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TITLE

Sliding Mode Control with Adaptive Gain for Nonlinear MIMO Systems

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"وَقُل رَّبِّ زِدْنِي عِلْمًا"

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Praise be to Allah for reaching the end. "Whoever says 'I can do it,' will achieve it."

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Summary

Chapter II: sliding mode control for nonlinear SISO systems

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List of figures:

Notations and Abbreviations

Notations:

$u(t)$: The control signal.

θ: The angle of the pendulum.

m_p: The mass of the pendulum.

m_c: The weight of the trolley.

$m_t = m_c + m_p$: the total mass of the pendulum trolley.

L: Half-length of the pendulum.

g: Gravity.

R : set of real numbers.

$X(t)$: System State Vector.

$X_d(t)$: Desired State Vector.

$\forall$: Whatever.

x_e : Equilibrium point of the system.

V : Lyapunov function.

$\dot{V}$: Derived from the function of Lyapunov.

$B(\rho)$: Boule dans R^n on définit une boule fermée dans R^n comme l'ensemble

$$B(\rho) = \{x \in R^n / \|x\| \prec \rho\}$$

$\|\cdot\|$: Euclidean norm.

$y(t)$: Output size (size to adjust).

S : sliding surface.

$e(x)$: The difference between the status variable to be adjusted x and the reference x_d.

u_{eq} : the equivalent control.

u_{eq}^* : The optimal equivalent control.

K : Sliding gain.

$\langle f, g \rangle$: The scalar product of f and g.

$d(t)$: Perturbation.

$\hat{K}$: Adaptative gain.

u_n: The discontinuous control.

Abbreviations:

SMC: Sliding mode control

MPC: Model Predictive Control

VSS: variable structure system

SISO: single-input single-output

MIMO: multi-input multi-output

General introduction:

In Automatic, the mathematical modeling of a system consists in representing the dynamic behavior of the system by mathematical equations with the desired precision. The mathematical models obtained are generally in the form of linear (linear systems) or nonlinear (nonlinear systems) differential equations.

Over the last two decades, a large number of publications have been dedicated to the problem of controlling nonlinear systems. A problem that presents many challenges since nonlinear systems, unlike linear systems for which the automatic provides a panoply of methods for the synthesis of the control, does nonlinear control law. This is due to the fact that nonlinear systems have extremely varied structures, complex dynamics and can exhibit all kinds of strange behavior.[27]

In general, control techniques are required in order to solve the problem of parametric variations, with almost zero static error and fast response, so we obtain a stable and robust control system. Among these techniques, one finds the control by sliding mode with an adaptive gain, for example, known by its simplicity and robustness.

The control of non-linear systems by sliding mode with adaptive gain is a significant area of study in control systems. This approach combines sliding mode control with adaptive gain to enhance system stability and performance. It aims to mitigate the effects of external disturbances, uncertainties, and the "chattering" phenomenon associated with classical sliding mode control. By utilizing Lyapunov's approach, the stability of the closed-loop system is analytically proven. This method offers advantages in terms of robustness against disturbances and model uncertainties, addressing issues like chattering and high control efforts. The integration of sliding mode control with adaptive gain results in a robust and smooth control strategy, combining the speed and ease of implementation with the stability and robustness of sliding mode control. [32]

This brief is organized into a general introduction, three chapters and a conclusion:

The first chapter is devoted to some reminders on the state models of nonlinear systems, the stability theory of nonlinear systems and the different methods of control of nonlinear systems.

In the second chapter, we present fundamental notions of variable structure control and some basic concepts on the theory of sliding modes of monovariable systems (SISO) with an adaptive gain.

In the third chapter, we present the technique of sliding mode control of multivariable nonlinear systems (MIMO) with an adaptive gain.

Chapter I:

Generalities on the control of nonlinear systems

I.1.Introduction

Approaches used to study linear systems are extremely effective due to the many available tools, such as linear algebra, differential equations, and linear differential systems. However, despite their power, these methods have several limitations:

- No physical system is perfectly linear, meaning that linear methods are only applicable in restricted operating ranges.
- Some phenomena cannot be adequately described by linear models.
- Some systems are inherently impossible to model using linear techniques. [1]

In this context, various tools are proposed in the literature, such as the use of differential geometry for system linearization, control based on Lyapunov theory for system stability approach, variable structure control, adaptive control, …etc.

I.2. Nonlinear Systems

A nonlinear system is defined as a system that cannot be described by linear differential equations with constant coefficients. Unlike linear systems, nonlinear systems do not follow the principle of superposition and can exhibit characteristics such as distortions due to nonlinearities, such as saturation or thresholds, which can affect the relationship between the input and output of the system [1].

Nonlinear systems are varied and complex, necessitating specific approaches for their analysis and control. Instead of a single general theory, there are several methods adapted to different classes of nonlinear systems. For instance, in the field of nonlinear control systems, the emphasis is on studying control systems that contain one or more nonlinear elements belonging to specific types of nonlinearities.

I.3. Characteristics of a nonlinear system

- The principle of superposition does not apply: The behavior of a non-linear system cannot be predicted by adding up the behavior of its components.
- It is described by non-linear differential equations: These equations include coefficients that depend on the variables.

- It is difficult to analyze: There is no general theory for analyzing non-linear systems, which makes their study more complex.

I.4. State Space Representation

provides a powerful mathematical framework for modeling physical systems as first-order differential equations, offering a flexible and efficient approach for the analysis and design of complex systems in diverse fields such as economics, statistics, computer science, electrical engineering, and neuroscience.

The general representation of a nonlinear system is of the form [2]:

$$\begin{cases} \dot{x} = f(x) + g(x) \cdot u(t) \\ y = h(x) \end{cases} \quad \text{(I.1)}$$

y is the system output, x is the state vector, and u is the control vector.

$f(x)$, $g(x)$, and $h(x)$ are nonlinear functions of the state vector describing the system.

I.4.1. Autonomous System

The nonlinear system (I.1) is said to be autonomous if:

- The evolution of a system can be defined by a differential equation of the form

$$\dot{x}(t) = f(x(t), u(t), t), \forall t \geq 0 \quad \text{(I.2)}$$

where:

- $x(t)$is the state vector
- $u(t)$ is the control vector

In this chapter, we will restrict ourselves to considering unforced systems (zero input; (t) = 0) given by: $\dot{x}(t) = f(x(t), t)$ (I.3)

A system is said to be autonomous if $f(t)$ does not explicitly depend on time t:

$$\dot{x}(t) = f\big(x(t)\big) \tag{I.4}$$

Otherwise, the system is said to be non-autonomous [3].

I.4.2. Variable Structure Systems (VSS)

A system is said to be a variable structure if it admits a representation by differential equations of the type:

$$\dot{x} = \begin{cases} f_1(x) & \text{if condition } \mathbf{1} \text{ is verified} \\ f_n(x) & \text{if condition } \boldsymbol{n} \text{ is verified} \end{cases} \tag{I.5}$$

Where fi the functions belong to a set of subsystems of class C^k. Consequently, variable structure systems are characterized by the choice of a function and a switching logic [4].

