MEMOIRE

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Sujet

Etude analytique et numérique des contraintes d'un réservoir cylindrique de gasoil

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Acknowledgments

First of all:

All praise is due to Allah lord of the worlds,
without his aid we can't do this work

And thanks to **all my teacher how do real educate me** thanks to them
a lot.

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the member of the mechanical engineering department it helps us
a lot during our study at the university

we ask god to guide all of them to what he loves and satisfies.

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Abstract

To study the distribution of stresses in a complex shape such as flat bottom tank or hemispheric bottom require an advanced method to find the solution of the equations that describe such distribution. And that's what we did in this thesis by a brief introduction to tanks and their types follow that by the development of the equilibrium equations in three coordinate systems and applying the solution of this equations to study the stresses in the tanks wall and bottom and finely by comparing the results by a numerical result in SolidWorks.

ملخص

لدراسة توزيع الإجهادات في أشكال معقدة مثل خزان بقعر مسطح أو نصف كروي، نحتاج إلى طرق متقدمة لإيجاد حلول للمعادلات التي تصف توزيع هذه الإجهادات, وهذا ما قمنا به في هذه المذكرة، وذلك بمقدمة مختصرة حول الخزانات و أنواعها، وشرح حالات وكيفية تضررها، وأتبعنا هذه المقدمة بإيجاد معادلات التوازن في ثلاثة أنظمة إحداثية، وتطبيق حلول هذه العادلات لدراسة الإجهادت في سطح الخزان الجانبي و كذا قعر الخزان المسطح والنصف كروي، ثم قارنا النتائج النظرية مع النتائج المتحصل عليها من محاكاة الخزان على برنامج سوليدوركس.

Résumés

Pour étudies la distribution des contraints en un géométrie complexe come un réservoir à fond plat ou hémisphérique ce l'exige des méthodes avancer pour trouver la solution des équations qui exprimé comme ces distributions. Et ça ce que nous faire dans ce mémoire, par une petite introduction à les réservoirs et ces types et ces modes d'endommagement, après ça nous développer les équations d'équilibre en trois system de coordonner et appliquer la solution du ces équations a la paroi et le fond plat même hémisphérique de ce réservoir pour obtenu des résultats numériques et comparer ces résultats par des autre résultat obtenu par un simulation en SolidWorks.

Introduction

1.1.   Introduction

1.1.1. History of Storage Tank Systems

In august 1859, the first oil well was constructed in Titusville, PA. The visionaries who financed and developed the primitive derrick and drill believed that "rock oil" would provide an excellent source of energy for illuminating buildings. And for a few years, it did—until Thomas Edison found a way during the early 1880s to harness electricity. …other world-changing inventors in Europe and the United States had already begun the first steps toward redefining transportation—through development of a four-stroke engine and adaptation of the motor to power a buggy. The seeds of an automotive industry had been shown. And, from that point on, a need for storing petroleum products grew.

The first service stations required minimal tank storage capacity. In fact, it was common for product to be stored within the dispenser itself. As the need for hydrocarbons grew, the ability to store the product safely became an important growth factor for the petroleum and automotive industries. The storage tank industry traces its start to these events that have altered society. **[1]**

1.1.2. Definition

Industrial storage tanks are containers used for storage of gas, oil, water, and petrochemical products, employed for industrial uses. It come in different sizes and shapes, they can be underground, horizontal, and vertical, and be made from concrete, stone, fiberglass, steel or plastic. The term can be used for reservoirs (artificial lakes and ponds), and for manufactured containers. The usage of the word tank for reservoirs is uncommon in American English but is moderately common in British English. In other countries, the term tends to refer only to artificial containers. [2]

Figure 1.3.1Industriel vertical cylindrical tank

1.2   Types of storage tanks

Industrial storage tanks can be categorized into several types based on the substance they hold and some other factors. Industrial fuel storage tanks, known as

petroleum tanks also, can store various fluids. In general, they are used for storing non-organic and organic liquids. They can also hold vapor as well as different flammable fluids. Fuel storage tanks are manufactured in various designs and sizes. They are designed to store a variety of fuels, vapor, and industrial liquids. There are two main types of fuel storage tanks – Aboveground tanks and Underground tanks.

1.1.3. *Aboveground Fuel Tanks*

Aboveground fuel tanks (AST) are quite popular because of their lower long-term maintenance and upfront costs. These tanks are more cost-effective to install compared to underground tanks since you don't need to spend for backfilling, deep excavation, and pavement of more involved piping.

Aboveground fuel tanks offer greater ease of maintenance compared to the below ground tanks. You can check them easily for leaks and access for repairs. This is the reason aboveground fuel storage tanks are preferred for storing fuels and chemicals.

Figure 1.3.2 underground Storage tanks

1.1.4. *Underground Storage Tanks*

In Underground storage tank (UST), at least 10% of the tank's stored volume is buried underground. Such tanks that are used for storing hazardous materials or fuels are regulated, and must have registration with the EPA.

These tanks are suitable for people wanting to maximize the space and/or value of their property. Underground storage tanks can be put beneath lawns and driveways, where they are not visible. Arguably, these types of tanks are safer as the chance of explosion is very little. However, the chance of leaks as well as that of generating pollution is increased in case of these tanks as they cannot be inspected often.

1.2.   Advantages of Using Fuel Storage Tanks

If your business requires safe storage of fuel and other inflammable liquids, then it's important to use an industrial fuel storage tank. tanks are standards certified containers which provide safe storage of chemicals, solvents, oil, petrol, diesel, and other hazardous and flammable liquids. Fuel storage tanks restrict evaporative emissions as well as prevent any leakage of the substance contained in it.

These storage tanks are manufactured and designed for meeting industry standards, which makes them an efficient and reliable option for addressing your storage needs of hazardous substances. Using industrial fuel storage tanks offers several advantages, which are listed below:

1.2.1. Cost Efficient

Using fuel storage tank is cost efficient since your staff need not leave the business facility for getting equipment or vehicles refueled. This saves both money and time that's otherwise spent to refuel off your business premises. These tanks have a storage capacity lying between 1,000 and 110,000 liters, allowing you to store petrol and fuel according to the amount required on a weekly or daily basis.

1.2.2. Variety

There are various types of fuel storage tanks you can choose from. The chief types are – above ground fuel storage tanks and underground fuel storage tanks. These types have already been discussed above.

Another type of industrial fuel storage tank is – **self bunded tank**. This type of tank is employed widely because of its durability. Having double steel walls, the self-bunded storage tank prevents spillage of the substance contained in it. The capacity of storage of the self-bunded tanks and above ground tanks usually lay between 1,000 and 150,000 liters.

If your storage requirements are small, you may choose the minor storage wrap tank. This type of fuel storage tank has a storage capacity between 1,000 and 1,450 liters. Similar to self-bunded tank, minor storage wrap tank has no need for any complex bunding requirements as well as provides maximum versatility and effectiveness.

1.2.3. Versatility

An industrial fuel storage tank has the ability to hold a variety of inflammable liquids. If your storage needs are very specific, inform your manufacturer about them since they may customize

or adjust tank specifications and models so as to fulfill your special requirements. Moreover, industrial fuel storage tanks are installed easily. Some of the models are portable too to allow easy relocation when necessary.

1.3   Single Skin & Double Skin Tanks

Single skin tank has one layer and double skin tank has two layers of plastic or steel. Double skin tanks are also known as twin-walled tanks.

1.4   Vertical cylindrical tank

The Storage Tanks are superiorly utilized in divergent industries for containing various liquefied and viscous materials and solutions. [2]

1.4.1   advantage of vertical cylindrical tank

The Vertical Storage Tanks are high-handedly suitable for stocking up a large volume of liquids, chemicals and solvents for various industrial processing. These vertical shaped tanks are designated to handle pressure distribution. The cylindrical shape of vertical storage tanks makes a fewer stress points, creating a more even distribution of pressure. This allows vertical tanks to store greater volumes of liquid without compromising safety. Vertical tanks usually take up less space, making them a good choice for smaller areas. The exception is spaces with low roofs or ceilings, where a horizontal tank will be a better fit. [3]

Chapter 1

1   Damage of Storage tanks

The safe and long service life of structures could be assured only when adequate design and construction are combined with a proper and regular maintenance. And this is also true for the storage tanks because it is a very important static equipment for the oil and gas in industry. Even though various codes and standards stipulate its design to avoid failure of storage tanks, still there are many incidents of storage tank failures. So, storage tank failure is not at all a new phenomenon. In the following section, we will explore the types of such tank damages and solution to prevent this.

1.1   Type of damage

1.1.1   damage of the tank bottom

The floors of aboveground storage tanks remain a most difficult part of the vessel to inspect for corrosion damage.

Figure 1.1 Bottom damage of a tank

1.1.2   Damage of tank Shell

- paint coating deterioration and initiation of intensive corrosion processes
- local deformation of the tank shell

1.1.3   damage of the tank roof

- paint degradation and intense corrosion on the roof plates and attached nozzles for equipment
- poorly implemented attempts to repair the roof plates [4]

Figure 1.2 Example of damages in elements of the tank

1.1.4   Corrosion

Corrosion is defined as the degradation of a material or its properties due to a reaction with its environment. This includes not only "steel," but also metals and plastics that are incompatible with certain chemical compounds. However, for the purposes of this research, we will limit the discussion to corrosion in metals.

Corrosion in metals is an electrochemical reaction, which means a chemical reaction takes place that creates direct electric current flow. Electrochemical corrosion is the corrosion caused by the reaction of metals in an electrolyte solution to form a cell. The contact of impure metals with the electrolyte causes an electrochemical reaction to take place, in which the more active metals are oxidized. The corrosion process of a storage tank made of carbon steel will generally take place as follows: the anode, which acts as an electrochemical reaction, loses electrons and becomes divalent iron ions, while the cathode gains electrons as hydrogen ions and produces hydrogen gas. **[5]**
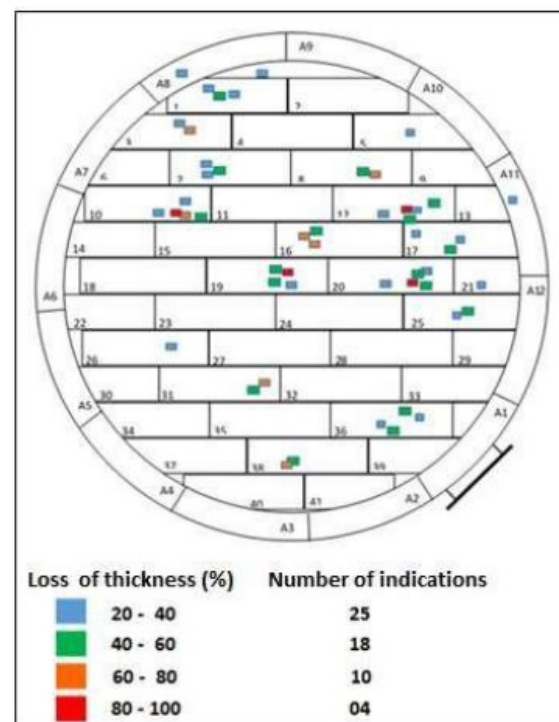
Figure 1.3 Loss of thickness (%) in carbon steel plates of the bottom of a diesel storage tank by the non-destructive inspection (method Magnetic Flux Leakage).

