \delta q_{2}-k_{z x} f^{2}(y) \cdot q_{1} \cdot \delta q_{2}-$ $c_{x x} f^{2}(y) \cdot \dot{q}_{1} \cdot \delta q_{1}-c_{x z} f^{2}(y) \cdot \dot{q}_{2} \cdot \delta q_{1}-c_{z z} f^{2}(y) \cdot \dot{q}_{2} \cdot \delta q_{2-} c_{z x} f^{2}(y) \cdot \dot{q}_{1} \cdot \delta q_{2}$

Or Under forme

$$
\begin{align*}
& \delta w=F_{q_{1}} \delta q_{2}+F_{q_{2}} \delta q_{2}  \tag{3.27}\\
& \left\{\begin{array}{l}
\frac{d}{d t}\left(\frac{d T}{d \dot{q}_{1}}\right)-\frac{d U}{d q_{1}}=F_{q_{1}} \\
\frac{d}{d t}\left(\frac{d T}{d \dot{q}_{2}}\right)-\frac{d U}{d q_{2}}=F_{q_{2}}
\end{array} \rightarrow \begin{array}{l}
w_{1} \\
w_{2}
\end{array}\right.
\end{align*}
$$

Kinetic energy of the unbalance
$\left.T_{b}=m_{b} d \Omega \dot{(\dot{u}} \cdot \cos \Omega t-\dot{w} \cdot \sin \Omega t\right)$
$\dot{u}=\mathrm{f}(y) \cdot \dot{q}_{1}$
With $\mathrm{y}=l_{1}$
$\dot{u}=f\left(l_{1}\right) \cdot \dot{q}_{1}$
Then
$T_{b}=m_{b} d \Omega f\left(I_{1}\right) \dot{q}_{1} \cdot \cos \Omega t-\dot{q}_{2} \cdot \sin \Omega t$

## Forces

The two components of the forces are supposed to be
$\left\{F_{\dagger}=F_{1}(t)\right.$
$\left\{F_{w}=F_{2}(t)\right.$
And as their action is supposed to act at $y=l_{3}$
$\delta W=F_{1}(t) \delta u\left(l_{3}\right)+F_{2}(t) \delta w\left(l_{3}\right)$
Which can be written as
$\delta W=F_{1}(t) f\left(l_{3}\right) \delta q_{1}+F_{2}(t) f\left(l_{3}\right) \delta q_{2}$
A simple identification of expression (3.30) and (3.27) shows that the two components of the forces are
$\left\{\begin{array}{c}F_{q_{1}}=F_{1}(t) f\left(l_{3}\right) \\ F_{q_{2}}=F_{2}(t) f\left(l_{3}\right)\end{array}\right.$

## Numerical data

## Geometric characteristic

For the disc
Inner radius $R_{1}=0.08 \mathrm{~m}$
Outer radius $R_{2}=1.1275 \mathrm{~m}$
Thickness $h=0.225 m$

$$
l_{1}=L / 3
$$

## For the shaft

Length $\mathrm{L}=2.585 \mathrm{~m}$
Cross-sectional radius $R_{1}=0.08 \mathrm{~m}$

## Mechanical characteristic

## For Disc and Shaft:

Mass volume $\rho=7800 \mathrm{~kg} / \mathrm{m}^{3}$
Yong module $E=2 e 11 p a$
Then $\mathrm{M}_{\mathrm{D}}, \mathrm{I}_{\mathrm{Dx}}=\mathrm{I}_{\mathrm{Dz}}$ are obtained
$M_{D}=\pi\left(R_{2}^{2}-R_{1}^{2}\right) h \rho=6973 k g$
$I_{D x}=I_{D z}=\frac{M_{D}}{12}\left(3 R_{2}^{2}+3 R_{1}^{2}+h^{2}\right)=2256 \mathrm{~kg} \cdot \mathrm{~m}^{2}$
$I_{D y}=\frac{M_{D}}{2}\left(R_{2}^{2}+R_{1}^{2}\right)=4454.55 \mathrm{~kg} \cdot \mathrm{~m}^{2}$
Also S and I are obtained
$S=\pi R_{1}^{2}=0.02 \mathrm{~m}^{2}$
$I=\frac{\pi R_{1}^{4}}{4}=3.21 \times 10^{-5} \mathrm{~m}^{4}$
Basic data for mass unbalance
Mass $m_{u}=0.043 \mathrm{~kg}$
Distance $d=R_{2}=1,1275 \mathrm{~m}$

## Basic data for the Bearings

In this rotor model we are using bearings of (22328/C3) this makes the rotor asymmetric and causes viscous damping which causes a spring that situated at $l_{2}=2 L / 3$

Stiffness $k_{z z}=1,2 \times 10^{7} \mathrm{~N} / \mathrm{m}$

Viscous damping $C_{z z}=1,5 \times 10^{3} \mathrm{~N} / \mathrm{m}$ by replacing in the (3.26) we get
$\delta \mathrm{W}=-1,2 \times 10^{7} f^{2}(2 \mathrm{~L} / 3) \cdot q_{2} \cdot \delta q_{2}-1,5 \times 10^{3} f^{2}(2 \mathrm{~L} / 3) \cdot \dot{q}_{2} \cdot \delta q_{2}$