I.5. Equilibrium Points of a Nonlinear System

Linear systems have only one equilibrium point, but nonlinear systems can have multiple equilibrium points. [1]

- **Example**

Consider a physical system described by the following differential equation:

$$\begin{gathered} \dot{x}(t) = -x(t) + x^2(t) \\ x(0) = x_0 \end{gathered} \tag{I. 6}$$

The nonlinear system has the following characteristics

$$\dot{x}(t) = -x(t) + x^2(t) \begin{cases} \text{Equilibrium points } x = 0 \text{ and } 1 \\ \\ \text{Solution } x(t) = \dfrac{x_0 e^{-t}}{1 - x_0 + x_o e^{-t}} \end{cases} \tag{I. 7}$$

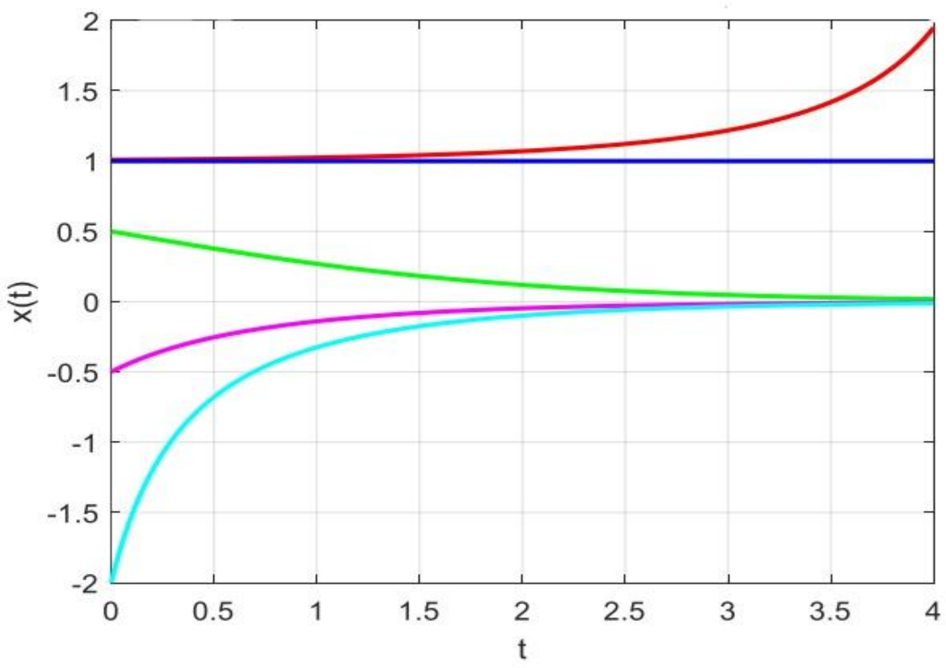

Figure I.1. Equilibrium points of a nonlinear system

The nonlinear system has two equilibrium points, X = 0 and X = 1. The point X = 1 is unstable and constitutes a kind of stability boundary. The axis is indeed divided into two regions of initial conditions for which the trajectories are convergent to the equilibrium state 0 or are divergent [1].

I.6. Stability of Nonlinear Systems

Stability: is considered to be the most sought-after concept in the study of a dynamic system. There are different ways of expressing stability in automatic control. We distinguish between the stability of an equilibrium point, input-output stability, etc. In general, this study has undergone a very important development since the use of the results of the stability theory deduced from the work of Lyapunov, which takes into account the stability of the dynamic models of linear or nonlinear systems [5].

I.6.1. Lyapunov stability

Powerful tools for analyzing and ensuring the stability of nonlinear systems use specific functions to evaluate the convergence of trajectories towards equilibrium points [6].

Let be a function such that [7]: $V: R^n \to R^+$

1. V is continuously differentiable in all its arguments,
2. V is positive definite,
3. There exist two scalar functions a and b from R^+ to R^+, continuous, monotonic, non-decreasing such that:

$$a(0) = b(0) = 0$$

$$\forall x \in R^n: a(\|x\|) \leq V(x) \leq b(\|x\|) \quad (I.8)$$

Then V is a Lyapunov candidate function.

I.6.2. Stability Theorems

I.6.2.1. Local Asymptotic Stability

If there exists a scalar function V(x) of the state whose first partial derivatives are continuous and such that:

1. V is a Lyapunov candidate function.
2. $\dot{V}$ is a locally semi-definite negative in a neighborhood of the origin, Ω.

Then the origin equilibrium point is stable and a domain of stable initial conditions is delimited by any Lyapunov equipotential contained in Ω. If V is locally definite negative in Ω, then the stability is said to be locally asymptotically stable in the part of the space delimited by any Lyapunov equipotential contained in Ω [3].

I.6.2.2. Global Asymptotic Stability

If there exists a function V such that [3]:

1. V is a Lyapunov candidate function.
2. $\dot{V}$ is definitely negative.
3. The condition $\|x\|\rightarrow+\infty$ implies $V(x)\rightarrow+\infty$.

Then the equilibrium point (origin) is a globally asymptotically stable point.

Example

Consider the following system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 \end{cases}$$

The equilibrium point of this system is: $(x_1, x_2) = (0,0)$.

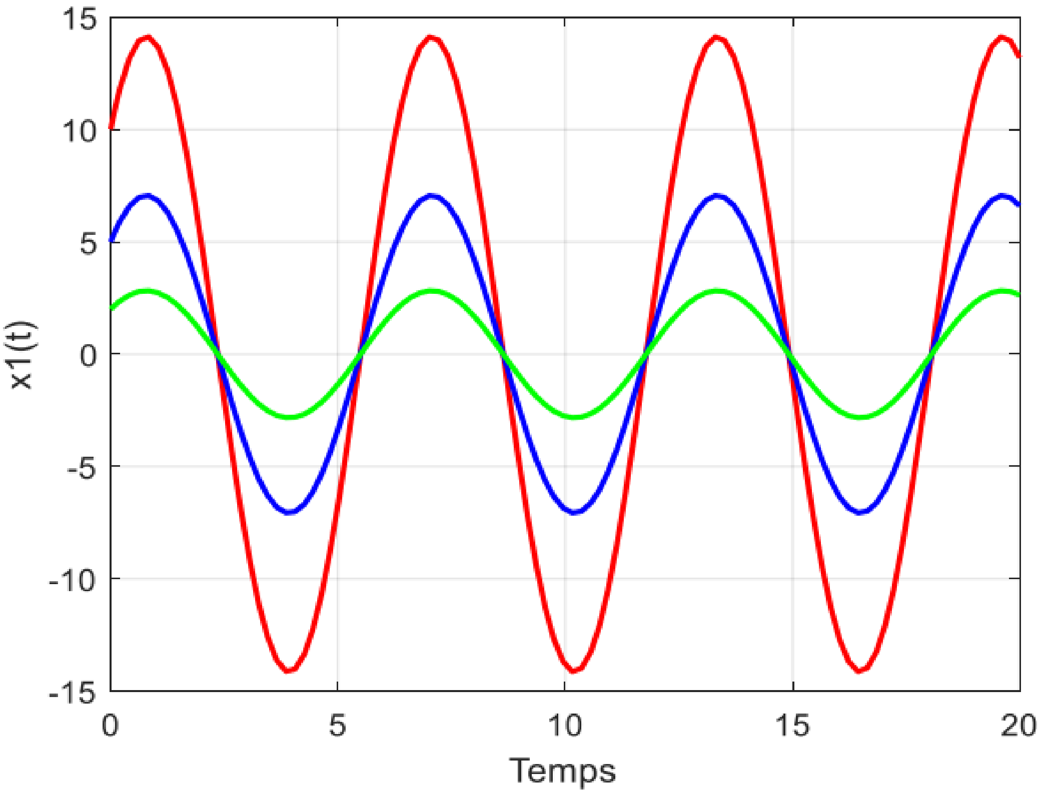

Figure I.2. The evolution of $x1$ with several initial conditions close to the equilibrium point