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1.1.5   Oxygen concentration cell

This macroscopic corrosion cell occurs mainly on the underside of the tank floor, in the tank wall panels. It is formed by contact between the tank's constituent material, carbon steel, and a solution with a different oxygen content and is also known as an uneven-filling cell. Immersion of the carbon steel in a neutral solution containing oxygen results in the formation of an oxygen electrode, and the following reactions take place:

$$O_2 + 2H_2O + 4e^- \rightarrow 4OH^-$$
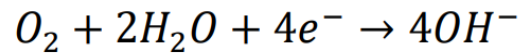
figure (1.3) shows the results of the loss of thickness of the bottom plate of a 40-m diameter diesel storage tank supported on compacted soil by non-destructive inspection, the so-called MFL (Magnetic Flux Leakage). [6]

Chapter 2

2   Equilibrium equations

In engineering world sometimes, the engineers dealing with bodies in equilibrium, which mean that the summation of forces and moments is zero. because the body is in equilibrium, the applied loadings satisfy the equations second law of Newton which is **static equilibrium**. If the entire body is in equilibrium, then all parts must also be in equilibrium. Thus, we can partition any solid into an appropriate subdomain and apply the equilibrium principle to the region.
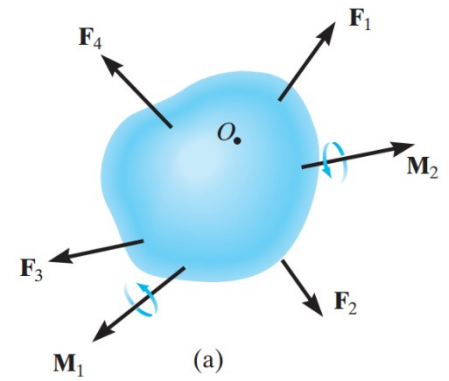
Figure 2.1 equilibrium of the rigid body

$$\sum F = 0$$
$$\sum M_O = 0$$

(3.1)

2.1   Body and Surface Forces

Body forces: are proportional to the body's mass and are reacted with an agent outside of the body. Example of these include gravitational-weight forces, magnetic forces, and inertial forces.

Body force density (force per unit volume) $F(x)$ can be defined such that the total resultant body force of an entire solid can be written as a volume integral over the body
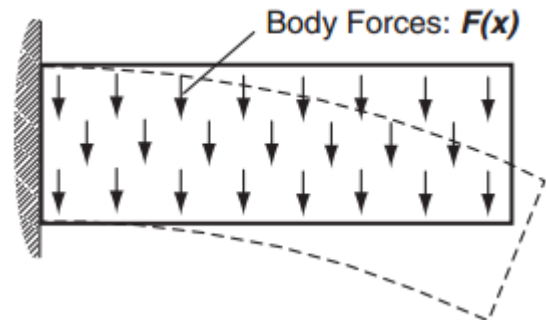
Figure 2.2 Cantilever Beam Under Self-Weight Loading

$$F_R = \iiint_V F(x)\,dV \qquad \textbf{(3.2)}$$

Surface forces: always act on a surface and result from physical contact with another body.

$$F_S = \iint_S T^n(x)\,dS \qquad \textbf{(3.3)}$$

Figure (3.4) illustrates surface forces existing in a beam section that has been created by sectioning the body into two pieces.
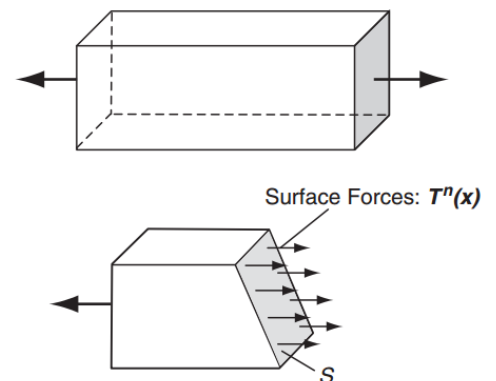
Figure 2.3 Sectioned Axially Loaded Beam

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We use an arbitrary finite subdomain, then we can partition the solid into a subdomain and apply the equilibrium principle to that region
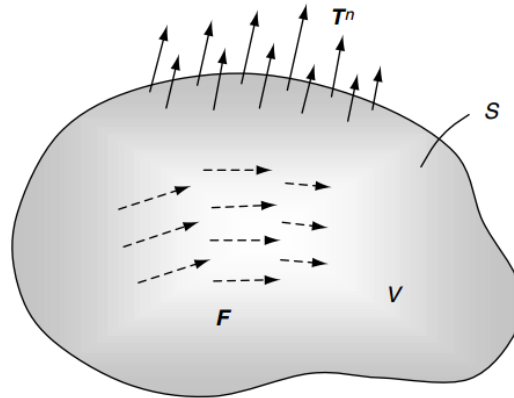
Figure 2.4 Body and surface forces acting on an arbitrary portion of a continuum.

Tⁿ : traction forces
S : surface within a body
V : volume of the body
F : body forces

$$\iint_S T_i{}^n dS + \iiint_V F_i\, dV = 0 \qquad \text{(3.4)}$$

from the principles of elasticity, we can write the traction vector as following:

$$T_i{}^n = \sigma_{ji}\, n_j \qquad \text{(3.5)}$$

Then the express of the equilibrium statement in terms of stress will be:

$$\iint_S \sigma_{ji}\, n_j\, dS + \iiint_V F_i\, dV = 0 \qquad \text{(3.6)}$$

Applying the divergence theorem to the surface integral allows the conversion to a volume integral, and relation **(3.6)** can then be expressed as

$$\iiint_V (\sigma_{ji} + F_i)dV = 0 \qquad \text{(3.7)}$$

Because the region V is arbitrary (any part of the medium can be chosen) and the integrand in **(3.7)** is continuous, then by the zero-value theorem, the integrand must vanish: **[5]**

$$\sigma_{ji,j} + F_i = 0 \qquad \text{(3.8)}$$

2.2   Equilibrium equations in Coordinates System

In order to solve many elasticity problems, formulation must be done in curvilinear coordinates typically using cylindrical or spherical or plan systems, we now wish to develop expressions for the equilibrium equations in this coordinate systems.

2.2.1   Equilibrium equations in Cylindrical coordinate

Cylindrical coordinates are useful in connection with objects and phenomena that have some rotational symmetry about the longitudinal axis, such as water flow in a straight pipe with round cross-section, heat distribution in a metal cylinder, electromagnetic fields produced by an electric current in a long, straight wire, accretion disks in astronomy, and so on. **[6]**

Many problems are such that it is advantageous to use cylindrical coordinates (r, θ, z) instead of Cartesian (x, y, z) coordinates
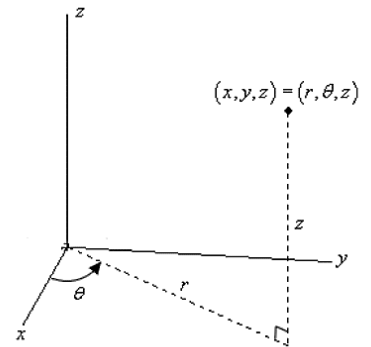
Figure 2.5 Cylindrical coordinat

2.2.1.1 Geometrical Axi-symmetry

A large number of practical engineering problems involve geometrical features which have a natural axis of symmetry, such as the solid cylinder, show, in **figure (3.7)**. The axis of symmetry is an axis of revolution; the feature which possesses axi-symmetry (axial symmetry) can be generated by revolving a surface (or line) About this axis
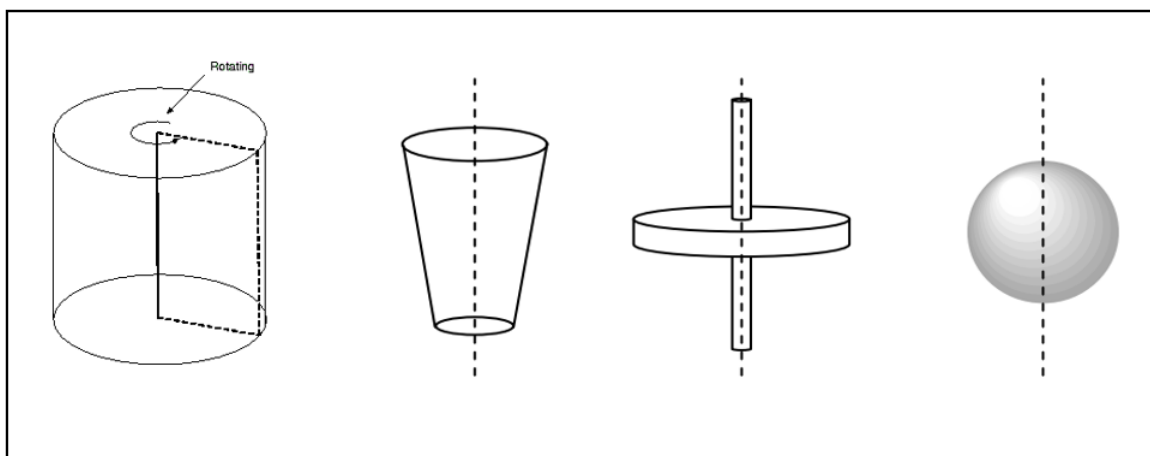
Figure 2.6 axi-symmetry shapes

Some features are not only axisymmetric – they can be represented by a plane, which is similar to other planes right through the axis of symmetry.

In our case we shall begin to developpe the equilibrium equation in cylindrical system, and we start by choosing a small element from the cylindrical shape, and represent the state of stresses in cylindrical coordinates, **figure (3.9)** and then applying Newton secand law. [7]

axisymmetric plane
representative of
feature
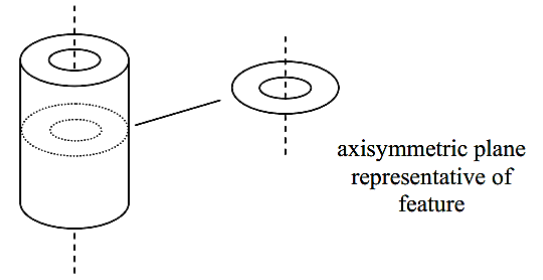
Figure 2.7 axisymmetric plane

Figure 2.8 stress components in cylindrical coordinates

$$\Sigma F_r = 0$$

$$= \left(\sigma_{rr} + \frac{\partial \sigma_{rr}}{\partial r} dr \right) (r+dr) d\theta dz - \sigma_{rr} r d\theta dz$$

$$+ \left(\sigma_{r\theta} + \frac{\partial \sigma_{r\theta}}{\partial \theta} d\theta \right) dr dz \cos\left(\frac{d\theta}{2}\right) - \sigma_{r\theta} dr dz \cos\left(\frac{d\theta}{2}\right)$$

$$+ \left(\sigma_{rz} + \frac{\partial \sigma_{r\theta}}{\partial \theta} d\theta \right) \left(r + \frac{dr}{2} \right) dr d\theta - \left(r + \frac{dr}{2} \right) dr d\theta$$

$$- \left(\sigma_{\theta\theta} + \frac{\partial \sigma_{r\theta}}{\partial \theta} d\theta \right) dr dz \sin\frac{d\theta}{2} - \sigma_{r\theta} dr dz \sin\frac{d\theta}{2} + F_r = 0$$

(3.10)

To simplify the equation, we do the followings:

    1- cancel the equal opposite forces

    2- we know for a very small value of (dθ) then: $\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2}$; $\cos\left(\frac{d\theta}{2}\right) = 1$

    3- dividing by the volume r dr dθ dz

Similarly, we find for the (θ and z) directions:

we find that the equation will be like this:

$$\sum F_r = 0 \qquad \frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta r}}{\partial \theta} + \frac{\partial \sigma_{zr}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + F_r = 0$$

$$\sum F_\theta = 0 \qquad \frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{z\theta}}{\partial z} + \frac{2\sigma_{r\theta}}{r} + F_\theta = 0 \qquad \textbf{(3.11)}$$

$$\sum F_z = 0 \qquad \frac{\partial \sigma_{rz}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta z}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} + F_z = 0$$

2.2.2 *Equilibrium equations in Polar Coordinates*

While discussing the problems with circular boundaries, it is more convenient to use the cylindrical co-ordinates such as (r, θ, z). In the case of plane stress or plane strain problems, we have $\tau_{rz} = \tau_{\theta z} = 0$ and the other stress components as functions of r and θ only. Hence, the cylindrical co-ordinates reduce to polar co-ordinates in this case. In general, polar co-ordinates are used advantageously where a degree of axial symmetry exists. Examples include a cylinder, a disc, a curved beam and a large thin plate containing a circular hole.[8]

In two dimensions, the Cartesian coordinates (x, y) specify the location of a point P in the plane. Another two-dimensional coordinate system is polar coordinates. Instead of using the signed distances along the two coordinate axes, polar coordinates specify the location of a point P in the plane by its distance r from the origin and the angle θ made between the line segment from the origin to P and the positive x-axis [9]
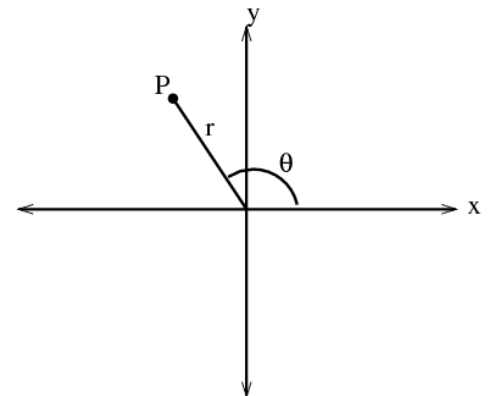
Figure 2.9 Polar Coordinats