## Displacement function

The displacement function chosen is the exact first mode shape of a beam with constant crosssection in bending, simply supported at both ends

$$
\text { P a g e } 69 \text { | } 97
$$

$f(y)=\sin \frac{\pi y}{L}=\sin \frac{\pi y}{2,585}$
Then
$g(y)=\frac{\pi}{L} \cos \frac{\pi y}{L}=\frac{\pi}{2.585} \cos \frac{\pi y}{2.585}$
And
$h(y)=-\left(\frac{\pi}{L}\right)^{2} \sin \frac{\pi y}{L}=-\left(\frac{\pi}{2.585}\right)^{2} \sin \frac{\pi y}{2,585}$
Kinetic energy the kinetic energy of the disk $\mathrm{T}_{\mathrm{D}}$ is given by (3.17)
$T_{d}=3031.1\left(\dot{q}_{1}^{2}+\dot{q}_{2}^{2}\right)+1643.73 \Omega \cdot \dot{q}_{1} \cdot q_{2}$
The kinetic energy of the shaft $\mathrm{T}_{\mathrm{s}}$ is given by
$\int_{0}^{L} f^{2}(y) d y=\int_{0}^{L} \sin ^{2} \frac{\pi y}{2,585} d y=1.2925 m$
and

$$
\int_{0}^{L} g^{2}(y) d y=\int_{0}^{L} \frac{\pi}{2.585} \cos ^{2} \frac{\pi y}{2,585} d y=1.91 m^{-1}
$$

Then
$T_{s}=101.09\left(\dot{q}_{1}^{2}+\dot{q}_{2}^{2}\right)+0.478 \Omega \dot{q}_{1} \cdot q_{2}$
The kinetic energy of mass unbalance $T_{u}$ is given by (3.22)
$T_{b}=0.42 \Omega\left(\dot{q}_{1} \cdot \cos \Omega t-\dot{q}_{2} \cdot \sin \Omega t\right)$
The kinetic energy of the system is
$T=T_{D}+T_{S}+T_{b}$
$T=3132.19\left(\dot{q}_{1}^{2}+\dot{q}_{2}^{2}\right)+1644.21 \Omega \cdot \dot{q}_{1} \cdot q_{2}+0.42 \Omega\left(\dot{q}_{1} \cdot \cos \Omega t-\dot{q}_{2} \cdot \sin \Omega t\right)$
Strain energy The strain energy is given by equation
$\int_{0}^{L} h^{2}(y) d y=\int_{0}^{L}\left(\frac{\pi}{2.585}\right)^{2} \sin \frac{\pi y}{2,585} d y=2.82 m^{-3}$
Then
$U_{a}=9050.9\left(q_{1}^{2}+q_{2}^{2}\right)$
Forces due to stiffness $\mathrm{k}_{\mathrm{zz}}$ the expression becomes
$\delta \mathrm{W}=-9 \times 10^{6} \cdot q_{2} \cdot \delta q_{2}$
And from we have $F_{q_{1}}=0$
$F_{q_{2}}=-9 \times 10^{6} . q_{2}$
Equations of the rotor The two rotor equations come from
$\left\{\begin{array}{l}\frac{d}{d t}\left(\frac{d T}{d q_{1}}\right)-\frac{d T}{d q_{1}}+\frac{d U}{d q_{1}}=F_{q_{1}} \\ \frac{d}{d t}\left(\frac{d T}{d q_{2}}\right)-\frac{d T}{d q_{2}}+\frac{d U}{d q_{2}}=F_{q_{2}}\end{array}\right.$

## Asymmetric Rotor

The asymmetry is introduced by a spring $k_{z z}$ as shown in equations then
$\left\{\begin{array}{c}k_{1}=k \\ k_{2}=k+k_{z z} f^{2}\left(l_{2}\right)\end{array}\right.$
Then equations (3.32) and (3.33) become
$\left\{\begin{array}{l}m \ddot{q}_{1}+a \Omega \dot{q}_{2}+k_{1} q_{1}=m^{*} d \Omega^{2} \sin \Omega t \\ m \ddot{q}_{2}+a \Omega \dot{q}_{1}+k_{2} q_{2}=m^{*} d \Omega^{2} \cos \Omega t\end{array}\right.$
Then we have

$$
\begin{align*}
& 6264,38 \ddot{q}_{1}+1644,21 \Omega \dot{q}_{2}+18101.8 q_{1}=0.42 \Omega^{2} \sin \Omega t  \tag{3.42}\\
& 6264,38 \ddot{q}_{2}-1644,21 \Omega \dot{q}_{2}+18101.8 q_{1}=0.42 \Omega^{2} \cos \Omega t \tag{3.43}
\end{align*}
$$