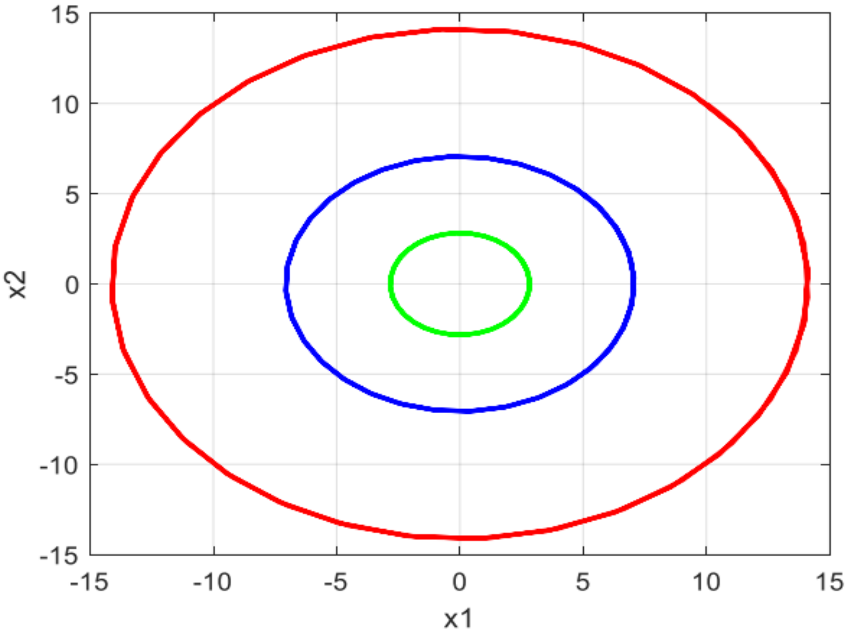

Figure I.3. Phase plane with several initial conditions close to the equilibrium point (0,0)

From the two figures above, we can see that the trajectories of the system remain close to the equilibrium point (0,0), therefore, the system is stable.

I.6.3. Stability of a Trajectory

In some cases, systems do not have equilibrium points, or the equilibrium point is not stable. However, the trajectories do not diverge. Various cases can occur [3]:

- **The system has a stable domain:** There is a domain of initial conditions (basin of attraction) such that all trajectories remain within the stable domain.
- **The system has an attractive domain:** There is a domain of initial conditions such that all trajectories are included in the attractive domain after a certain time.
- **The system has a stable trajectory.** The stability of a trajectory can be demonstrated by applying Lyapunov's second theorem.

I.6.4. The phase plane

The phase plane method is a graphical approach used to analyze and study the behavior, typically the stability, of both linear and nonlinear second-order systems.

It involves graphically solving the second-order differential equation without seeking the analytical solution. The main concept is to create trajectories in the state space of a second-order dynamical system (a two-dimensional plane known as the phase plane) for different initial conditions and then observe and analyze the qualitative characteristics of these trajectories.

I.6.5. Input/Output Stability

Input/output stability is a special form of stability of dynamical systems studied in automatic control. A system is stable if a bounded input corresponds to a bounded output or the free response of the system tends to zero at infinity. By simplification, if the output remains of finite energy as long as the input is of finite energy. A necessary and sufficient condition is that all poles of the transfer function have strictly negative real parts.

Current approaches to defining system stability focus on the stability of the equilibrium state around a point, within a domain, or along a trajectory. Input-output stability is another possible perspective.

Consider the nonlinear system [7]**:**

$$\begin{aligned} \dot{x}(t) &= f\big(x(t), u(t)\big) \\ y(t) &= g\big(x(t), u(t)\big) \end{aligned} \tag{I. 9}$$

Or u is the system input and y its output. The input/output stability of an equilibrium point (u_e, y_e) is defined as:

Whatever $\varepsilon \succ 0$ there is $\alpha \succ 0$ and an initial condition domain of the system state such that if $\|u(t) - u_e\| \prec \alpha$ for any t and $x(0)$ belongs to the initial condition domain then $\|y(t) - y_e\| \prec \varepsilon$for any t .

It is worth noting that input/output stability is very rarely used. It is indeed essential to know the evolution of the entire state of the system. It is not uncommon, in fact, for non-observable systems, for the output to have stable behavior and yet for the system state to diverge [7].

I.7. Types of Control of a Nonlinear System

Many different types of control can be used for nonlinear systems. Some of the most common types include:

I.7.1. Adaptive control

Adaptive control is a branch of control theory that deals with designing control laws for systems with parameters that vary over time or are initially unknown. The goal is to ensure the stability and performance of the system despite these variations.

It relies on estimating the system's parameters online and adapting the control law accordingly. This estimation is usually done using machine learning techniques like Kalman filters or neural networks [9].

applications:

Adaptive control finds use in various applications, including:

- **Robotics:** Controlling robots where parameters change based on payload or environment.
- **Aerospace:** Controlling airplanes where parameters vary with altitude and speed.

I.7.2. Model Predictive Control (MPC)

Has become increasingly popular in recent years in the industrial field due to its tolerance for different types of systems and respect for imposed constraints, as well as its compatibility with hardware. It can be used to control complex systems with multiple inputs and outputs where the simple PID controller is insufficient. This technique is particularly interesting when systems have significant delays, inverse responses, and many disturbances. The main users of predictive control are oil refineries, the chemical and agri-food industry, metallurgy, and aerospace [8].

I.7.3. Robust control

Is a method or approach used to design a system that can handle uncertainty, specifically in terms of stability or performance measures. Uncertainty can arise from various sources, such as unknown plant dynamics, disturbances, or sensor noise.

The goal of robust control is to create a control system that remains stable and performs well, even when these uncertainties are present. This is achieved by designing the system in a way that accounts for potential variations and disturbances, ensuring that it can handle a range of possible scenarios. Robust control theory is a branch of control theory that focuses on designing controllers that can handle plant uncertainties, disturbances, and noise2. This involves modeling the plant and its uncertainties, as well as understanding the properties of observability, controllability, and stability. Robust control methods seek to bind the uncertainty rather than express it in the form of a probability distribution, as in stochastic control. This allows for a worst-case analysis, ensuring that the system meets its control requirements in all cases, even if some performance is sacrificed [10].

I.7.4. Backstepping

Is a recursive control design for nonlinear systems that uses the Lyapunov stability theory to synthesize the control law. It is applied to nonlinear triangular systems and has been used in various applications, such as the control of a class of nonlinear dynamic systems, the control of manipulator robot with two degrees of freedom, and the control of a vehicle's movement along the X and Y axes. It has several advantages over other control methods for nonlinear systems but also has some limitations, such as the determination of regression matrices and the assumption of linearity in parameters.