We will begin by considering the state of stress on an infinitesimal element of unit thickness described by the polar coordinates as shown in **figure (3.11)**. Resolving the forces in the r-direction, we have for equilibrium:
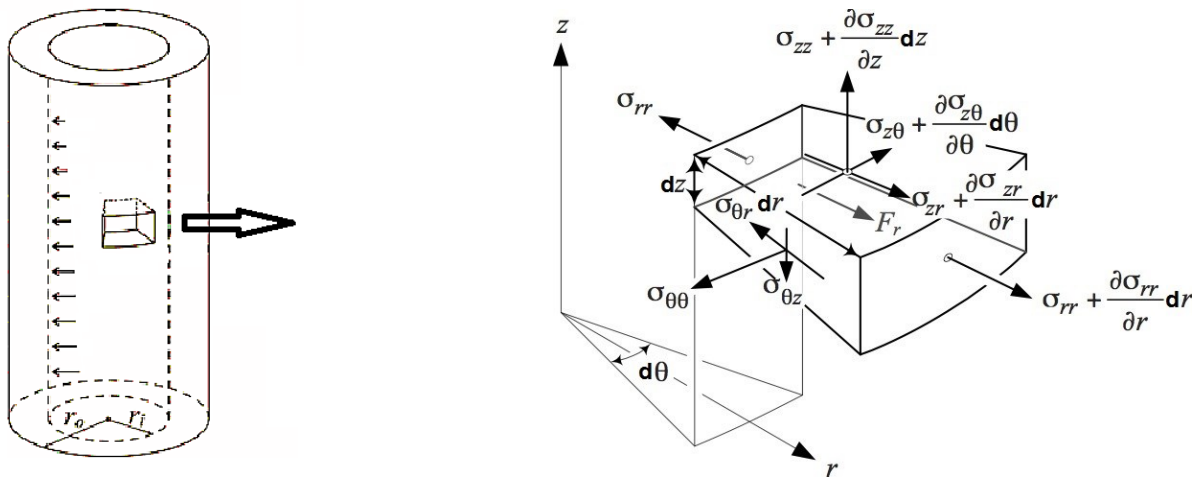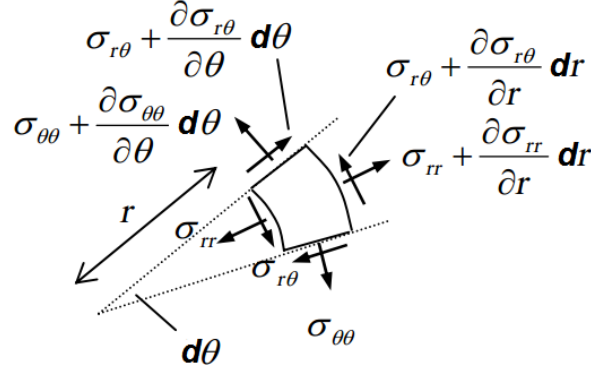

Figure 2.10 Stress components in polar coordinates

$$\Sigma F = 0 \qquad = \left(\sigma_{rr} + \frac{\partial \sigma_{rr}}{\partial r} dr \right)\left(r + dr \right) d\theta - \sigma_{rr} r \, d\theta$$

$$- \sin\frac{d\theta}{2}\left(\sigma_{\theta\theta} + \frac{\partial \sigma_{\theta\theta}}{\partial\theta} dx \right) dr - \sin\frac{d\theta}{2}\left(\sigma_{\theta\theta} \right) dr \qquad (3.12)$$

$$+ \cos\frac{d\theta}{2}\left(\sigma_{r\theta} + \frac{\partial \sigma_{r\theta}}{\partial\theta} d\theta \right) dr - \cos\frac{d\theta}{2}\left(\sigma_{r\theta} \right) dr \; = 0$$

For small element, $\mathbf{sin}\left(\frac{\mathbf{d\theta}}{\mathbf{2}}\right) = \frac{\mathbf{d\theta}}{\mathbf{2}}$; $\mathbf{cos}\left(\frac{\mathbf{d\theta}}{\mathbf{2}}\right) = \mathbf{1}$ and so, dividing through by dr dθ we get

$$\frac{\partial \sigma_{rr}}{\partial r}\left(r + dr \right) + \sigma_{rr} - \sigma_{\theta\theta} - \frac{d\theta}{2}\left(\frac{\partial \sigma_{\theta\theta}}{\partial\theta} \right) + \frac{\partial \sigma_{r\theta}}{\partial\theta} = 0 \qquad (3.13)$$

In the limit as (dr, dθ) → 0, Then we do the same method for the tangential direction we get:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial\theta} + \frac{1}{r}\left(\sigma_{rr} - \sigma_{\theta\theta} \right) = 0 \qquad (3.14)$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial\theta} + \frac{2\sigma_{r\theta}}{r} = 0 \qquad (3.15)$$

2.2.3   Spherical coordinate system

Spherical coordinates can be a little challenging to understand at first. Spherical coordinates determine the position of a point in three-dimensional space based on the distance R from the origin and two angles θ and φ. If one is familiar with polar coordinates, then the angle φ isn't too difficult to understand as it is essentially the same as the angle θ from polar coordinates.[9]
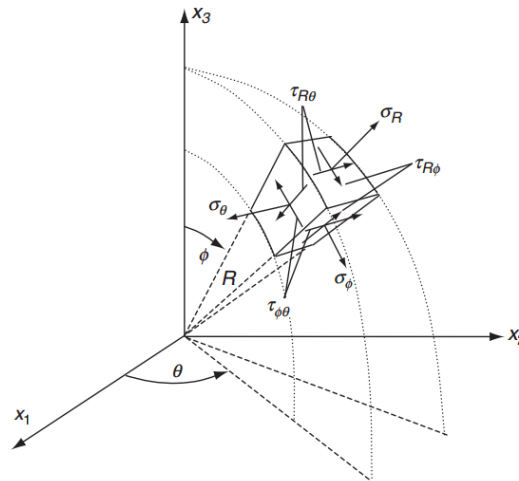
Figure 2.11 Stress components in spherical coordinates

We now wish to repeat the development of equilibrium equations for the spherical coordinate system, the stress components in spherical coordinates are defined on the differential element illustrated in **figure (3.12)**, and we do the same steps as before we start by equilibrium law of Newton and simplifying the equations like we did in the cylindrical system and we get the following equations :

$$\sum F_r = 0 \qquad \frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \sigma_{\phi r}}{\partial \phi} + \frac{1}{r}\left(2\sigma_{rr} - \sigma_{\theta\theta} - \sigma_{\phi\phi} + \sigma_{r\theta}\cot\theta\right) = 0$$

$$\sum F_\theta = 0 \qquad \frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \sigma_{\phi\theta}}{\partial \phi} + \frac{1}{r}\left[(\sigma_{\phi\phi} - \sigma_{\theta\theta})\cot\theta + 3\sigma_{r\theta}\right] = 0 \qquad \textbf{(3.16)}$$

$$\sum F_\phi = 0 \qquad \frac{\partial \sigma_{r\phi}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\phi}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \sigma_{\phi\phi}}{\partial \phi} + \frac{1}{r}\left(3\sigma_{r\phi} + 2\sigma_{\theta\phi}\cot\theta\right) = 0$$

2.3   The application of coordinate systems in stress calculation in the tank

to calculate the stress in the tank we have to model it, so we choose SolidWorks to model it
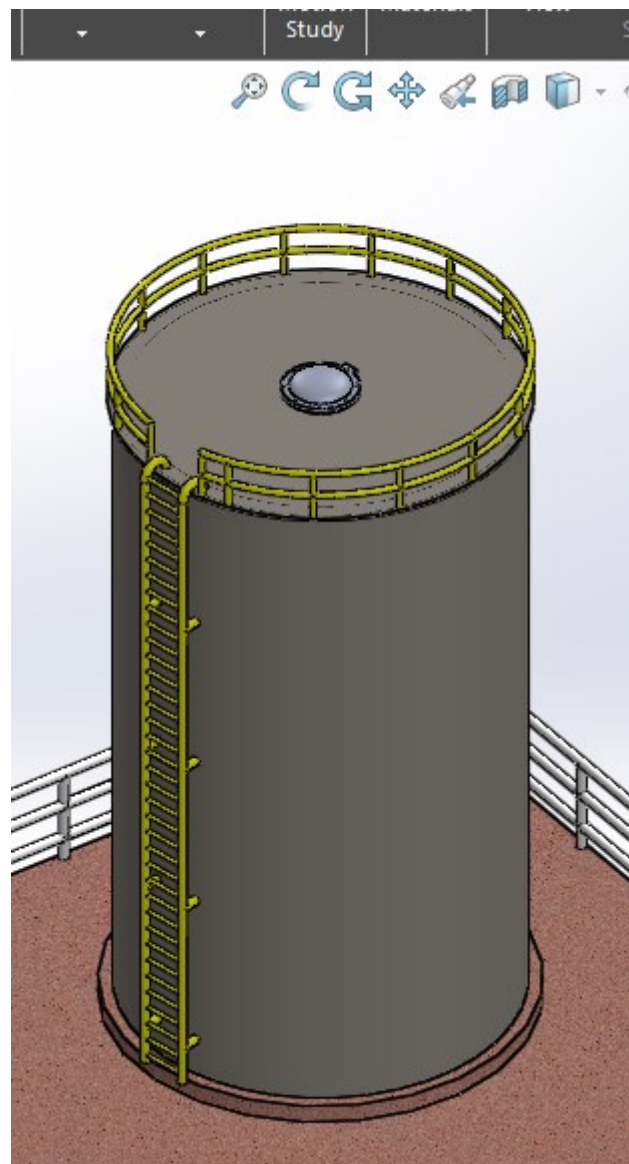
Figure 2.12 Tank model in SolidWorks

1.4.2   Cylindrical face

To calculate the stresses in the cylindrical wall of the tank we consider it as a thick-walled cylinder subjected to internal load (pressure) then we simplify it, because we have an axisymmetric plan, it is convenient to express these problem in terms of the cylindrical coordinates, but because of symmetry the stress components are independent of the angular (θ) coordinate, so, all derivatives with respect to vanish and the components σ_{zz} , $\tau_{r\theta,}$ $\tau_{\theta z}$ are zero, and we neglect the body forces, then the equation will be:

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The next figure show the analytic model we choose to study which represent an axisymmetric plan and that's simplify the calculation
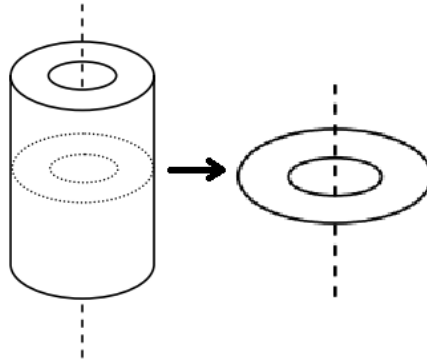
Figure 2.13 Axi-symmetric plane

So after applying all the simplification to the equation **(3.11)**, it reduces to:

$$r\frac{\partial \sigma_{rr}}{\partial r} + \sigma_{rr} - \sigma_{\theta\theta} = 0 \tag{3.17}$$

Since (r) is the only independent variable, the above equation can be written as

$$r\frac{\partial \sigma_{rr}}{\partial r} + \sigma_{rr} - \sigma_{\theta\theta} = 0$$

$$\frac{d}{dr}(r\,\sigma_r) - \sigma_\theta = 0 \tag{3.18}$$

For tow-dimensional state of stresses and strains the Hooks law will be:

$$\varepsilon_r = \frac{1}{E}\left(\sigma_r - v\,\sigma_\theta\right), \quad \varepsilon_\theta = \frac{1}{E}\left(\sigma_\theta - v\,\sigma_r\right) \tag{3.19}$$

Also

$$\varepsilon_r = \frac{du}{dr}, \quad \varepsilon_\theta = \frac{u}{r} \tag{3.20}$$

The stresses in terms of strains are

$$\sigma_r = \frac{E}{\left(1 - v^2\right)}\left(\varepsilon_r - v\,\varepsilon_\theta\right), \quad \sigma_\theta = \frac{E}{\left(1 - v^2\right)}\left(\varepsilon_\theta - v\,\varepsilon_r\right) \tag{3.21}$$

Substituting the values of and in the above expressions, we get

$$\sigma_r = \frac{E}{\left(1-v^2\right)}\left(\frac{du}{dr} - v\frac{u}{r}\right) \qquad \sigma_\theta = \frac{E}{\left(1-v^2\right)}\left(\frac{u}{r} - v\frac{du}{dr}\right) \tag{3.22}$$

Substituting these in the equilibrium Eq., then

$$\frac{d}{dr}\left(r\,\frac{E}{\left(1-v^2\right)}\left(\frac{du}{dr} - v\frac{u}{r}\right)\right) - \frac{E}{\left(1-v^2\right)}\left(\frac{u}{r} - v\frac{du}{dr}\right) = 0 \tag{3.23}$$

Dividing by $\dfrac{E}{\left(1-v^2\right)}$ it will be:

$$\frac{d}{dr}\left(r\left(\frac{du}{dr} - v\frac{u}{r}\right)\right) - \left(\frac{u}{r} - v\frac{du}{dr}\right) = 0$$

$$\frac{du}{dr} + r\frac{d^2u}{dr^2} + v\frac{du}{dr} - \frac{u}{r} - v\frac{du}{dr} = 0 \tag{3.24}$$

Canceling the opposite terms and dividing by r will be:

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} = 0 \tag{3.25}$$

The above equation is called equidimensional equation in radial displacement.