Natural frequencies as a function of the speed rotation
Again the rotors is studied in free motion equations

$$
\begin{align*}
& m \ddot{q}_{1}+a \Omega \dot{q}_{2}+k_{1} q_{1}=0  \tag{3.44}\\
& m \ddot{q}_{2}-a \Omega \dot{q}_{1}+k_{2} q_{2}=0 \tag{3.45}
\end{align*}
$$

These equations can be written in the following matrix form
$\left[\begin{array}{cc}m & 0 \\ 0 & m\end{array}\right]\left[\begin{array}{l}\ddot{q}_{1} \\ \ddot{q}_{2}\end{array}\right]+\Omega\left[\begin{array}{cc}0 & a \\ -a & 0\end{array}\right]\left[\begin{array}{l}\dot{q}_{1} \\ \dot{q}_{2}\end{array}\right]+\left[\begin{array}{cc}k_{1} & 0 \\ 0 & k_{2}\end{array}\right]\left[\begin{array}{l}q_{1} \\ q_{2}\end{array}\right]=0$
The middle $2 * 2$ matrix multiplied by $\Omega$ gives the gyroscopic (coriolis) effect. The other two matrices are respectively the mass and stiffness matrices.

The solution(3.35) must be sought in the form

$$
\begin{align*}
q_{1} & =Q_{1} e^{r t}  \tag{3.47}\\
q_{2} & =Q_{2} e^{r t} \tag{3.48}
\end{align*}
$$

Substituting (3.47) and (3.48) in (3.46) gives us the next equation
$\left[\begin{array}{cc}k_{1}+m r^{2} & a \Omega r \\ -a \Omega r & k_{1}+m r^{2}\end{array}\right]\left[\begin{array}{l}Q_{1} \\ Q_{2}\end{array}\right]=0$
The expansion of the determinant gives the characteristic equation
$\left(k_{1}+m r^{2}\right)\left(k_{2}+m r^{2}\right)+a^{2} \Omega^{2} r^{2}=0$
This can be written as
$m^{2} r^{4}+\left(k_{1} m+k_{2} m+a^{2} \Omega^{2}\right) r^{2}+k_{1} k_{2}=0$
At rest $(\Omega=0)$ the roots of
$r_{1_{0}}^{2}=j^{2} \omega_{1_{0}}^{2}=-\frac{k_{1}}{m}$
$r_{2_{0}}^{2}=j^{2} \omega_{2_{0}}^{2}=-\frac{k_{2}}{m}$

And the angular frequencies at rest are
$\omega_{1_{0}}=\sqrt{\frac{k_{1}}{m}}$
$\omega_{2_{0}}=\sqrt{\frac{k_{2}}{m}}$
Under rotating conditions $(\Omega \pm 0)$ the roots of (3.52)are $r_{1}$ and $r_{2}$ and corresponding angular frequencies are $\omega_{1}$ and $\omega_{2}$ the expression of the first root is

$$
\begin{align*}
& r_{1}^{2}=-\left[\frac{\omega_{1_{0}}^{2}}{2}+\frac{\omega_{2_{0}}^{2}}{2}+\frac{a^{2} \Omega^{2}}{2 m^{2}}-\sqrt{\left(\frac{\omega_{10}^{2}}{2}+\frac{\omega_{2_{0}}^{2}}{2}+\frac{a^{2} \Omega^{2}}{2 m^{2}}\right)^{2}-\omega_{1_{0}}^{2} \omega_{2_{0}}^{2}}\right] \\
& =j^{2} \omega_{1}^{2} \tag{3.53}
\end{align*}
$$

Then
$\omega_{1}=\sqrt{\frac{\omega_{1_{0}}^{2}}{2}+\frac{\omega_{2_{0}}^{2}}{2}+\frac{a^{2} \Omega^{2}}{2 m^{2}}-\sqrt{\left(\frac{\omega_{1_{0}}^{2}}{2}+\frac{\omega_{2_{0}}^{2}}{2}+\frac{a^{2} \Omega^{2}}{2^{2}{ }^{2}}\right)^{2}-\omega_{1_{0}}^{2} \omega_{2_{0}}^{2}}}$
$r_{2}^{2}=-\left[\frac{\omega_{1_{0}}^{2}}{2}+\frac{\omega_{2_{0}}^{2}}{2}+\frac{a^{2} \Omega^{2}}{2 m^{2}}+\sqrt{\left(\frac{\omega_{1_{0}}^{2}}{2}+\frac{\omega_{2_{0}}^{2}}{2}+\frac{a^{2} \Omega^{2}}{2 m^{2}}\right)^{2}-\omega_{1_{0}}^{2} \omega_{2_{0}}^{2}}\right]$
$=j^{2} \omega_{2}^{2}$
$\omega_{2}=\sqrt{\frac{\omega_{1_{0}}^{2}}{2}+\frac{\omega_{2_{0}}^{2}}{2}+\frac{a^{2} \Omega^{2}}{2 m^{2}}+\sqrt{\left(\frac{\omega_{10}^{2}}{2}+\frac{\omega_{2_{0}}^{2}}{2}+\frac{a^{2} \Omega^{2}}{2 m^{2}}\right)^{2}-\omega_{1_{0}}^{2} \omega_{2_{0}}^{2}}}$
The expressions $d \omega_{1} / d \Omega$ and $d \omega_{1} / d \Omega$ can be calculated and they show that
$\frac{d \omega_{1}}{d \Omega}<0$ then $\omega_{1}<\omega_{1_{0}}$
$\frac{d \omega_{2}}{d \Omega}>0$ then $\omega_{2_{0}}<\omega_{2}$
Then
$\omega_{1}<\omega_{1_{0}}<\omega_{2_{0}}<\omega_{2}$
The equations (3.54) and (3.55) shows that $r_{1}^{2}$ and $r_{2}^{2}$ are negative quantities which means that $\mathrm{r}_{1}$ and $\mathrm{r}_{2}$ and $\omega_{1}, \omega_{2}$ are real quantities which means :the rotor is stable The representation of the angular frequencies as a function of the speed of rotation
$\omega_{1}=\omega_{1}(\Omega)$ And $\omega_{2}=\omega_{2}(\Omega)$ is the Campbell diagram. In general, the diagram shows $F_{1}=\frac{\omega_{1}}{2 \pi}$