I.7.5. Fuzzy control

Fuzzy logic control is a method of controlling systems that uses fuzzy logic to determine the control action. Fuzzy logic is a mathematical approach that allows for the representation of imprecise information and the manipulation of uncertain or vague concepts. In fuzzy control, the input variables are mapped to output variables through a set of rules that are based on linguistic variables and fuzzy sets. These rules are typically defined by a human expert and are used to create a fuzzy controller that can handle complex systems with non-linear behavior. Fuzzy control has been used in various applications, such as the control of a class of non-linear dynamic systems, the control of a manipulator robot with two degrees of freedom, and the control of a vehicle's movement along the x and y axes. It has also been used in adaptive control for non-linear systems, such as the control of a suspension system with input saturation. The implementation of fuzzy control is typically done using a computer or microcontroller. There are libraries of programs in languages such as C that can be used to implement fuzzy control, and some microcontrollers, such as the 68HC12 of Motorola, have instructions for implementing fuzzy control directly in assembly. Fuzzy control has several advantages over other control methods for non-linear systems. It does not require the system to become linear and allows for the cancellation of non-linearities that could be useful. It also provides a systematic and iterative method for designing controllers for non-linear systems of any order and guarantees the stability of the controller-system couple. However, fuzzy control also has some limitations. One of the main difficulties is the determination of the regression matrices and the number of unknown parameters, which increases with each step of the Backstepping development process. Additionally, the assumption of linearity in parameters may not always be true in practice [12].

I.7.6. Sliding mode control

Is a control technique that aims to bring the state trajectory of a system to a sliding surface, where it remains until equilibrium. This method is divided into several parts: the convergence mode, the sliding mode, and the steady-state mode.

Sliding mode control is a particularly interesting technique. It dates back to the 1970s with the work of Utkin [9]. The principle is to bring, regardless of the initial conditions, the representative point of the evolution of the system on a hyperplane of the phase space by integrating switching elements into the control law. In addition, the control guarantees that the representative point of the system reaches the hyperplane in a finite time. The system enters sliding mode when this point has reached the hyperplane, called the sliding surface. Its behavior then becomes insensitive to output disturbances and parameter variations. However, the problems of "chattering" inherent to this type of discontinuous control quickly appear.

Note that chattering can excite neglected high-frequency dynamics sometimes leading to instability. Methods to reduce this phenomenon have been developed [11].

I.8.Conclusion

This chapter provided a general overview of the properties of nonlinear systems, addressing the most commonly used methods to analyze and ensure their stability. We also reviewed several control approaches applicable to these systems.

In the next chapter, we will go into detail with the sliding mode control and we focus to ensure the stability and robustness of nonlinear systems and explore its applications in detail. We will analyze its fundamental principles, and its practical implementations through various examples.

Chapter II:

Sliding mode control

For nonlinear SISO systems

II.1. Introduction

Conventional control techniques are very effective for controlling systems Constant parameter linear systems. For linear (or nonlinear) systems with non-parameter, these control techniques will be insufficient.

Sliding mode control (SMC) is a nonlinear control method that alters the dynamics of a system by applying a discontinuous control signal, forcing the system to "slide" along a cross-section of its normal behavior. This technique is robust against disturbances and uncertainties, making it suitable for real-time control without requiring a linearly parameterized dynamic model of the system. In SMC, the control law switches between different continuous structures based on the system's current state, ensuring trajectories move towards adjacent regions with different dynamics. This variable structure control method allows trajectories to slide along boundaries of control structures, known as sliding modes, Leading to robust and optimal control of various dynamic systems. The main strength of sliding mode control lies in its robustness and simplicity, as it can Handle parameter variations and uncertainties effectively. By using discontinuous control laws, the sliding mode can be reached in finite time, offering better performance than asymptotic behavior. This approach has applications in Electric drives, robotics, and other fields where robust and efficient control is essential [14].

II.2. Introduction to variable structure control systems

II.2.1. Historic

Considerable attention has been paid to the control of the non-linear uncertainty dynamics, often subject to perturbations and parametric variations. The Theory of variable structure systems and associated sliding modes has been studied in Detailed studies over the past 30 years. Controllers with variable structure Have made their application in the Soviet literature (Emelyanov 1967, Utikin 1974), and have been widely identified as a potential approach to this problem (Gao and Hung 1993).

Searches on the variable structure control were given by DeCarlo and Other (1998), and Hung and Other (1993), control action forces the trajectory of systems to Intercept the surface of the slide. The system trajectories are Then confused with the sliding surface during the use of High switching speed. The salient advantage of variable structure control with the sliding mode is robustness against changes in parameters or disturbances. The "chattering" phenomenon associated with the sliding mode control presents a major drawback because it can excite the dynamics of high switching frequency which makes it undesirable. Several methods to reduce this phenomenon have been proposed [23] [24].

II.2.2. Objective of sliding mode control

The objective of the sliding mode control is summarized in two essential points [14]:

- Synthesize a surface $S(x,t)$, such that all trajectories of the system obey A desired behavior of pursuit, regulation, and stability.
- Determine a control law (switching) $u(x,t)$ that can attract All state trajectories toward the sliding surface and maintain them on this surface.

II.2.3.the advantages and disadvantages of sliding mode control

Advantages [14]

- Simple control implementation.
- Control converges in finite time.
- Robust control of system parameter variations.
- Robust control of external disturbances.

Disadvantages

- Presence of oscillations (chattering phenomenon) caused by the discontinuous part of the control.
- The system (controlled by sliding mode control) is constantly subjected to a high-frequency control to ensure its convergence to the desired state, which is not desirable or possible for certain systems.

II.2.4. Principle

A variable structure system (VSS) is a system whose structure changes during Its functioning. It is characterized by the choice of a structure and logic of Switching. This choice allows the system to switch from one structure to another to any instance. In addition, such a system may have new properties that do not exist in Each structure [18].

II.2.5. Structure by switching at the control organ level

The change in structure is done by switching at the control unit level, by the figure shown below:

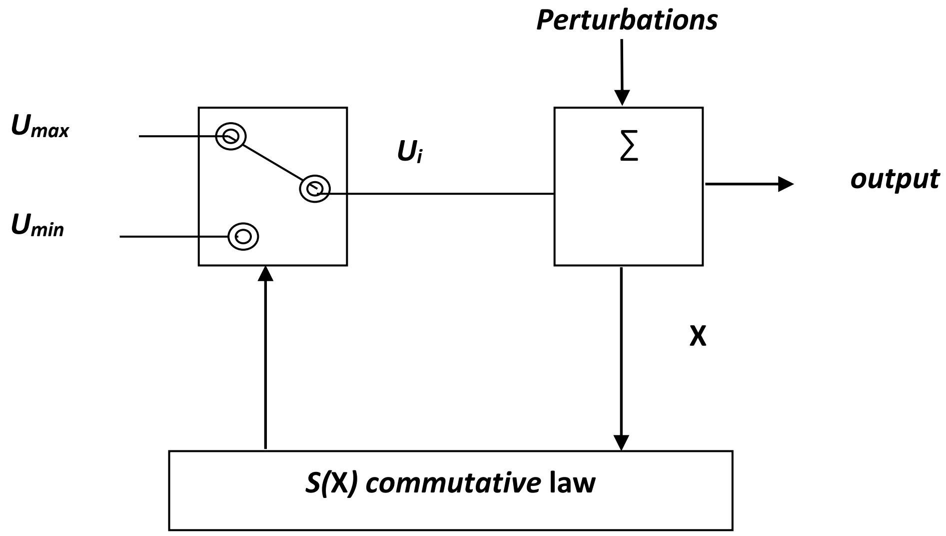

Figure II.1. Configuration of the structure by switching at the control organ [16].