The solution of the above equation is

$$U = C_1 r + \frac{C_2}{r}$$
$$\frac{du}{dr} = C_1 - C_2\frac{1}{r^2} \tag{3.26}$$

Where C_1 and C_2 are constants.

The radial and tangential stresses are written in terms of constants of integration C_1 and C_2

Therefore,

$$\sigma_r = \frac{E}{\left(1-v^2\right)}\left(\frac{du}{dr} + v\frac{u}{r}\right)$$

$$= \frac{E}{\left(1-v^2\right)}\left(C_1 - C_2\frac{1}{r^2} + v\,C_1 + v\frac{C_2}{r^2}\right) \tag{3.27}$$

$$\sigma_r = \frac{E}{\left(1-v^2\right)}\left(C_1(1+v) - C_2\left(\frac{1-v}{r^2}\right)\right)$$

With the same way we do to second equation:

$$\sigma_\theta = \frac{E}{\left(1-v^2\right)}\left(C_1(1+v) - C_2\left(\frac{1-v}{r^2}\right)\right) \tag{3.28}$$

The constants are determined from the boundary conditions. When:

$$r = R_1 \quad \Rightarrow \quad \sigma_r = -p_{in} = \rho g\,z$$
$$r = R_2 \quad \Rightarrow \quad \sigma_r = 0$$

Substituting this conditions in the following:

$$\sigma_r = \frac{E}{\left(1-v^2\right)}\left[C_1(1+v) - C_2\left(\frac{1-v}{R_1^{\,2}}\right)\right] = -\rho g\,z$$

$$\sigma_\theta = \frac{E}{\left(1-v^2\right)}\left[C_1(1+v) - C_2\left(\frac{1-v}{R_2^{\,2}}\right)\right] = 0 \tag{3.29}$$

If we devise both of the equations by $\frac{E}{(1-v^2)}$ we get:

$$C_1\left(1+v\right) - C_2\left(\frac{1-v}{R_1^{\,2}}\right) = -\rho g\, z\, \frac{\left(1-v^2\right)}{E}$$

$$C_1\left(1+v\right) - C_2\left(\frac{1-v}{R_2^{\,2}}\right) = 0$$

(3.29.1)

After that we subtract the second equation from the first we get:

$$-C_2\left(\frac{1-v}{R_1^{\,2}}\right) + C_2\left(\frac{1-v}{R_2^{\,2}}\right) = -\rho g\, z\, \frac{\left(1-v^2\right)}{E}$$

(3.29.2)

Then we multiply by (-1) for each side of the eq. **(3.29)**, and take $C_2(1-v)$ as a common factor we get:

$$C_2\left(1-v\right)\left(\frac{1}{R_1^{\,2}} - \frac{1}{R_2^{\,2}}\right) = \rho g\, z\, \frac{\left(1-v^2\right)}{E}$$

(3.29.3)

If we simplify the formula and make $(1-v^2) = (1-v)(1+v)$, we get:

$$C_2\left(1-v\right)\left(\frac{R_2^{\,2}-R_1^{\,2}}{R_1^{\,2}R_2^{\,2}}\right) = \rho g\, z\, \frac{\left(1-v\right)\left(1+v\right)}{E}$$

(3.29.4)

And if we cancel the term $(1-v)$ from the equation and organize the formula we get:

$$C_2 = \frac{\left(1+v\right)}{E}\left(\frac{R_1^{\,2}R_2^{\,2}\,\rho g\, z}{R_2^{\,2}-R_1^{\,2}}\right)$$

(3.29.5)

Then we can find C_1 and C_2

$$C_1 = \frac{\left(1-v\right)}{E}\left(\frac{R_1^{\,2}\,\rho g\, z}{\left(R_2^{\,2}-R_1^{\,2}\right)}\right)$$

(3.30)

Substituting C_1 and C_2 in **Eq. (3.29)**. We get [10]

$$\sigma_r = \frac{R_1^{\,2}\,\rho g\, z}{\left(R_2^{\,2}-R_1^{\,2}\right)}\left(1-\frac{R_2^{\,2}}{r^2}\right) \qquad \sigma_\theta = \frac{R_1^{\,2}\,\rho g\, z}{\left(R_2^{\,2}-R_1^{\,2}\right)}\left(1+\frac{R_2^{\,2}}{r^2}\right)$$

(3.31)

to have a good understanding for the equation, we show the plot of it: [10]
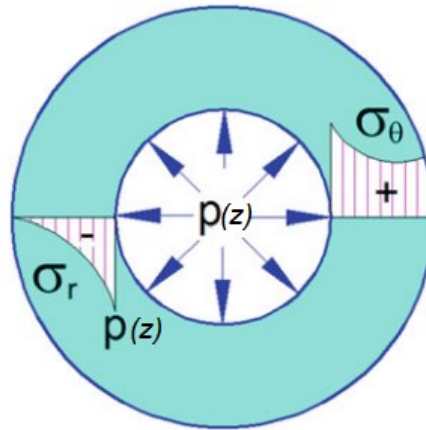
Figure 2.14 Plot of radial stress and tangential stress

2.3.1   Flat Bottom

We can model the bottom of the tank by a Circular plate under a uniform load (q) which represent the hydrostatic pressure duo to the liquid in the tank.
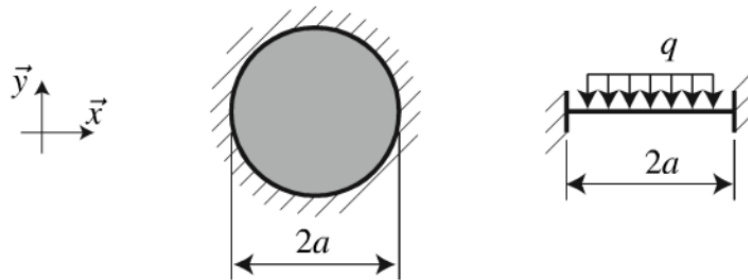
Figure 2.15 uniform load on circular plate

If we restrict the model with some assumption, we can actually make a better solution for the problem and that's done by the following assumption:

- **The mid-plane is a "neutral plane":**

Figure 2.16 A plate under lateral load

- The middle plane of the plate remains free of in-plane stress/strain. Bending of the plate will cause material above and below this mid-plane to deform in-plane as shown in the figure

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- **Line elements remain normal to the mid-plane**

Line elements lying perpendicular to the middle surface of the plate remain perpendicular to the middle surface during deformation, this is similar the "plane sections remain plane" assumption of the beam theory.

- **Vertical strain is ignored**

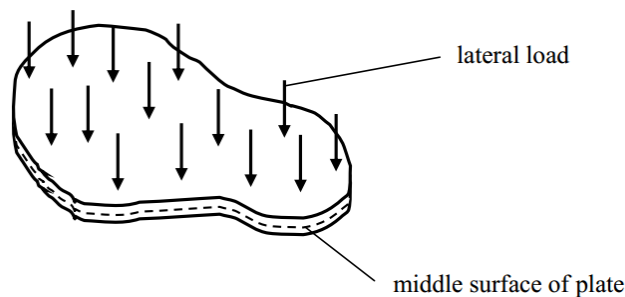Line elements lying perpendicular to the mid-surface do not change length during deformation, so that $\varepsilon_{zz} = 0$ throughout the plate. Again, this is similar to an assumption of the beam theory. These three assumptions are the basis of the **Classical Plate Theory** or the **Kirchhoff Plate Theory**. The second assumption can be relaxed to develop a more exact theory. [11]

2.3.1.1 Plat theory

Plates subjected only to in-plane loading can be solved using two-dimensional plane stress theory on the other hand, plate theory is concerned mainly with lateral loading, one of the differences between plane stress and plate theory is that in the plate theory the stress components are allowed to vary through the thickness of the plate, so that there can be bending moments In-plane normal forces and bending moments are:

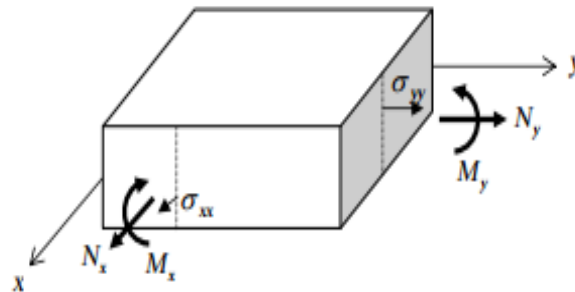*Figure 2.17   in-plane normal forces and bending moments*

$$N_x = \int_{-h/2}^{+h/2} \sigma_{xx}\,dz \quad N_y = \int_{-h/2}^{+h/2} \sigma_{yy}\,dz$$

$$M_x = \int_{-h/2}^{+h/2} z\,\sigma_{xx}\,dz \quad M_y = \int_{-h/2}^{+h/2} z\,\sigma_{yy}\,dz$$

(3.32)

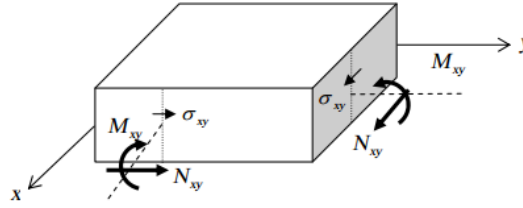In-plane shear force and twisting moment, **figure (3.19)**:

Figure 2.18 in-plane shear force and twisting moment

$$N_{xy} = \int\limits_{-h/2}^{+h/2} \sigma_{xy} dz \quad M_{xy} = \int\limits_{-h/2}^{+h/2} z\, \sigma_{xy} dz \tag{3.33}$$

In classical plate theory the strains will be [11]

$$\varepsilon_{xx} = -z\,\frac{\partial^2 w}{\partial x^2}, \;\; \varepsilon_{yy} = -z\,\frac{\partial^2 w}{\partial y^2}, \;\; \varepsilon_{xy} = -z\,\frac{\partial^2 w}{\partial x\,\partial y} \tag{3.34}$$

2.3.1.2 Stresses and the Curvatures in a linear Elastic Plate:

From Hooke's law, taking $\sigma_{zz} = 0$

$$\varepsilon_{xx} = \frac{1}{E}\sigma_{xx} - \frac{1}{E}\sigma_{yy}, \;\; \varepsilon_{yy} = \frac{1}{E}\sigma_{yy} - \frac{1}{E}\sigma_{xx}, \;\; \varepsilon_{xy} = \frac{1+v}{E}\sigma_{xy} \tag{3.35}$$

So from **(3.34)**, and solving **(3.35)** for the normal stresses

$$\sigma_{xx} = -\frac{E}{1-v^2}\,z\left(\frac{\partial^2 w}{\partial x^2} + v\,\frac{\partial^2 w}{\partial y^2}\right)$$

$$\sigma_{yy} = -\frac{E}{1-v^2}\,z\left(\frac{\partial^2 w}{\partial x^2} + v\,\frac{\partial^2 w}{\partial y^2}\right) \tag{3.36}$$

$$\sigma_{xy} = -\frac{E}{1-v}\,z\,\frac{\partial^2 w}{\partial x\,\partial y}$$

2.3.1.3 The moment-Curvature Equations

$$M_x = D\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right), \; M_y = D\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2}\right), \; M_{xy} = -D\left(1+v\right)\frac{\partial^2 w}{\partial x\,\partial y} \quad \textbf{(3.37)}$$

Where

$$D = \frac{E\,h^2}{12\,(1-v^2)} \quad \textbf{(3.38)}$$

The factor **D** is called the **plate stiffnes**s or **flexural rigidity** and plays the same role in the plate theory as does the flexural rigidity term **EI** in the beam theory.