$$
F_{2}=\frac{\omega_{2}}{2 \pi}
$$

As a function of the speed of rotation N given in revolutions per minute ( rpm ), which is related to $\Omega(\mathrm{rad} / \mathrm{sec})$ by
$N=30 \Omega / \pi$
Here $\omega$ and $\Omega$ are angular frequencies, in all that follows this distinction in terminology will be dropped and all three with be called frequencies.

The mode shapes have to be considered now
The first equation (3.48)gives
$Q_{1}=\frac{a r \Omega Q_{2}}{k+m r^{2}}$
Note
The calculation corresponding to free motion are not given here as they are much complex than those for the symmetric rotor. It is easy to observe that $Q_{1} \neq Q_{2}$, and in the next steps we are going to use the symmetric rotor to show and explain the method to calculate them

The modes corresponding to (3.53) (i.e.to $\left.\pm j \omega_{1}\right)$ are if ((3.50)is used

$$
\left\{\begin{aligned}
Q_{1}\left(j \omega_{1}\right) & =-j Q_{2}\left(j \omega_{1}\right) \\
Q_{1}\left(-j \omega_{1}\right) & =-j Q_{2}\left(-j \omega_{1}\right)
\end{aligned}\right.
$$

The modes corresponding to(3.53) (i.e.to $\pm j \omega_{2}$ ) are if (3.50) is used
$\left\{\begin{array}{c}Q_{1}\left(j \omega_{1}\right)=-j Q_{2}\left(j \omega_{1}\right) \\ Q_{1}\left(-j \omega_{1}\right)=-j Q_{2}\left(-j \omega_{1}\right)\end{array}\right.$
The general expression of the free motion of the system can now be written. Using (3.48), (3.49) and (3.58), (3.63)
$\left\{\begin{array}{l}q_{1}=j A_{1} e^{j \omega_{1} t}-j B_{1} e^{-j \omega_{1} t}-j A_{2} e^{j \omega_{2} t}+j B_{2} e^{-j \omega_{2} t} \\ q_{2}=j A_{1} e^{j \omega_{1} t}+j B_{1} e^{-j \omega_{1} t}+j A_{2} e^{j \omega_{2} t}+j B_{2} e^{-j \omega_{2} t}\end{array}\right.$
The four constants $A_{1}, B_{1}, A_{2}, B_{2}$ are determined by the initial conditions. First we will choose initial conditions which only shows the frequency $\omega_{1}$. At $t_{0}=0$
$q_{1}=0$
$q_{2}=q_{2_{0}}$

$$
\left\{\begin{array}{c}
\dot{q}_{1}=0 \\
\dot{q}_{2}=-\omega_{1} q_{20}
\end{array}\right.
$$

Then
$0=A_{1}-B_{1}-A_{2}+B_{2}$
$q_{2_{0}}=A_{1}+B_{1}+A_{2}+B_{2}$
$0=\omega_{1} A_{1}+\omega_{1} B_{1}-\omega_{1} A_{2}-\omega_{1} B_{2}$
$-\omega_{1} q_{2_{0}}=\omega_{1} A_{1}-\omega_{1} B_{1}+\omega_{1} A_{2}-\omega_{1} B_{2}$
Then we get
$\left\{\begin{array}{c}A_{2}=B_{2}=0 \\ A_{1}=B_{1}=q_{2_{0}} / 2\end{array}\right.$
Equations (3.60) and (3.61) becomes
$\left\{\begin{array}{c}q_{1}=\frac{q_{2_{0}}}{2}\left(j e^{j \omega_{1} t}-j e^{-j \omega_{1} t}\right) \\ q_{2}=\frac{q_{2_{0}}}{2}\left(e^{j \omega_{1} t}+e^{-j \omega_{1} t}\right)\end{array}\right.$