In this case, the control unit between two constant values u_{max} and u_{min}, according to the $sign$ of the function $S(x)$, the switching logic is given by:

$$f(x) = \begin{cases} u = u_{max}, & S(x) > 0 \\ u = u_{min}, & S(x) < 0 \end{cases} \tag{II.1}$$

This configuration corresponds to a two-position setting, with a more efficient switching law. When the sliding speed is reached, the state variables are connected by the relationship: $S(x) = 0$ [22].

II.2.6. Structure by commutation at a state feedback level

The change in structure occurs when switching the U control. The control unit receives in this case a control voltage u_{Cm} that rapidly switches between two variable values, u_{Cm_1} and u_{Cm2} which can cause strong solicitations of the control organ, therefore, a practical realization impossible [25].

The representation of this configuration is given by the figure below. The system then operates in slip mode and the dynamic behavior of the system is determined by the condition: $S(x) = 0$.

$$f(x) \begin{cases} U = u_{Cm_1} = -k_1 x, & S(x) > 0 \\ U = u_{Cm_2} = -k_2 x, & S(x) < 0 \end{cases} \tag{II.2}$$

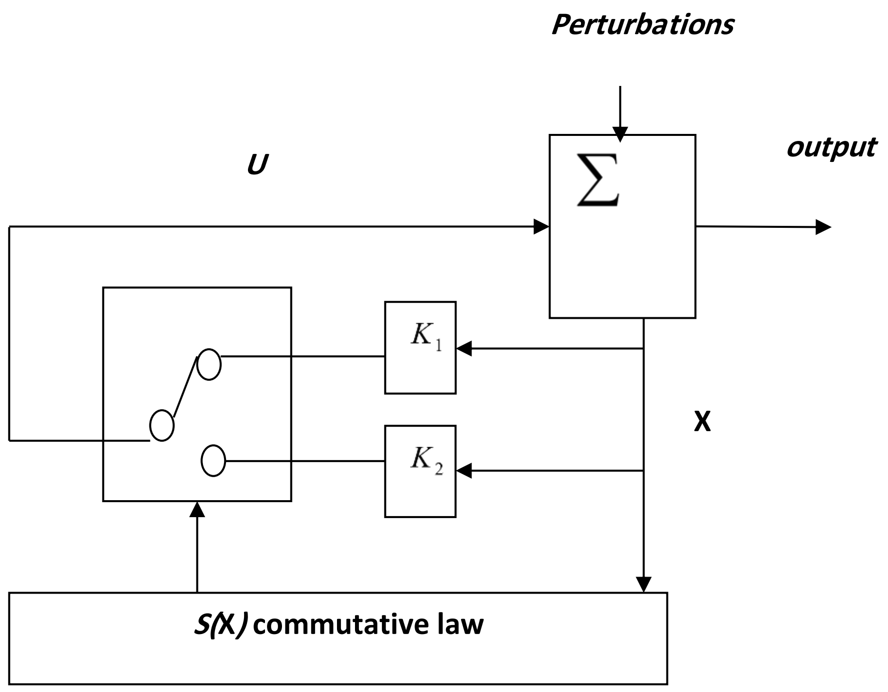

Figure. II.2. Configuring the structure by switching a variable state feedback [16].

II.2.7. Switching structure at the control organ with equivalent control

Such a structure, the principle of which is shown in Figure. II.3 has a real Advantage. It allows pre-positioning of the future state of the system through the equivalent control, which is nothing other than the desired value of the system in a steady state.

The control element is much less solicited, but we are more dependent on parametric variations due to the expression of this equivalent control [21].

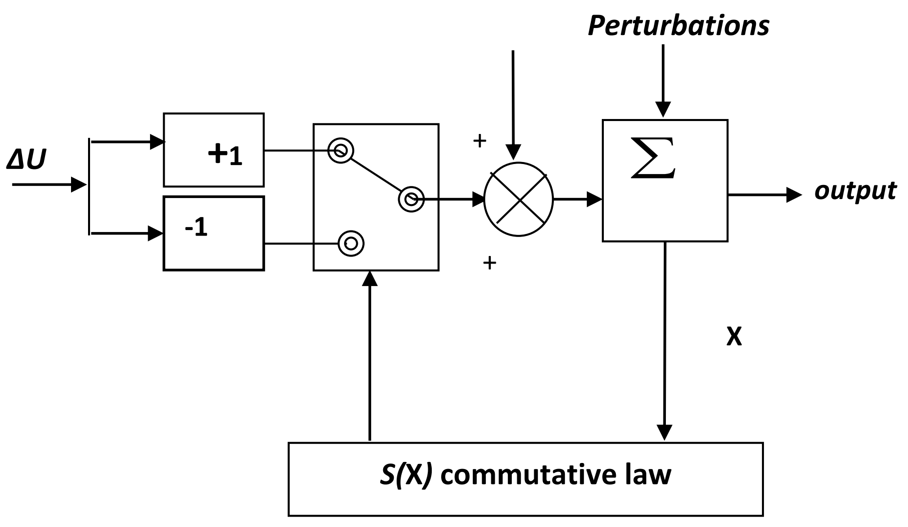

Figure II.3. Control structure by adding equivalent control [16].

We have chosen to base our study on this type of configuration, such a structure by its constitution is very simple and allows for less solicitation of the controller. It seems natural to add the equivalent control to pre-position the system in a desired permanent and stable state and then play on the switching term to ensure convergence towards this state and to stay there afterward.

II.3. Principle of sliding mode control

The principle of sliding mode control involves constraining a system to stay on a sliding surface using a control strategy. This surface forces system trajectories into a reduced-order subspace, referred to as a sliding surface. The control law in sliding mode control is not continuous; it switches between states based on the system's position in the state space. By selecting a hyper-surface or manifold (sliding surface) and finding feedback gains to keep the system trajectory on this surface, sliding mode control can drive trajectories to the sliding mode in finite time. Once trajectories reach the sliding surface, the system exhibits the characteristics of the sliding mode, with trajectories confined to this surface by high-gain feedback. The sliding mode control scheme ensures trajectories approach the sliding surface,

offering stability superior to asymptotic stability. The control law's discontinuity allows for trajectories to move across the sliding surface efficiently, enhancing the system's robustness against disturbances and uncertainties. So, this order is controlled in two phases [14]:

- **The convergence phase:** The state trajectory of the system moves from an initial state $(x_0, \dot{x}_0)$ and converges to the sliding surface in finite time. During this phase, the system remains sensitive to parametric variations, uncertainties, and external disturbances.

- **The sliding phase:** The state trajectory of the system has reached the sliding surface and is moving toward the desired state. The system's behavior during this phase no longer depends on the system itself or disturbances, but solely on the properties of the sliding surface.

The figure below shows the principle of sliding mode control.