Now this is similar to the beam formula $\sigma = -\frac{M}{I}y$ the stresses in the plate will be:

$$\sigma_{xx} = -\frac{M_x}{h^3/12}z, \; \sigma_{yy} = -\frac{M_y}{h^3/12}z, \; \sigma_{xy} = \frac{M_{xy}}{h^3/12}z \quad \textbf{(3.39)}$$

2.3.1.4 Deflection of a Circular plate under a uniform lateral load

A circular plate with boundary

$$x^2 + y^2 = a^2 \quad \textbf{(3.40)}$$

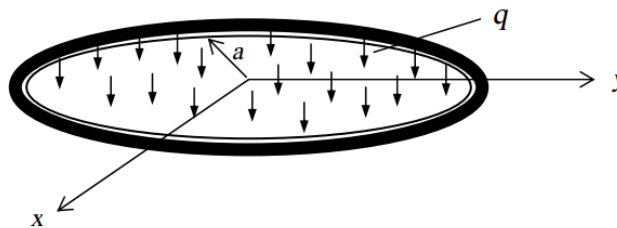Welded at its edges and subjected to a uniform lateral load P

Figure 2.19 circular plate subjected to a uniform lateral load

The differential equation for the problem is given by:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = -\frac{q}{D} \quad \textbf{(3.41)}$$

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This equation is called the equation of Sophie Germain after the French investigator who first obtained in 1815 [11]. This partial differential equation is solved subject to the boundary conditions of the problem. The boundary conditions are that the slope and deflection are zero ate the boundary, along $x^2 + y^2 = a^2$

$$w = 0, \quad \frac{\partial w}{\partial x} = 0, \quad \frac{\partial w}{\partial y} = 0 \tag{3.42}$$

It will be shown that the deflection

$$w = c\,(x^2 + y^2 - a^2)^2 \tag{3.43}$$

Is a solution to the problem. First, this function certainly satisfies (). Further, letting

$$f(x,y) = x^2 + y^2 - a^2 \tag{3.45}$$

The relevant partial derivatives are

$$\frac{\partial w}{\partial x} = 4c\,x\,f, \qquad\qquad \frac{\partial w}{\partial y} = 4c\,y\,f$$

$$\frac{\partial^2 w}{\partial x^2} = 4c\left(2x^2 + f\right), \qquad \frac{\partial^2 w}{\partial x\,\partial y} = 8c\,x\,y, \qquad \frac{\partial^2 w}{\partial y^2} = 4c\left(2y^2 + f\right)$$

$$\frac{\partial^3 w}{\partial x^3} = 24c\,x, \qquad \frac{\partial^3 w}{\partial x^2\partial y} = 8c\,y, \qquad \frac{\partial^3 w}{\partial x\,\partial y^2} = 8c\,x, \qquad \frac{\partial^3 w}{\partial y^3} = 24c\,y$$

$$\frac{\partial^4 w}{\partial x^4} = 24c, \qquad \frac{\partial^4 w}{\partial x^2\partial y^2} = 8c, \qquad \frac{\partial^4 w}{\partial y^4} = 24c \tag{3.46}$$

Substituting these into the differential equation **(3.41)**, yields to:

$$c = -\frac{q}{64D} \tag{3.47}$$

So, the deflection is

$$w = -\frac{q}{64D}\left(x^2 + y^2 + a^2\right)^2 \tag{3.48}$$

The maximum deflection occurs at the plate center, where

$$W_{max} = -\frac{q\,a^4}{64\,D} \tag{3.49}$$

The moments occurring in the plate are, from the moment-curvature equations ( ) and ( )

$$M_x = -\frac{q}{16}\left[(3+v)x^2 + (3v+1)y^2 - (1+v)a^2\right]$$

$$M_y = -\frac{q}{16}\left[(3+v)x^2 + (3v+1)y^2 - (1+v)a^2\right] \tag{3.50}$$

$$M_{xy} = \frac{q}{16}(1+v)\,x\,y$$

1.4.2.1 Stress in the Plate

From (3.50), (3.39), the stresses in the plate will be:

$$\sigma_x = -\frac{3q\,z}{4h^3}\left[(3+v)x^2 + (3v+1)y^2 - (1+v)a^2\right]$$

$$\sigma_y = -\frac{3q\,z}{4h^3}\left[(3+v)x^2 + (3v+1)y^2 - (1+v)a^2\right]$$

$$\sigma_{xy} = \frac{3q\,z}{2h^3}(1+v)\,x\,y$$

$$\sigma_{zx} = \frac{3q\,z}{4h}\left[1-\left(\frac{z}{h/2}\right)^2\right] \tag{3.51}$$

$$\sigma_{zy} = \frac{3q\,z}{4h}\left[1-\left(\frac{z}{h/2}\right)^2\right]$$

Converting to polar coordinates (r, θ) through

$$x = r\cos\theta,\; y = r\sin\theta \tag{3.52}$$

And using a stress transformation [11],

$$\sigma_{rr} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + \sigma_{xy}\sin 2\theta$$

$$\sigma_{\theta\theta} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta - \sigma_{xy}\sin 2\theta \tag{3.53}$$

$$\sigma_{r\theta} = \cos\theta\sin\theta\left(\sigma_{yy} - \sigma_{xx}\right) + \sigma_{xy}\cos 2\theta$$

Leads to the axisymmetric stress field:

$$\sigma_{rr} = \frac{3q\,z}{4h^3}\left[(3+v)r^2 - (1+v)a^2\right]$$

$$\sigma_{\theta\theta} = \frac{3q\,z}{4h^3}\left[(3v+1)r^2 - (1+v)a^2\right] \qquad \textbf{(3.54)}$$

$$\sigma_{r\theta} = 0$$

Where: **q:** lateral pressure
              z: perpendicular distance from the neutral plan to a point away from it.
              h: the thickness of the plate

At the center of the plate

$$\sigma_{rr} = \sigma_{\theta\theta} = \frac{3q\,z\,a^2}{4h^3}(1+v)a^2 \qquad \textbf{(3.55)}$$

At the edge of the plate **r = a**,

$$\sigma_{rr} = \frac{3\,q\,z\,a^2}{2h^3}, \quad \sigma_{\theta\theta} = \frac{3\,q\,z\,a^2}{2h^3}v \qquad \textbf{(3.56)}$$

The maximum stress in the plate is

$$\sigma_{rr} = \sigma_{rr}\left(a, \frac{h}{2}\right) = \frac{3q}{4}\left(\frac{a}{h}\right)^2 \qquad \textbf{(3.57)}$$

2.3.2   Hemisphere Bottom

A sphere is a very strong structure and this make it the best shape of a pressure vessel. but a spherical shape is tough to manufacture, therefore more expensive. The even distribution of stresses on the sphere's surfaces, both internally and externally, generally means that there are no weak points. Spheres however, are much more costly to manufacture than cylindrical vessels. [12]

So, we want to analyze this shape which is a spherically symmetric, and determine the stress formula of it in every point of the hemisphere as shown in fig. **(3.21)**

Figure 2.20 Hemispheric bottom of the tank

We return to the equation **(3.16)** and simplify it, because we have an axisymmetric case where:

$$u_r = u(r), \quad u_\varphi = u_\theta = 0 \tag{3.57.1}$$

We use this relationship between strain and displacement, form Appendix of the book **[5]**

$$\varepsilon_r = \frac{\partial u_r}{\partial r}, \quad \varepsilon_\varphi = \frac{1}{r}\left(u_r + \frac{\partial u_\varphi}{\partial \varphi}\right), \quad \varepsilon_\theta = \frac{1}{r\sin\varphi}\left(\frac{\partial u_\theta}{\partial \theta} + \sin\varphi u_r + \cos\varphi u_\varphi\right)$$

$$\varepsilon_{r\varphi} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \varphi} + \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r}\right)$$

$$\varepsilon_{\varphi\theta} = \frac{1}{2r}\left(\frac{1}{\sin\varphi}\frac{\partial u_\varphi}{\partial \theta} + \frac{\partial u_\theta}{\partial \varphi} - \cot\varphi u_\theta\right) \tag{3.58}$$

$$\varepsilon_{\theta r} = \frac{1}{2}\left(\frac{1}{r\sin\varphi}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right)$$

Substitute the eq. **(3.57.1)** in **(3.58)**

$$\varepsilon_r = \frac{\partial u_r}{\partial r}, \quad \varepsilon_\varphi = \varepsilon_\theta = \frac{u_r}{r}, \quad \varepsilon_{r\varphi} = \varepsilon_{\varphi\theta} = \varepsilon_{\theta r} = 0 \tag{3.59}$$

We know that hook's law in spherical coordinates is:

$$\sigma_r = \lambda(\varepsilon_r + \varepsilon_\varphi + \varepsilon_\theta) + 2\mu\varepsilon_r$$

$$\sigma_\varphi = \lambda(\varepsilon_r + \varepsilon_\varphi + \varepsilon_\theta) + 2\mu\varepsilon_\varphi$$

$$\sigma_\theta = \lambda(\varepsilon_r + \varepsilon_\varphi + \varepsilon_\theta) + 2\mu\varepsilon_\theta$$

$$\tau_{r\varphi} = 2\mu\varepsilon_{r\varphi} \tag{3.60}$$

$$\tau_{\varphi\theta} = 2\mu\varepsilon_{\varphi\theta}$$

$$\tau_{\theta r} = 2\mu\varepsilon_{\theta r}$$

Then we substitute the equation **(3.59)** in **(3.60)** and we get:

$$\sigma_r = \lambda\left[\left(\frac{\partial u_r}{\partial r}\right) + \left(\frac{u_r}{r}\right) + \left(\frac{u_r}{r}\right)\right] + 2\mu\left(\frac{\partial u_r}{\partial r}\right) = (\lambda + 2\mu)\left(\frac{\partial u_r}{\partial r}\right) + 2\lambda\left(\frac{u_r}{r}\right)$$

$$\sigma_\theta = \sigma_\varphi = \lambda\left[\left(\frac{\partial u_r}{\partial r}\right) + \left(\frac{u_r}{r}\right) + \left(\frac{u_r}{r}\right)\right] + 2\mu\left(\frac{u_r}{r}\right) = \lambda\left(\frac{\partial u_r}{\partial r}\right) + (2\lambda + 2\mu)\left(\frac{u_r}{r}\right) \qquad \textbf{(3.61)}$$

$$\tau_{r\varphi} = \tau_{\varphi\theta} = \tau_{\theta r} = 0$$

If we substitute this equation in eq. **(3.16)** we get:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{2}{r}\left(\sigma_{rr} - \sigma_{\theta\theta}\right) = 0 \qquad \textbf{(3.62)}$$

Replacing $(\boldsymbol{\sigma_{rr}}, \boldsymbol{\sigma_{\theta\theta}})$ with their form we get

$$\begin{aligned}
\frac{\partial \sigma_{rr}}{\partial r} &= \frac{\partial}{\partial r}\left[(\lambda + 2\mu)\left(\frac{\partial u_r}{\partial r}\right) + 2\lambda\left(\frac{u_r}{r}\right)\right] \\
&= (\lambda + 2\mu)\frac{\partial^2 u_r}{\partial r^2} + 2\lambda\left(\frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2}\right) \qquad \textbf{(3.63)} \\
&= (\lambda + 2\mu)\frac{\partial^2 u_r}{\partial r^2} + 2\lambda\frac{1}{r}\frac{\partial u_r}{\partial r} - 2\lambda\frac{u_r}{r^2}
\end{aligned}$$