And as
$e^{ \pm j \omega_{1} t}=\cos \omega_{1} t \pm \sin \omega_{1} t$

Then the two expressions become
$\left\{\begin{array}{c}q_{1}=-q_{2_{0}} \sin \omega_{1} t \\ q_{2}=q_{2_{0}} \cos \omega_{1} t\end{array}\right.$
By replacing these two equations in $u$ and $w$ equations when $y=1$ we get
$u(l, t)=-q_{2_{0}} \sin \frac{\pi l}{L} \sin \omega_{1} t=-R \sin \omega_{1} t$
$w(l, t)=q_{2_{0}} \sin \frac{\pi l}{L} \cos \omega_{1} t=R \cos \omega_{1} t$
These two expressions give us
$R=\sqrt{u^{2}(l, t)+w^{2}(l, t)}=q_{2_{0}} \sin \frac{\pi}{L}$
The characteristic equation of the rotor defined by (3.42) and (3.43) is
$r^{4}+\left(1442.47+0.0689 \Omega^{2}\right) r^{2}+1.6324 \times 10^{11}$
And the frequencies at rest are

$$
\begin{aligned}
& F_{1_{0}}=\frac{\omega_{1_{0}}}{2 \pi}=0.27 \mathrm{~Hz} \\
& F_{2_{0}}=\frac{\omega_{2_{0}}}{2 \pi}=6.04 \mathrm{~Hz}
\end{aligned}
$$

The frequencies in rotation are

$$
\begin{gathered}
F_{1}=\frac{\omega_{1}}{2 \pi}=\frac{1}{2 \pi} \sqrt{721.167+0.0345 \Omega^{2}-\sqrt{\left(721.167+0.0345 \Omega^{2}\right)^{2}-4159.454}} \\
F_{2}=\frac{\omega_{2}}{2 \pi}=\frac{1}{2 \pi} \sqrt{721.167+0.0345 \Omega^{2}+\sqrt{\left(721.167+0.0345 \Omega^{2}\right)^{2}-4159.454}}
\end{gathered}
$$

The Campbell diagram for $F_{1}=F_{1}(N)$ and $F_{2}=F_{2}(N)$ is shown in.
Where the intersection of $F_{1}(N)$ and $F_{2}(N)$ with two straight lines are shown points A and B correspond to the intersection with $\mathrm{F}=\mathrm{N} / 60$ at the corresponding points frequency of the rotor equals the frequency of the rotation .points C and D correspond to the intersection with $\mathrm{F}=0.5 \mathrm{~N} / 60$ at the corresponding points a frequency of the rotor equals half of the frequency of rotation


Fig.3.9 Campbell diagram

It is of interest to get a general expression for the frequencies corresponding to points $A, B, C, D$ the general relationship between $\omega$ and $\Omega$ is $\omega=\mathrm{s} \Omega$

Where $s=1$ in A and B, $s=0.5$ in C and D. Expression (3.45) and (3.46) give $r= \pm j \omega= \pm j s \omega$
which substituted (3.41)
$m^{2} s^{4} \Omega^{4}+\left(k_{1} m+k_{2} m+a^{2} \Omega^{2}\right) s^{2} \Omega^{2}+k_{1} k_{2}=0$
And $s^{2}\left(m^{2} s^{2}-a^{2}\right) \Omega^{4}+m\left(k_{1}+k_{2}\right) s^{2} \Omega^{2}+k_{1} k_{2}=0$

For the example considered here

In $\mathrm{A}, \mathrm{F}_{1}=0.2951 \mathrm{~Hz}, \mathrm{~N}=374.2988 \mathrm{rpm}$. 36.2703 rpm

In $\mathrm{B}, \mathrm{F}_{1}=7.0733 \mathrm{~Hz}, \mathrm{~N}=851.2573 \mathrm{rpm}$
In $\mathrm{C}, \mathrm{F}_{1}=, 6.2400 \mathrm{~Hz} \mathrm{~N}=374.2988 \mathrm{rpm}$.
In $\mathrm{D}, \mathrm{F}_{1}=0.3189 \mathrm{~Hz}, \mathrm{~N}=17.6467 \mathrm{rpm}$

## Chapter :4

## Matlab simulation

## Chapter 04 Matlab simulation

### 4.1 Introduction

In the industry domain many theories and new ideas are developed every day and the process to use and improve those theories begins which may let these new ideas to see the day light or fail and never be heard but this doesn't means always that these ideas have flaws because the true hard part is to prove the effectiveness of these theories on the production field .which requires a lot of time effort and resources to prove it . But many theories requires a lot of resources and the results are not verified they don't get funding, this was a huge problem for the industry domain because the true experiments require a lot of money with no certain results, until the simulation process was created this simulation mad a huge change in the industry domain it allowed to create digital experiments that simulate true experiments with negligible cost . in this chapter we are going to define many simulation programs that uses different types of simulation after that we will use Matlab program to simulate a rotor and plot the Campbell diagram calculating the needed frequencies

### 4.2 Simulation programs

The simulation domain contains many different programs that uses different methods to simulate such as

### 4.2.1 Solidworks

The SOLIDWORKS® CAD software is a mechanical design automation application that lets designers quickly sketch out ideas, experiment with features and dimensions, and produce models and detailed drawings.