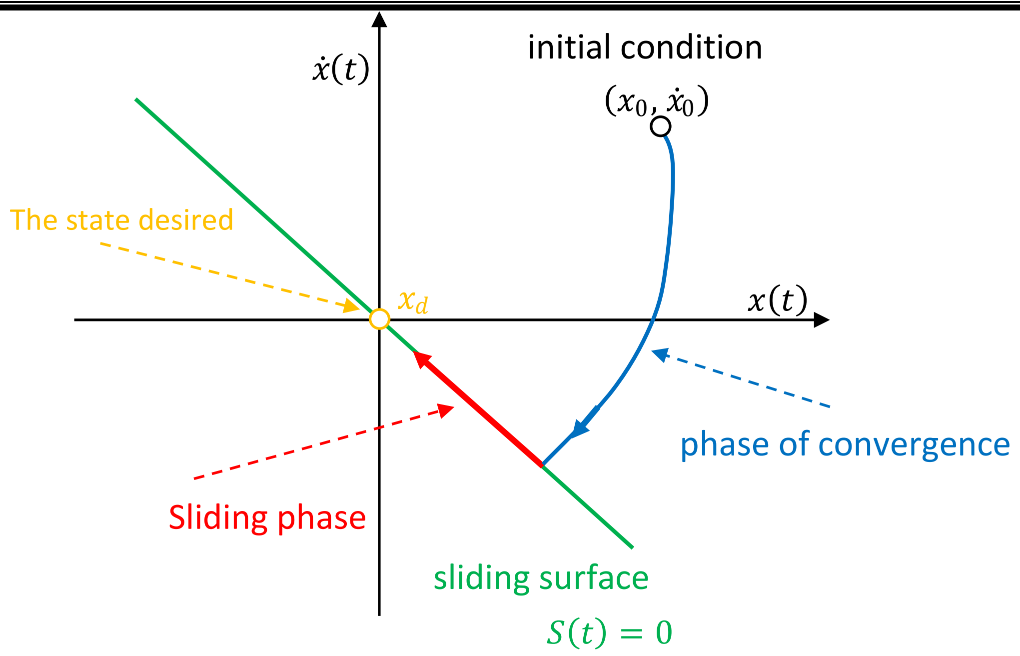

Figure II.4. principle of sliding mode control [14].

In summary, a sliding mode control is mainly done in three parts:

1. The choice of sliding surface.
2. The establishment of the conditions for convergence.
3. The determination of the control law.

II.4. Reference

Let the system be a non-linear affine system whose dynamic behavior is described by the following differential equation [20] [14]:

$$\dot{x} = f(x) + g(x)u \tag{II.3}$$

With:

$x = [x_1 x_2 \dots \dots x_n]^T$: represents the state vector of the system.

f and g: are linear or nonlinear functions.

u : represents the control vector.

The steps for constructing the sliding mode control can be summarized in three main steps: determining a suitable sliding surface, establishing the conditions for the system trajectory to

converge to the chosen sliding surface, and determining the control law that allows the system trajectory to be brought back to this surface and remain there.

II.4.1. Choice of sliding surface

The sliding surface is a scalar function on which the state trajectory of the system to be controlled slides on this surface and tends towards the desired state. There are several forms of the sliding surface, but the most commonly used surface to ensure convergence towards the desired state is given by [14]:

$$S = \left(\frac{d}{dt} + \alpha\right)^{r-1} e \qquad (\text{II}.4)$$

With:

e: represents the error between the current and desired value.

α: is a positive constant.

r: represents the relative degree, which is equal to the number of times the system output needs to be differentiated to reveal the control input u.

- If $r = 1$: $S = e$.
- If $r = 2$: $S = \dot{e} + \alpha e$.
- If $r = 3$: $S = \ddot{e} + 2\alpha\dot{e} + \alpha^2 e$.

The sliding surface must satisfy the following two conditions:

1-If: $S = 0 \Rightarrow x = x_d$.

2- $\dot{S}$ contains the control vector u.

The derivative with respect to time of the sliding surface is written as:

$$\dot{S} = \frac{\partial S}{\partial t} = \frac{\partial S}{\partial x}\frac{\partial x}{\partial t} = \frac{\partial S}{\partial x}\dot{x} = \frac{\partial S}{\partial x}\left(f(x) + g(x)u\right) \qquad (\text{II}.5)$$

II.4.2. Establishment of convergence conditions

The convergence conditions are the criteria that allow the system dynamics to converge towards the sliding surface. To do this, the Lyapunov method is used to ensure the stability and convergence of the control law.

Let $V(S)$ be the candidate Lyapunov function, which is a scalar function defined as positive, that is: $V(0) = 0$ and $V(S) > 0, S \neq 0$.

$$V(S) = \frac{1}{2}S^2 \qquad (\text{II}.6)$$

The derivative of the Lyapunov function is:

$$\dot{V}(S) = S(x)\dot{S}(x) \qquad (\text{II}.7)$$

To determine a control law capable of ensuring the convergence of the system's state trajectory to the sliding surface, it is sufficient to ensure that the time derivative of the Lyapunov function $V(S)$ is defined as negative: $\dot{V}(S) < 0$.

Therefore, in order for the Lyapunov function $V(S)$ to decrease and converge to zero, it is sufficient to ensure that [19]: $S(x)\dot{S}(x) \leq 0$.

II.4.3. Determination of the control law

The sliding mode control law is divided into two parts: equivalent control and discontinuous control [17]:

$$u = u_{eq} + u_{dis} \tag{II.8}$$

u_{eq}: is the equivalent control, it allows to drive the state trajectory of the system to the sliding surface. It is obtained by solving the equation $\dot{S} = 0$.

$$\dot{S} = 0 = \frac{\partial S}{\partial x}\left(f(x) + g(x)u_{eq}\right) \Rightarrow u_{eq} = -\left(\frac{\partial S}{\partial x}g(x)\right)^{-1}\left(\frac{\partial S}{\partial x}f(x)\right) \tag{II.9}$$

u_{dis}: is the discontinuous control, once the system's state trajectory reaches the sliding surface, the discontinuous control brings it to the desired state. There are several options for discontinuous ordering. But the simplest function is the sign function (simple relay):

$$sign(S) = \begin{cases} 1, & S > 0 \\ 0, & S = 0 \\ -1, & S < 0 \end{cases} \tag{II.10}$$

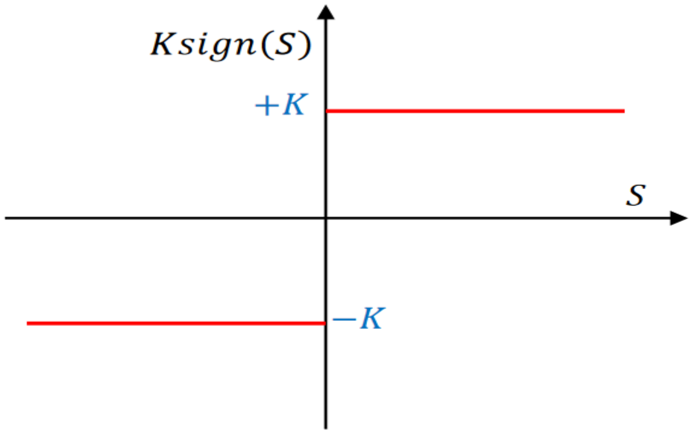

Figure II.5. Sign function

The gain K must be positive to verify the stability condition. The value of K is very influential, because if it is chosen very small, the response time will be very long, if it is chosen too large, there will be strong oscillations (chattering phenomenon) at the level of the control organ [19].

II.5. Chattering phenomenon

During the sliding phase, the use of the sign function $sign(S)$ means that the control u switches between two values $\pm K$ at a high frequency and is manifested by strong oscillations around the slip surface S.