Substitute **(3.63)** in **(3.62)**, we get:

$$\begin{aligned}
&\left[(\lambda + 2\mu)\frac{\partial^2 u_r}{\partial r^2} + 2\lambda\frac{1}{r}\frac{\partial u_r}{\partial r} - 2\lambda\frac{u_r}{r^2}\right] + \\
&\frac{2}{r}\left[(\lambda + 2\mu)\left(\frac{\partial u_r}{\partial r}\right) + 2\lambda\left(\frac{u_r}{r}\right) - \lambda\left(\frac{\partial u_r}{\partial r}\right) + (2\lambda + 2\mu)\left(\frac{u_r}{r}\right)\right] = 0
\end{aligned} \qquad \textbf{(3.64)}$$

$$(\lambda + 2\mu)\frac{\partial^2 u_r}{\partial r^2} + \left(\frac{2\lambda}{r} + 2\frac{(\lambda + 2\mu)}{r} - \frac{2\lambda}{r}\right)\frac{\partial u_r}{\partial r} + \left(-\frac{2\lambda}{r^2} + \frac{4\lambda}{r^2} + 2\frac{(2\lambda + 2\mu)}{r^2}\right)u_r = 0 \quad \textbf{(3.65)}$$

After simplification and dividing by $(\lambda + 2\mu)$, and because we have only one variable **(r)**

we can change the partial derivative to ordinary one, and we get:

$$\frac{d^2 u_r}{dr^2} + \frac{2}{r}\frac{du_r}{dr} - \frac{2}{r^2}u_r = 0 \qquad \textbf{(3.66)}$$

The equation **(9)** is second order deferential equation the form of the solution is:

$$u = C_1 r + \frac{C_2}{r^2} \qquad (3.67)$$

So, if we substitute this equation into eq. **(3.61)** we get:

$$\sigma_r = K_1 - \frac{2K_2}{R^3}, \qquad \sigma_\theta = \sigma_\varphi = K_1 + \frac{K_2}{R^3} \qquad (3.68)$$

Where:

$$K_1 = \frac{2\mu(1+v)}{1-2v} C_1, \qquad K_2 = 2\mu C_2 \qquad (3.69)$$

To find the formula of stress we consider that the spherical shell is subjected to an internal pressure P1 and external pressure P2, and applying the boundary Conditions we get:

$$\sigma_r(r_1) = -P_1, \qquad \sigma_r(r_2) = -P_2 \qquad (3.70)$$

And if we substitute this in the equation **(3.68)**, we get:

$$K_1 = \frac{P_1 R_1^{\,3} - P_2 R_2^{\,3}}{R_2^{\,3} - R_1^{\,3}}, \qquad K_2 = \frac{(P_1 - P_2) R_1^{\,3} R_2^{\,3}}{2(R_2^{\,3} - R_1^{\,3})} \qquad (3.71)$$

And the stresses become:

$$\sigma_r = \frac{P_1 R_1^{\,3} - P_2 R_2^{\,3}}{R_2^{\,3} - R_1^{\,3}} - \frac{(P_1 - P_2) R_1^{\,3} R_2^{\,3}}{(r_2^{\,3} - r_1^{\,3})} \frac{1}{r^3}$$

$$\sigma_\phi = \sigma_\theta = \frac{P_1 R_1^{\,3} - P_2 R_2^{\,3}}{R_2^{\,3} - R_1^{\,3}} + \frac{(P_1 - P_2) R_1^{\,3} R_2^{\,3}}{2(R_2^{\,3} - R_1^{\,3})} \frac{1}{r^3} \qquad (3.72)$$

For our case we have $P_1 = P$ and $P_2 = 0$, then the equation **(3.72)** will be:

$$\sigma_r = \frac{P_1 R_1^{\,3}}{R_2^{\,3} - R_1^{\,3}} \left[1 - \frac{R_2^{\,3}}{r^3} \right]$$

$$\sigma_\phi = \sigma_\theta = \frac{P_1 R_1^{\,3}}{R_2^{\,3} - R_1^{\,3}} \left[1 + \frac{R_2^{\,3}}{2r^3} \right] \qquad (3.73)$$

2.4   Numerical application for the Tank

Our objective in this section is to apply the equation that we developpe in the previous sections. So, We chose for this purpose a 2 tank of the following form:
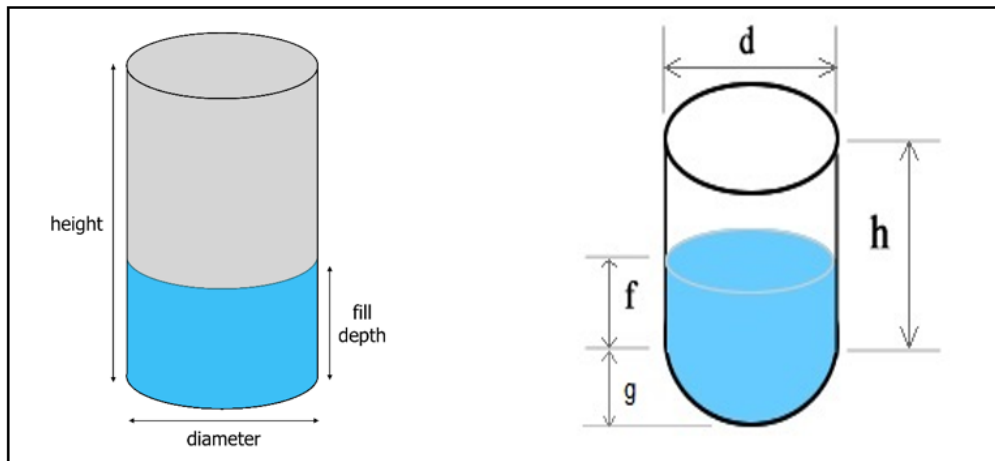

Figure 2.21dimension of the liquid in the tank

2.4.1   Assumptions

2.4.1.1   A Flat bottom cylindrical tank

We want to contain a volume of **187 m³** of diesel gasoline, which has the following characteristics

Fuel Characteristics	Diesel
Density at 35 °C (kg/m³)	812
Kinematic viscosity at 35 °C (mm²/s)	2.90

For our tank we choose the initial dimension to contain this volume which is:

- **Diameter: 5m**
- **Fill depth: 9.5m**
- **High: 10m**

3.4.1.2   Hemispheric bottom cylindrical tank

For this tank we choose the same dimension except we add the hemispheric volume and for this the initial dimension will be as following:

- **Diameter: 5m**
- **Fill depth: f = 9.5m plus hemispheric volume**
- **High: h=10m plus 2.5m**

We choose the material to be stainless steel which have the following characteristics: **[12]**

Materials	Specific Weight (lb/in³)	Moduls of Elasticity E (10³) ksi	Modulus of Rigidity G (10³) ksi	Yield Strength (ksi) σ_Y			Ultimate Strength (ksi) σ_u			%Elongation in 2 in. specimen	Poisson's Ratio ν	Coef. of Therm. Expansion α (10⁻⁶)/°F
				Tens.	Comp.ᵇ	Shear	Tens.	Comp.ᵇ	Shear			
Steel Alloy Stainless 304	0.284	28.0	11.0	30	30	–	75	75	–	40	0.27	9.60

Table 2-1 Stainless steel characteristics

And assume the following point in the properties of the material to be:

- **Homogenous**
- **Isotropy**
- **No crack**

2.4.2   Cylindrical wall

Cylinders are widely used for storage due to their being less expensive to produce than spheres. However, cylinders are not as strong as spheres due to the weak point at each end.

This weakness is reduced by hemispherical or rounded ends being fitted. If the whole cylinder is manufactured from thicker material than a comparable spherical vessel of similar capacity, storage pressure can be similar to that of a sphere.

Figure 2.22 cylindrical wall under hydrostatic pressure

To analyze the stresses in the cylindrical face in the tank we model it as shown in figure **(2.23)**. Then we calculate the hydrostatic pressure, applying the of hydrostatics.
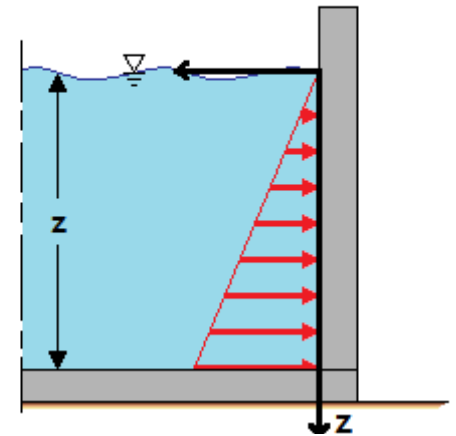
$$P(z) = \rho g z = 812 * 9.81 * 9.5 = 75674.34 \ Pa$$

And then using the equations **(3.31)**, we do the iterative calculation with Excel

$$\sigma_r = \frac{a^2 \rho g z}{\left(R_2^2 - R_1^2\right)}\left(1 - \frac{b^2}{r^2}\right), \quad \sigma_\theta = \frac{a^2 \rho g z}{\left(R_2^2 - R_1^2\right)}\left(1 + \frac{b^2}{r^2}\right)$$

To calculate the require thickness (e) of the cylindrical face we must change the form of the equation, to represent the thickness e in the formula, by changing the form of $\left(R_2^2 - R_1^2\right)$ by:

$$R_2^2 - R_1^2 = (R_2 - R_1)(R_2 + R_1) = e\,(R_2 + R_1)$$

33

Where e: the thickness of the tank. So, the form of the eq. (**3.31**) will be:

$$\sigma_r = \frac{a^2 \rho g z}{e(R_2 + R_1)}\left(1 - \frac{b^2}{r^2}\right), \quad \sigma_\theta = \frac{a^2 \rho g z}{e(R_2 + R_1)}\left(1 + \frac{b^2}{r^2}\right) \tag{1}$$

From the figure (), we know that the maximum stresses are in the point (**r =a)** and by

substitute this in the equation () we get

$$\left(\sigma_r\right)_{\text{max}} = \rho g z, \quad \left(\sigma_\theta\right)_{\text{max}} = \frac{(R_1 + R_2)}{e}\rho g z \tag{2}$$

$$\sigma_{r\,max} = \rho g z, \quad \sigma_{\theta\,max} = \frac{R_2 + R_1}{e}\rho g z \approx \frac{2R_1}{e}\rho g z, \quad \sigma_z = 0 \tag{3}$$

VON Mises yield criterion takes the form:

$$\sigma_{von} = \sqrt{\frac{1}{2}\left[\left(\sigma_r - \sigma_\theta\right)^2 + \left(\sigma_r - \sigma_z\right)^2 + \left(\sigma_\theta - \sigma_z\right)^2\right]}$$

$$= \sqrt{\frac{1}{2}\left[\left(\sigma_r - \sigma_\theta\right)^2 + \sigma_r^2 + \sigma_\theta^2\right]}$$

$$\sigma_{von} = \sqrt{\frac{1}{2}[(\sigma_r - \sigma_\theta)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_\theta - \sigma_z)^2]} = \sqrt{\frac{1}{2}[(\sigma_r - \sigma_\theta)^2 + \sigma_r^2 + \sigma_\theta^2]} \tag{4}$$

$$\sigma_{von} = \sqrt{\frac{1}{2}[(\sigma_r - \sigma_\theta)^2 + \sigma_r^2 + \sigma_\theta^2]} \leq R_e \tag{5}$$

the thickness e of the reservoir is given by the solution of this inequality:

$$\left(\frac{2R_1}{e}\right)^2 - \frac{2R_1}{e} + 1 \leq \left(\frac{R_e}{\rho g z}\right)^2 \tag{6}$$

We have to organize it with the following steps:

$$\left(\frac{2R_1}{e}\right)^2 - \frac{2R_1}{e} + 1 \leq \left(\frac{R_e}{\rho g z}\right)^2 \tag{7}$$

2.4.3 Hemispherical bottom:

for the hemispherical tank calculate the pressure is distribute nonlinearly at every point in the curve (a b), we know that this curve is a form of quad circular which has the formula of the form:

$$y = r_1 \sin \theta$$

$$P(z) = \rho g z + P_y$$
$$P(z) = \rho g z + \rho g r_1 \sin \theta \qquad (8)$$