Parts are the basic building blocks in the SOLIDWORKS software. Assemblies contain parts or other assemblies, called subassemblies. A SOLIDWORKS model consists of 3D geometry that defines its edges, faces, and surfaces. The SOLIDWORKS software lets you design models quickly and precisely. SOLIDWORKS models are: • Defined by 3D design • Based on components 3D Design SOLIDWORKS uses a 3D design approach. As you design a part, from the initial sketch to
the final result, you create a 3D model. From this model, you can create 2D drawings or mate components consisting of parts or subassemblies to create 3D assemblies. You can also create 2D drawings of 3D assemblies. When designing a model using SOLIDWORKS, you can visualize it in three dimensions, the way the model exists once it is manufactured.[10]


Fig 4.1 picture of solidworks interface [10]

### 4.2.2 Abaquis

Abaqus FEA (formerly ABAQUS) is a software suite for finite element analysis and computer-aided engineering, originally released in 1978. The name and logo of this software are based on the abacus calculation tool.[11]


Fig 4.2 abaquis interface[12]

### 4.2.3 MATLAB

MATLAB (matrix laboratory) is a multi-paradigm numerical computing environment and proprietary programming language developed by MathWorks. MATLAB allows matrix manipulations, plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages.

Although MATLAB is intended primarily for numerical computing, an optional toolbox uses the MuPAD symbolic engine allowing access to symbolic computing abilities. An additional package, Simulink, adds graphical multi-domain simulation and model-based design for dynamic and embedded systems.[13]

## Structure

MATLAB supports structure data types. ${ }^{[28]}$ Since all variables in MATLAB are arrays, a more adequate name is "structure array", where each element of the array has the same field names. In addition, MATLAB supports dynamic field names. (field look-ups by name, field manipulations, etc.).

## Functions

When creating a MATLAB function, the name of the file should match the name of the first function in the file. Valid function names begin with an alphabetic character, and can contain letters, numbers, or underscores. Variables and functions are case sensitive.

## Function handles

MATLAB supports elements of lambda calculus by introducing function handles, or function references, which are implemented either in .m files or anonymousnested functions.

## Classes and object-oriented programming

MATLAB supports object-oriented programming including classes, inheritance, virtual dispatch, packages, pass-by-value semantics, and pass-by-reference semantics. However, the syntax and calling conventions are significantly different from other languages. MATLAB has value classes and reference classes, depending on whether the class has handle as a super-class (for reference classes) or not (for value classes).

Method call behaviour is different between value and reference classes. For example, a call to a method

### 2.3 Matlab program

In this thesis we are going to use a matlab program which is going initial data such as [ $\mathrm{m}, \mathrm{a}, \mathrm{k}_{1}, \mathrm{k}_{2}$ ] to calculate the natural frequencies and plot the Campbell diagram then we will use a manual method via matlab find the critical frequencies from the diagram

The form of this program is as next

```
close all;
clear all;
clc
m = input('Donner les valeur de m: ');
a = input('Donner les valeur de a: ');
```

```
k1 = input('Donner les valeur de k1: ');
k2 = input('Donner les valeur de k2: ');
N = [0:1:2000];
```

W10 = sqrt(k1/m);
sprintf('w10 = \%g',w10)
$\mathrm{W} 20=\operatorname{sqrt}(\mathrm{k} 2 / \mathrm{m})$;
sprintf('W20 = \%g',W20)
F10 = W10/(2*pi);
sprintf('F10 = \%g',F10)
F20 = W20/(2*pi);
sprintf('F20 = \%g',F20)
$x=\left(W 10^{\wedge} 2 / 2\right)+\left(W 20^{\wedge} 2 / 2\right)+\left(\left(a^{\wedge} 2 * p i \wedge 2 * N . \wedge 2\right) /\left(1800 * m^{\wedge} 2\right)\right) ;$
F1 $=(1 /(2 *$ pi $)) *$ sqrt (x-sqrt (x.^2-W10^2*W20^2));
F2 $=(1 /(2 *$ pi $)) *$ sqrt (x+sqrt (x.^2-W10^2*W20^2));
F3 = N./60;
F4 $=$ N./120;
hold on
plot (N,F1)
plot(N,F2,'r')
plot(N,F3,'g')
plot(N,F4,'black')

```
legend('F1','F2','F3','F4')
axis([0 900 0 15])
grid on
[x,y]=ginput(1)
[x,y]=ginput (1)
[x,y]=ginput(1)
[x,y]=ginput(1)
```

After executing the program we interred the initiation data of the differential equations and got the next diagram