This phenomenon is known as reluctance or chattering. However, in practice, this phenomenon is undesirable because the switching frequency can deteriorate the control accuracy, damage actuators, and mechanical components, and cause an increase in temperature in electrical systems [13].

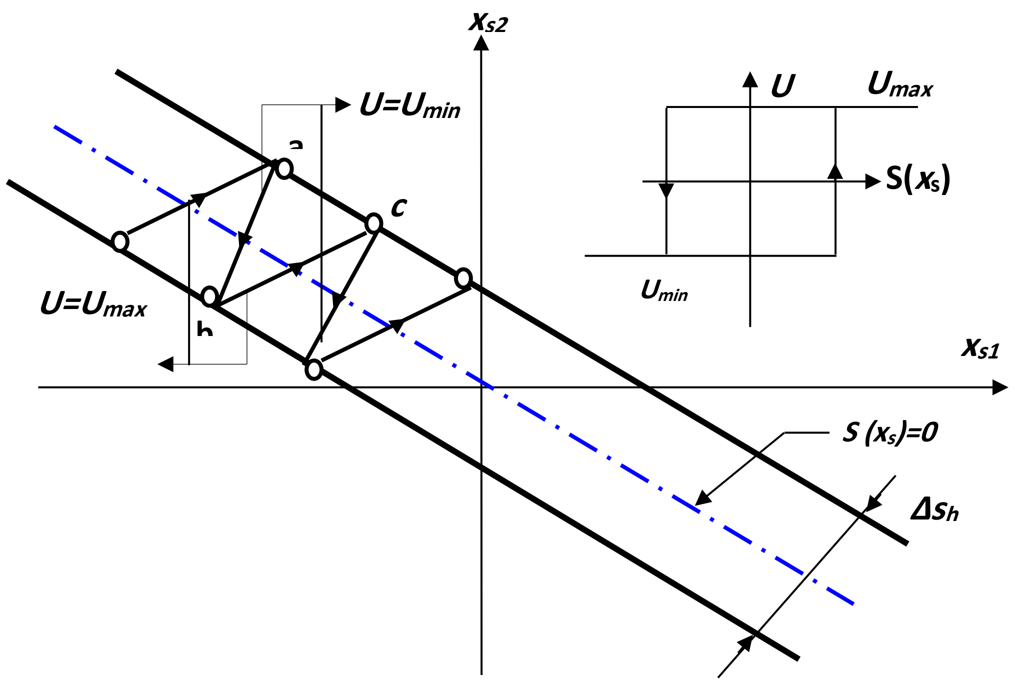

Figure. II.6. Chattering phenomenon

II.6. Solutions to mitigate the chattering phenomenon

To mitigate or eliminate the chattering phenomenon, several solutions have been proposed, among these proposals, we can mention [13] [14]:

- **Boundary layer solution:** This solution involves replacing the discontinuous part in the control with more appropriate functions that filter out high frequencies. Among the functions used are the saturation function, the sigmoid function, and the hyperbolic tangent function. These functions reduce the chattering phenomenon.

$$sat(S) = \begin{cases} 1, & S > a \\ \frac{1}{a}, & -a \le S \le a \\ -1, & S < -a \end{cases} \qquad (\text{II}.11)$$

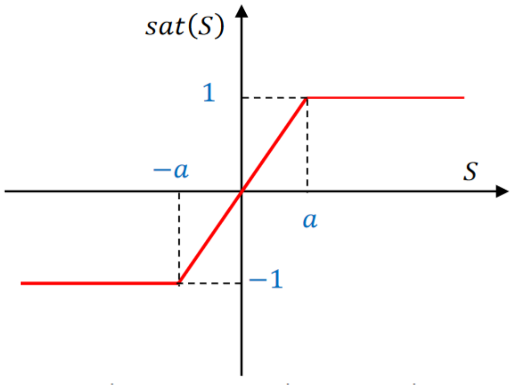

Figure II.7. Saturation function

$$sigm(S) = \frac{1}{1 + e^{-S}} \tag{II.12}$$

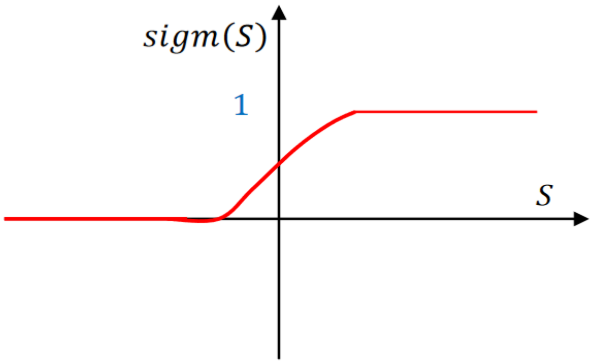

Figure II.8. Sigmoid function

$$tanh(S) = \frac{e^{S} - e^{-S}}{e^{S} + e^{-S}} \tag{II.13}$$

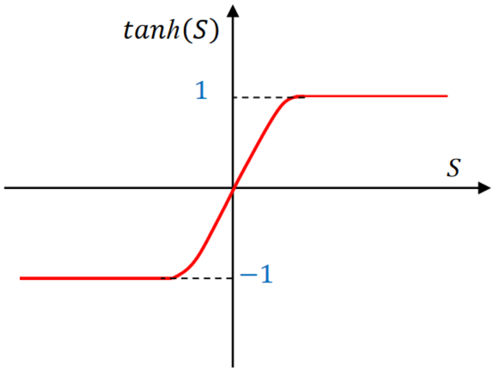

Figure II.9. Hyperbolic tangent function

II.7. Stability Analysis

Let's consider a nonlinear system described by the following state representation:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_n = f(x) + g(x)u + d(t) \\ y = x_1 \end{cases} \quad (II.14)$$

Where:

$$\begin{cases} x^{(n)} = f(x) + g(x)u + d(t) \\ y = h(x) \end{cases} \quad (II.15)$$

With:

$x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T = \left[x(t), \dot{x}(t), \dots, x^{(n-1)}(t)\right]^T$ is the state vector.

$f(x)$ and $g(x)$ are nonlinear functions of the state vector with $g \neq 0$.

u : the control.

$d(t)$: the disturbance considered to be bounded $d(t) \leq D$.

The purpose of the control is to find a control law such that, the trajectories of the state vector tend towards zero despite the presence of disturbances.

The implementation of a sliding mode control goes through three stages:

II.7.1. The choice of the sliding surface

The sliding surface S is given by [14]:

$$S(x) = c_1 x_1 + c_2 x_2 + \cdots + c_{n-1} x_{n-1} + x_n \quad (II.16)$$

where:

$$S(x) = \sum_{i=1}^{n-1} C_i x_i + x_n \quad (II.17)$$

With:

$C_i > 0$, $\quad i = \overline{1, n-1}$

II.7.2. The existence condition of a sliding surface

can be deduced from the Lyapunov function given by the following relationship:

$$\dot{V}(S) = \frac{1}{2} S^2 \quad (II.19)$$

A sufficient condition for the system (II.14) to be stable is:

$$\dot{V}(S) = S\dot{S} \leq -\eta |S| \quad (II.20)$$

where: $\eta > 0$.

II.7.3. The establishment of the control law

The sliding mode control law is given by the following formula:

$$u = u_{eq} + Ksign(S) \tag{II.21}$$

u_{eq} : the equivalent control.

$sign(.)$: the sign functions.