Using Excel, we calculate the pressure at every point in the curve (a b)

Θ (°)	Y (m)	P(y) (Pa)
0	0	75674.34
10	0.434120081	79132.41901
20	0.855049666	82485.42622
30	1.249999042	85631.48237
40	1.606967895	88474.9963
50	1.915109923	90929.56942
60	2.165062404	92920.62089
70	2.34923067	94387.65373
80	2.462018871	95286.09296
90	**2.5**	**95588.64**

And we calculate the stresses using the previous eq. **(3.73)**, and we have from the previous table that P(max) = 95588.64 Pa

$$\sigma_r = \frac{P_1 R_1^3}{R_2^3 - R_1^3}\left[1 - \frac{R_2^3}{r^3}\right], \qquad \sigma_\phi = \sigma_\theta = \frac{P_1 R_1^3}{R_2^3 - R_1^3}\left[1 + \frac{R_2^3}{2r^3}\right]$$

As we mention before the maximum stress is when **r = R₁** and if substitute that in the eq. (),
and change the form on $(R_2{}^3 - R_1{}^3)$ with the following:

$$R_2{}^3 - R_1{}^3 = (R_2 - R_1)(R_2{}^2 + R_1 R_2 + R_1{}^2) \approx 3e\,R_1{}^2$$

And the eq. () will be:

$$\sigma_r = \frac{P_{max} R_1^3}{3e\,R_1^2}\left[1 - \frac{R_2^3}{R_1^3}\right], \qquad \sigma_\phi = \sigma_\theta = \frac{P_{max} R_1^3}{3e\,R_1^2}\left[1 + \frac{R_2^3}{2R_1^3}\right] \qquad (9)$$
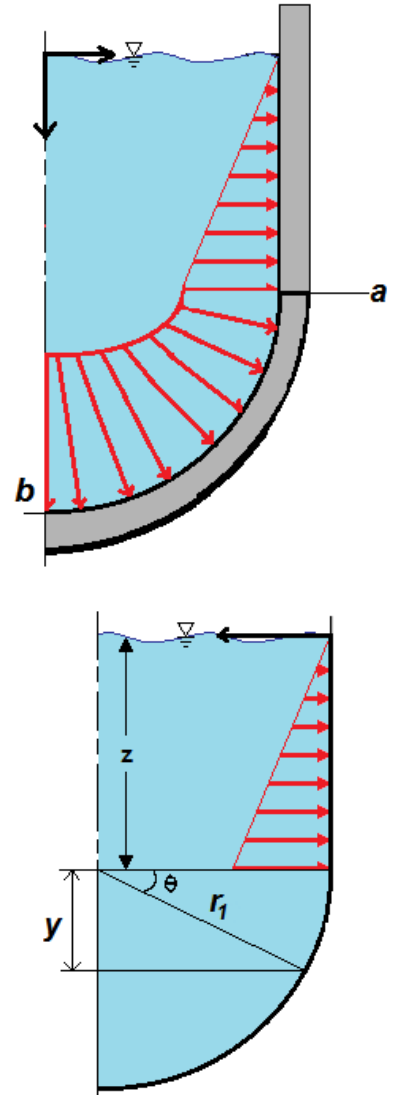
Chapter 3

3 Simulation with SolidWorks

SOLIDWORKS® Simulation is an easy-to-use portfolio of structural analysis tools that use Finite Element Analysis (FEA) to predict a product's real-world physical behavior by virtually testing CAD models. The portfolio provides linear, non-linear static and dynamic analysis capabilities.

And to do a simulation with it with the following of this steps

3.1 CAD Model

3.1.1 Cylindrical wall

To simulate the cylindrical wall, we start with sketching it's geometry in **front plan**, and we apply **revolve** feature in **features tab** as shown in figure ():
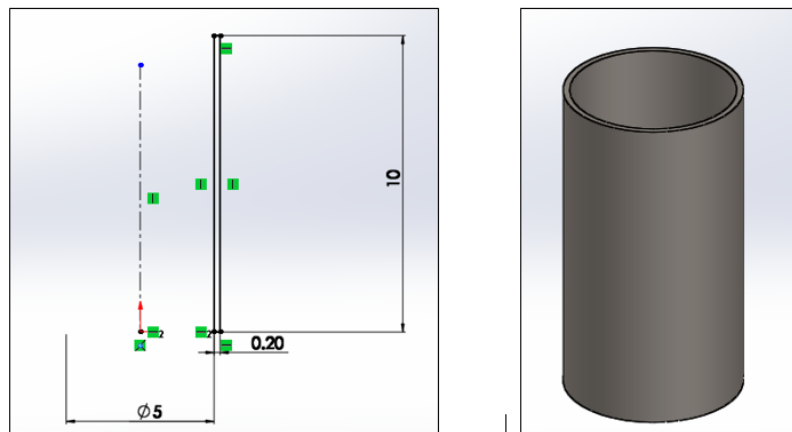
Figure 3.1 Sketch of the tank model

After that we start a new stady in **stady advisor,** and we choose **static** stady, the simulationn menu shown up fig. **(4.2)**, the first thing is applying **material** to cylindrical tank and sins in solidworks material library don't have stainless stell 304. We choose a closly material which is **alloy steel**, then we select the **fixture** for our model we choose a **fixed** fixture on the bottom of the cylindre, then we apply the **external loads** which is **pressure** load varie linierly in the direction of (y), but we must ser a **coordinate system** as a reference, cause the pressure load need a reference coordinate to start the distribution of the loas, and that's shown in the fig. **(4.5):**
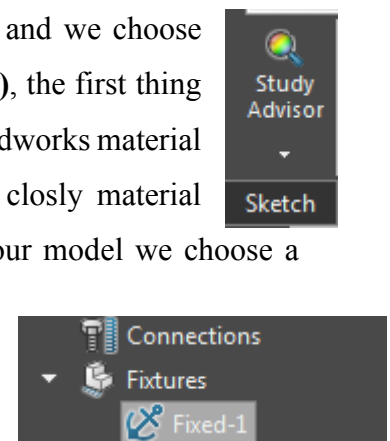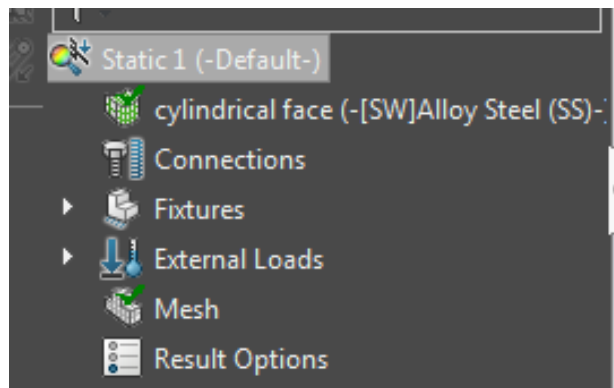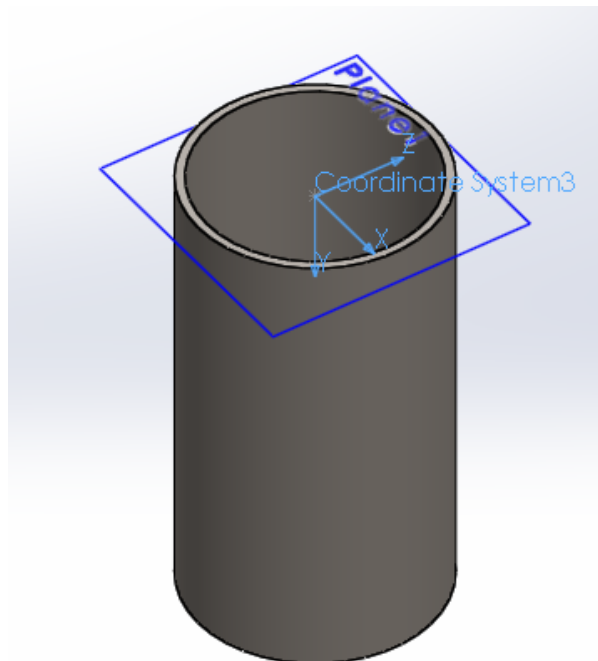

Figure 3.2 fixed fixture

Figure 3.3 Type of simulation menu
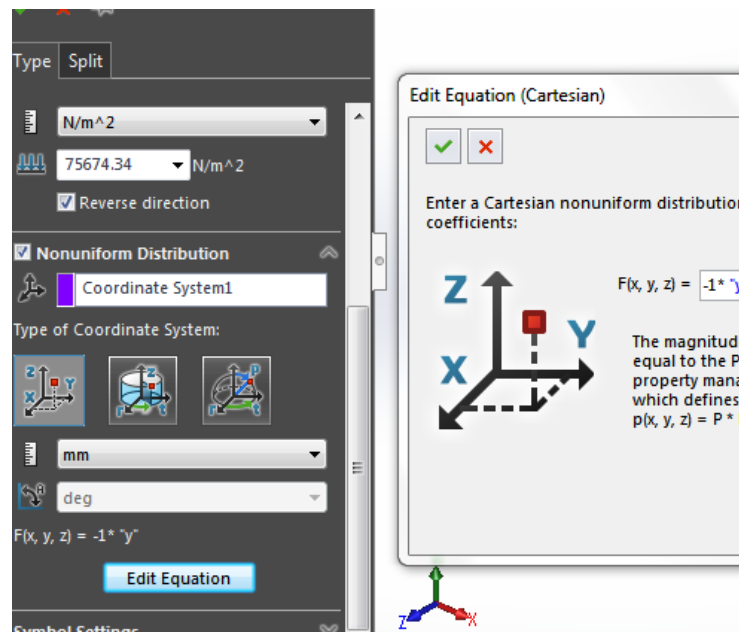
Figure 3.4 Simulation menu

Figure 3.5 Ceating a reference point in the model

We must select the following option to activate the nonuniform distribution fig. (4.6), so after applying the previous steps we get the following model fig. (4.7), which represent the distribution of pressure act on the internal surface and represent the fixture chosen for this model
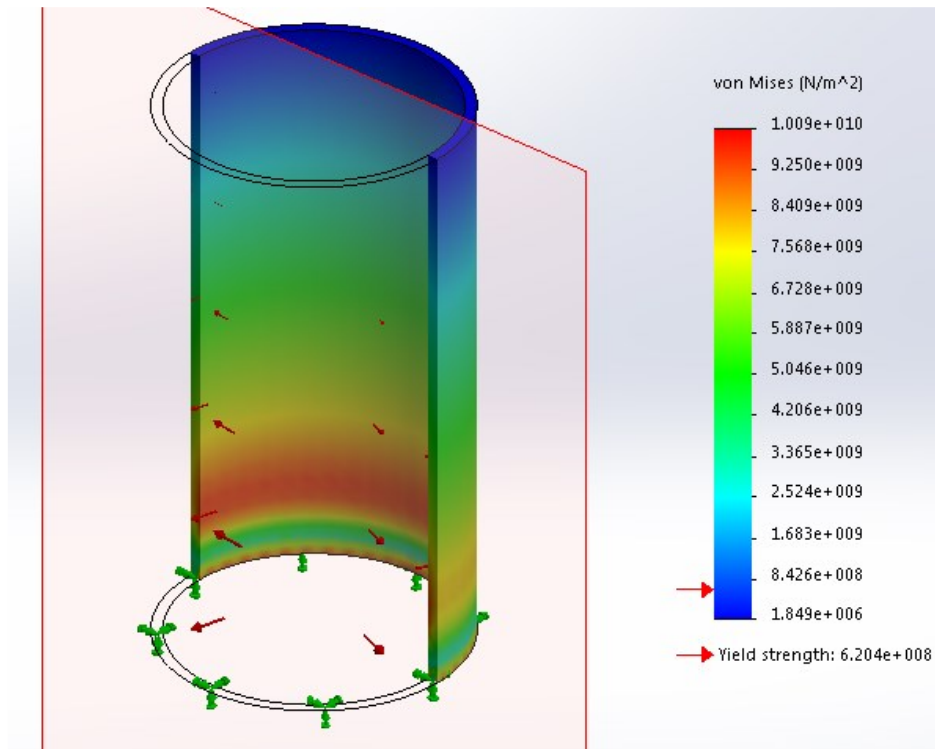
Figure 3.6 Non-uniform distribution option
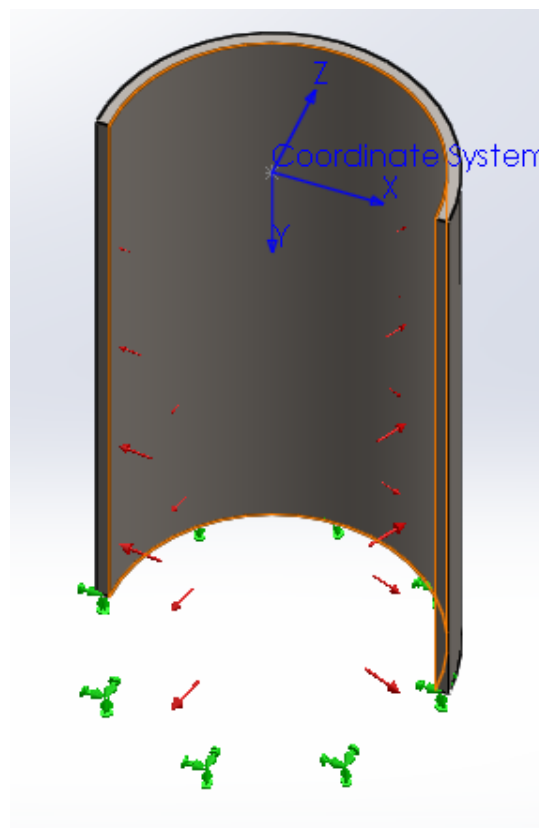
Figure 3.7 Applying distributed load and fixture