Fig.4.3 Campbell diagram via Matlab

After ploting the diagram we used the function input to find manually the critical speeds and frequencies and got the next results
$\omega_{10}=1.69989$
$\omega_{20}=37.9419$
$\mathrm{F}_{10=0.270547}$
$\mathrm{F}_{20=6.03863}$
In $A, F_{1}=0.2851 \mathrm{~Hz}, \mathrm{~N}=17.6267 \mathrm{rpm}$.
In $B, F_{1}=6.2500 \mathrm{~Hz}, N=376.3825 \mathrm{rpm}$
In C, $\mathrm{F}_{1}=0.3289 \mathrm{~Hz}, \mathrm{~N}=40.4378 \mathrm{rpm}$
In $\mathrm{D}, \mathrm{F}_{1}=7.0833 \mathrm{~Hz}, \mathrm{~N}=851.2673 \mathrm{rpm}$
Which means
If we use $B w$ then
$\omega_{1}=1.79$
$\omega_{2}=239.26$
And if we use then
$\omega_{1}=2.066$
$\omega_{2}=244.5$

## Conclusion

In this thesis we talked about the study of the dynamics of a rotor mono-Disc which contained 4 chapters, in the first chapter we got to know about the meaning of rotor dynamics and the full history of developing this domain then we talked about rotor and its component, the classification of rotors.

In the second chapter we defined maintenance, the different types of maintenance and its objectives, then we talked about the causes of damage that effect the rotor, the method to define them and how to repair it.

In the third chapter we started with the rotor elements characteristics the kinetic energies of each component, the deformation energy the unbalance ....ect we also went step by step from the characteristic into the differential equations and drawing the Campbell diagram and finding the critical frequencies in the case of an asymmetric rotor(there are other cases such as damped rotor which is more complicated or symmetric rotor which is much simpler), we used a true rotor data of a stone breaker and used the excel program to plt the diagram and finding the final results .

In our forth chapter we talked about different simulation programs including matlab which we used to create a program that made us plot the Campbell diagram and find the frequencies starting with data from the differential equations of the rotor and by comparing the results we find the error is $1 \%$ which is acceptable, and finally we finished the thesis with a short training ship report about the cement factory of
(in this year the factory didn't make any maintenance reparations so we used old reports from the archive ).

## L'annex

[1] Rotordynamicsprediction in engineering second edition by michel Lalane and duy Ferraris [2 ] Ziaei red .r. Tikani "Rotordynamics"
[3] Aimeur Noureddine »Etude dynamique d'un rotor par élément finis».
[4] on the cite The way firward"the maintenance of rotor assemblies from strategic rotating machine "
[5] Dr. Abed Schokry ,,, introduction to maintenance second semester 2010\2011
[6] Techniques De L'ingénieur - Surveillance Vibratoire Et Maintenance Prédictive
[7] https://my.solidworks.com/solidworks/guide/SOLIDWORKS Introduction_EN.pdf
[8]"ABAQUS CEO Interview". Archived from the original on 18 July 2014. Retrieved 4

December 2012
[9] "Dassault Systemes to acquire Abaqus Inc for 413 mln usd cash". Forbes.com. Forbes. 17 May
2005. Retrieved 7 July 2010..
[10] "An interview with CLEVE MOLER Conducted by Thomas Haigh On 8 and 9 March, 2004 Santa Barbara, California" (PDF). Computer History Museum. Archived from the original (PDF) on December 27, 2014. Retrieved December 6, 2016. So APL, Speakeasy, LINPACK, EISPACK, and PLO were the predecessors to MATLAB.

## ANNEXE

## Traineeshipreport:

## 1.Introduction

In the spring I've made a trip to the cement factory in IMA Alabyadh, but because the quarantine the trip lasted for only 2 days I've seen the way the stone breaker works and its components including the rotor but sadly i wasn't able to see a live maintenance for the breaker so i took old reports of maintenance which we are going to see and discuss in the

following report

Fig. factory stone breaker M. 1

## 2 .Maintenance report



Fig. maintenance report M. 2

The first page contains the name of the documents the year of production and the number of the report and a picture of the breaker
schémos snootique du CONCASSEUR CALCAREROTOR Cimenterie SCJ


Fig. maintenance report M. 3

In the second page we can see it contains the pattern of the breaker rotor with the motor and reducers with the frequencies of the motor and the rotor

Calcul des données cinématiques CONCASS AIMOI ROT $S C T$ El MA LABIOD

## Moteur

| MOTEUR | PUISS | NBR <br> TOUR <br> FREQ | PALIER AV ATT | PALIER AR OP |
| :---: | :---: | :---: | :---: | :---: |
| SIEMENS <br> 1LST-716- <br> 6HC-90-Z | 1180 | 988 <br> tr/mn | COUSSINET | COUSSINET |
|  |  | 16.46 <br> hz | COUSS $150 \times 180 \mathrm{~mm}$ | COUSS $150 \times 180 \mathrm{~mm}$ |

REDUCTETR

| DESIGNATION |  | ORGANE | $\begin{gathered} \text { NBR } \\ \text { DENTS } \end{gathered}$ | VITESSE ROTATION | FREQ DEFAUT | FREQ <br> ENGR HZ | $\underset{T}{\text { ROULEMEN }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Pignon 1 | 20 | 995 | 16.58 | 331.6 | $\begin{gathered} \text { SKF } \\ 24136 \end{gathered}$ |
|  |  | Roue 1 | 53 | 375.4 | 6.25 |  | $\begin{gathered} \text { SKF } \\ 24040 \end{gathered}$ |