Then we run the run the simulation, and getting the following results:

Figure 3.8 the results of simulation

If we examine the fig. (4.8) we that the stress distribution is linearly with depth, and this duo to hydrostatic forces, and we see that the maximum stress is in the bottom of the tank and the value of it is $1,009 \cdot 10^{10}$ Pa. and this value is under Von Mises method. The bottom of tank is a critical point cause the maximum stresses located in it and the displacement to.

So, for the designer of this type of tank made a great attention in that zone and this prevent a large deformation in the bottom and minimize the failure in the foundation of the tank.

3.1.2   Flat Bottom analysis

For the bottom plate of the tank we follow the same steps we do the previous model, we preper the CAD model which is a simple circul represent the bottom of the tank with a diametre of 5.2 m
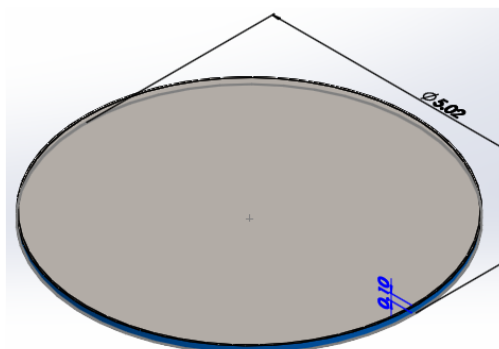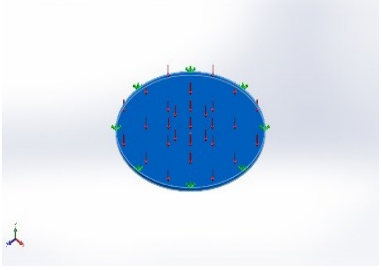

Figure 3.9 CAD model for the bottom of the tank

Material Properties

Model Reference	Properties	Components
	Name: **Alloy Steel (SS)**	**SolidBody 1(Split Line1)(3_flat bottom)**
	Model type: **Linear Elastic Isotropic**	
	Default failure criterion: **Unknown**	
	Yield strength: **6.20422e+008 N/m^2**	
	Tensile strength: **7.23826e+008 N/m^2**	
	Elastic modulus: **2.1e+011 N/m^2**	
	Poisson's ratio: **0.28**	
	Mass density: **7700 kg/m^3**	
	Shear modulus: **7.9e+010 N/m^2**	
	Thermal expansion coefficient: **1.3e-005 /Kelvin**	
Curve Data:N/A		

Affter selecting the material we apply the load and the fixtures in the model we choose the type of it as we do to the previous model as shown in the following fig.
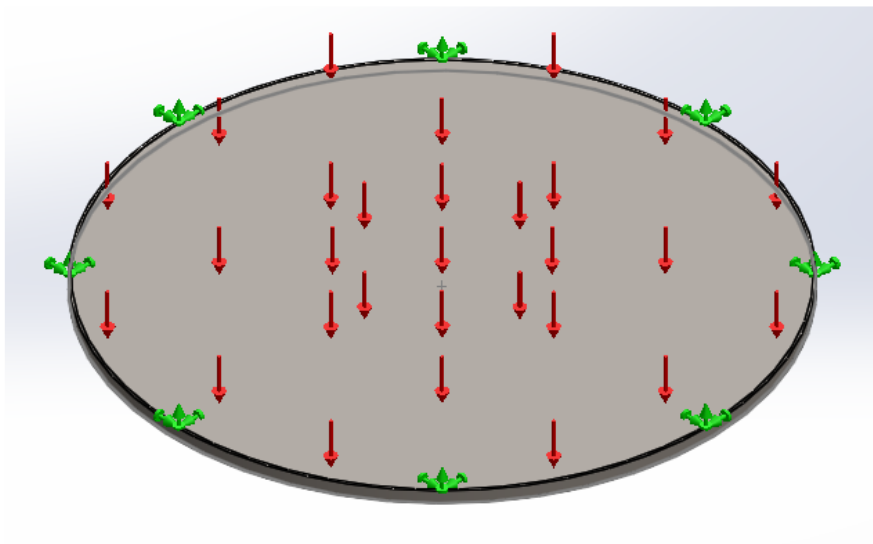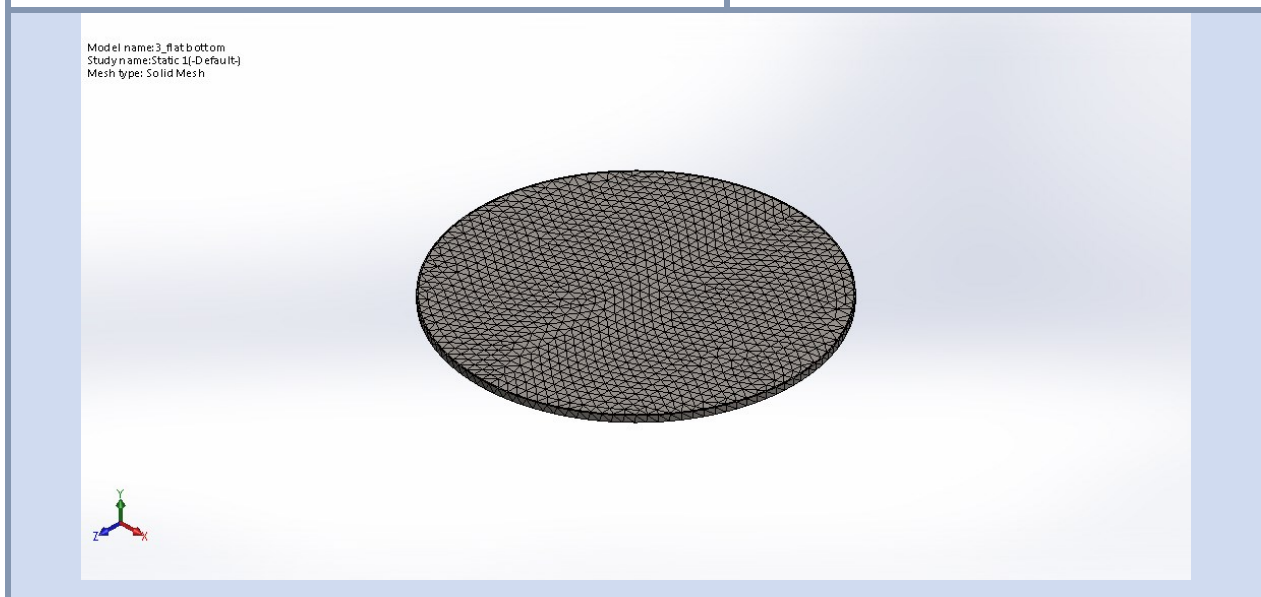

Figure 4.10 load and fixture conditions

Mesh properties

Total Nodes	14493
Total Elements	7129
Maximum Aspect Ratio	7.1252
% of elements with Aspect Ratio < 3	93.6
% of elements with Aspect Ratio > 10	0
% of distorted elements(Jacobian)	0
Time to complete mesh(hh;mm;ss):	00:00:02
Computer name:	SAMI

Model name:3_flat bottom
Study name:Static 1(-Default-)
Mesh type: Solid Mesh

And after the meshing we run the simulation and we get the following results as shown in fig. (4.11), the results is under Von Mises method and that the maximum stress in the plate is $1,223 \cdot 10^8$ Pa and this is in the boundary of the circular plat
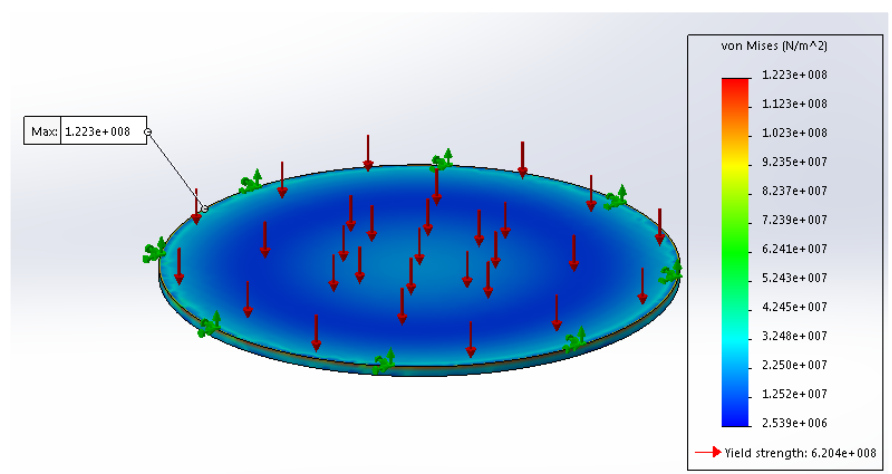
Figure 0.11 Results of simulation

3.5   Conclusion

In this brief simulation we understand that the tanks have a 2 critical zone that make it easy to fail duo the huge amount of stresses

- The bottom of the cylindrical wall
- The boundary of the plate

and if know that we can make a good solution to avoid that that's why the designer change the shape of the plat to spherical one or conical, and that's a better solution to avoid the excessive stress in the bottom of tank.
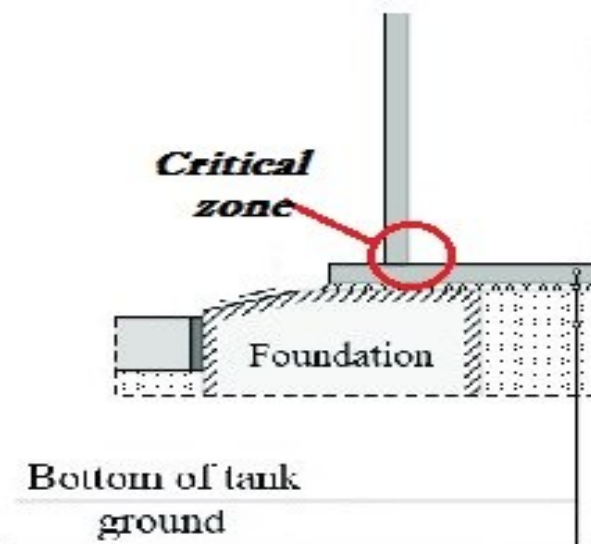
Conclusion

The need for a good design of a cylindrical tank require a good study of its shape and better understanding how the stresses are distribute in it and to achieve such results require a good understanding of the behavior of a material.

in this work we find that it's a superb method to develop the equilibrium equations in different coordinate systems which make it an easy way to find the distribution of the stresses in complex geometry such we study it.

in our research we find a critical point in the tank that make a huge amount of stresses in the tank which make great possibility for a failure of the flat bottom tank, and that's lead us to the hemispherical bottom which have a better performance in distribution of the stress because of its geometry, and that's make it a good choice in industries. And we prof that critical points in a simulation by solidWorks.

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