CONGASSEUR

| CONCAS | NBR <br> MART | VITESSE <br> tr/mn | FREQ | FREQ | ROUL <br> attaque | ROUL <br> opposé |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| FLS EV | $480 \times 200$ | 48 | 380 | 6.33 | 6.33 | SKF <br> 23164 CAK/C3 |  |

Fig. maintenance report M. 4

In the third page we find cinematic data of the breaker, the rotor, the breaker such as the torque , the rotation speed the frequencies $\qquad$ .ext

## Analyse spectrale

- Evolution des amplitudes des premières harmoniques de la fréquence de rotation du moteur dont le seuil est admissible.
- Amplitude acceptable à la fréquence électromagnétique $0,012 \mathrm{~g}$ à 100 Hz .



## REDUCTEUR

## Analyse en niveau global

- Les niveaux vibratoires globaux relevés en accélération sont acceptables.
- Evolution des niveaux vibratoires globaux relevés en vitesse à l'entrée
du réducteur surtout en axial dont le seuil est tolérable.
- Etat des roulements acceptables.

| PALIER | $3 A X$ | $3 R H$ | $3 R V$ | $6 A X$ | $6 R H$ | $6 R V$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NGACCEI[G] | 0.279 | 0.254 | 0.285 | 0.312 | 0.289 | 0.247 |
| NG Vit [mm/s] | 3.05 | 1.87 | 2.02 | 2.58 | 3.05 | 1.36 |
| DEF | 2.48 | 2.58 | 2.57 | 2.34 | 2.46 | 2.40 |

Fig. maintenance report M. 5

The forth page represent the surveillance report which contains the motor vibration analyses of the global level with the diagram of motor speed after comparing it with the last report they found the acceptable speed that doesn't causes critical vibration

## Maintenance conditionnelle

Courbe de tendance du NG vitesse à l'entrée du réducteur


Courbe de tendance du NG vitesse à la sortie du réducteur


## Analyse spectrale

- Evolution de l'amplitude à la fréquence d'engrènement réducteur dont le seuil reste acceptable $0,09 \mathrm{~g}$ à $329,96 \mathrm{~Hz}$.
- Evolution des amplitudes des premières harmoniques de la fréquence de rotation de l'arbre GV.


Fig. maintenance report M. 6

In the fifth page we can see it contains the spectral analyses of electromagnetic frequency amplitude and using it to determine the acceptable amplitude in both the motor and the reducer

## PALIERS CONCASSEUR

CHANGEMENT DES MARTEAUX ET DES BARREAUX
Analyse en niveau global

$$
\begin{aligned}
& \text { Evolution des niveaux vibratoires globaux relevés sur les paliers du rotor } \\
& \text { concasseur surtout en radial dont le seuil est juste tolérable } 8,27 \mathrm{~mm} / \mathrm{s} \text {. } \\
& \begin{array}{|c|c|c|c|c|c|c|c|}
\hline \text { PALIER } & 7 A X & 7 R H & 7 R V & 8 A X & 8 R H & 8 R V \\
\hline \text { NG Accel [G] } & 0.718 & 1.25 & 0.994 & 0.463 & 0.483 & 0.444 \\
\hline \text { NG Vit [mm/s] } & 6.64 & 7.23 & 7.27 & 8.14 & 6.46 & 8.27 \\
\hline \text { DEF } & 4.59 & 4.42 & 4.87 & 5.92 & 4.77 & 4.82 \\
\hline
\end{array}
\end{aligned}
$$

Facteur défaut de roulement acceptable
Courbe de tendance du NG vitesse palier 7


Courbe de tendance du NG vitesse palier 8


Fig. maintenance report M. 7

The sixth page contains the diagrams of the speeds that entering the reducer and exiting it and the spectral analyses of the reducer to determine the proper amplitude

## Analyse spectrale

- Amplitude acceptable à la fréquence de rotation du rotor concasseur (balourd rotor acceptable) 0.002 g à 6.17 Hz .
- Evolution des amplitudes des premières harmoniques (choc) à la fréquence de rotation du rotor
- Amplitude tolérable à la fréquence de passage des marteaux $0,053 \mathrm{~g}$ à $3 \times 6,17 \mathrm{~Hz}$ (en horizontal)


Fig. maintenance report M. 8
The seventh page includes the analyses of the rotor bearings for the goal of changing the rotor hammers

## ACTION PRECONISEES

- Serrage des bagues d'arrêt.
- Serrage des paliers du rotor
- Vérifier l'état de l'accouplement (élément élastique)

Fig. maintenance report M. 9

The eighth page includes the bearing spectral analyses to also define its proper amplitude


Fig. maintenance report M1.0

The last page contains the necessary actions for maintenance in order to keep the breaker functioning at peak conditions