K: a positive constant representing the gain of the control.

The objective is to determine a sliding mode control law of such the state of the system converges to the origin.

For the system (II.14) to be stable the coefficients of (II.16) must be chosen from such output that the roots are with negative real parts.

The function of Lyapunov is considered:

$$V(S) = \frac{1}{2}S^2 \tag{II.22}$$

According to Lyapunov's theorem if is negative the state trajectory will be attracted to the sliding surface and switches around it to the equilibrium point.

$$\dot{V}(S) = S\dot{S} = S\left(\sum_{i=1}^{n-1} c_i x_{i+1} + \dot{x}_n\right) \tag{II.23}$$

$$\dot{V}(S) = S\dot{S} = S\left(\sum_{i=1}^{n-1} c_i x_{i+1} + f(x) + g(x)u(t) + d(t)\right) \tag{II.24}$$

It is easy to conclude if the control u has the following form:

$$u = u_{eq} + Ksign\big(S\, g(x)\big), \qquad k > \frac{D}{g(x)} \tag{II.25}$$

with:

$$u_{eq} = \frac{-\sum_{i=1}^{n-1} c_i x_{i+1} - f(x)}{g(x)} \tag{II.26}$$

And:

$$sign(\varphi) = \begin{cases} 1, & \varphi > 0 \\ 0, & \varphi = 0 \\ -1, & \varphi < 0 \end{cases} \tag{II.27}$$

Note that the law of control (II.25)- (II.27) depends only on known parameters and functions and that the term causes a chattering phenomenon that can excite high frequencies and nonlinearities that cannot be modeled and damage the system. Thus, the control law (II.25) is effective but it is difficult to implement and can present risks to the process.

To reduce the effect of grazing the discontinuous function can be replaced by a saturation function, which consists in determining a boundary band around the sliding surface ensuring the smoothing of the control and the maintenance of the state of the system in this band.

$$u = u_{eq} - Ksat\left(\frac{S}{\phi}\right), \qquad \phi > 0 \tag{II.28}$$

With:

$$sat\left(\frac{S}{\phi}\right) = \begin{cases} sign\left(\frac{S}{\phi}\right), & |\varphi| \geq 1 \\ \frac{S}{\phi}, & |\varphi| < 1 \end{cases} \tag{II.29}$$

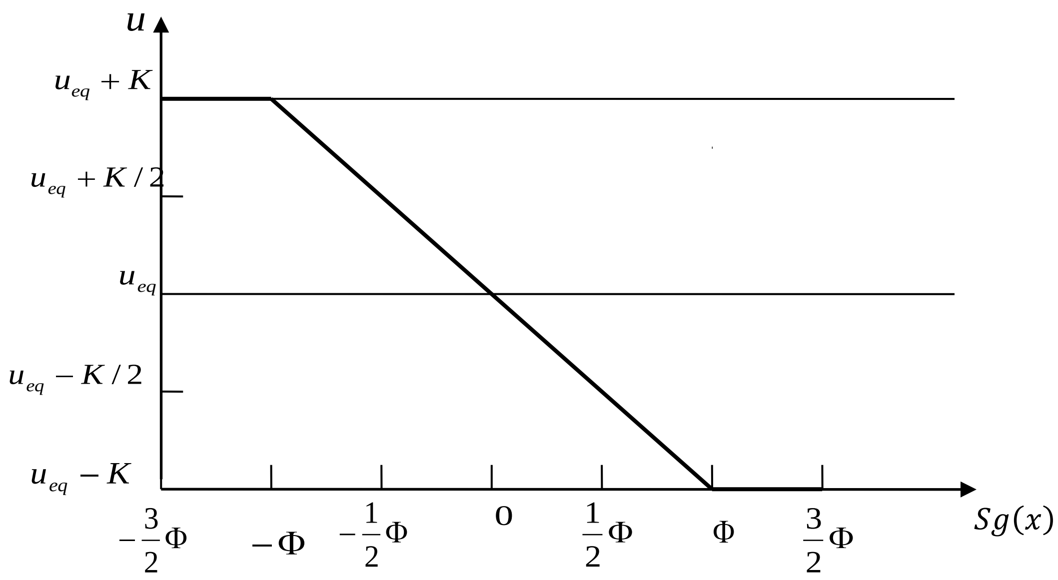

Figure II.10. control output by sliding mode [27] $u = u_{eq} - KSat(\frac{S}{\Phi})$

II.8. Sliding mode control with adaptive gain

The previous part assumed that the gain K of the control by sliding mode can be determined. However, in practice, there is no method for calculating this gain. To solve this problem, we use an adaptive gain control in this section.

The equivalent control can be obtained from the time derivative of the surface $\dot{S}_1 = 0$:

$$\dot{S}_1 = c_1\dot{e}_1 + \dot{e}_2 \tag{II.30}$$

$$\dot{S}_1 = c_1 e_2 + f_1(X) + g_1(X)u - \dot{x}_{2d} \quad \text{(II.31)}$$

We approach the equivalent control law optimal u_{eq}^{*} by the equivalent order u_{eq} given by:

$$u_{eq} = \frac{-c_1 e_1 - f(x) + x_{2d}}{g_1(X)} \quad \text{(II.32)}$$

$$u_{eq}^{*} = \frac{-c_1 e_1 - f_1(X) + \dot{x}_{2d}}{g_1(X)} \quad \text{(II.33)}$$

and is $K(t) = u_{eq} - u_{eq}^{*}$ given by:

$$K(t) = u_{eq} - u_{eq}^{*},\ 0 \le |K(t)| \le K \quad \text{(II.34)}$$

The uncertainty limit is a positive constant. However, this uncertainty limit cannot be measured in practice.

Let $\widehat{K}$ be the estimated value of K, the estimation error is considered:

$$\widetilde{K}(t) = K - \widehat{K}(t) \quad \text{(II.35)}$$

u_n : the discontinuous control whose purpose is to check the attractiveness conditions, an adaptive sliding mode control term is introduced to compensate for the difference between the optimal equivalent control u_{eq}^{*} is the equivalent control u_{eq}.

$$u_n = -\widehat{K} sign(Sg) \quad \text{(II.36)}$$

To ensure the objectives of the order, the following adaptation law is adopted:

$$\dot{\widehat{K}} = -\dot{\widetilde{K}} = n|Sg| \quad \text{(II.37)}$$

With: $n > 0$.

II.8.1. Stability Analysis

The objective is to ensure the stability of the control structures in the sense that all input and output signals remain bounded and the tracking error asymptotically tends to zero. In general, Lyapunov synthesis consists of selecting a candidate Lyapunov function V and then choosing control or adaptation laws that ensure its decrease. To demonstrate the stability of the system, we consider the following candidate Lyapunov function:

$$V = \frac{1}{2}S^2 + \frac{1}{2n}\widetilde{K}^2 \quad \text{(II.38)}$$

with: $\widetilde{K}(t) = K - \widehat{K}(t)$.

The time derivative of (II.41) is:

$$\dot{V} = S\dot{S} + \frac{1}{n}\widetilde{K}\dot{\widetilde{K}} \quad \text{(II.39)}$$

From (II.34) and (II.38), it comes:

$$\dot{V} = S\big(c_1 e_2 + f(x) + g(x)u + d(t)\big) + \frac{1}{n}\big(K - \widehat{K}\big)\dot{\widehat{K}} \qquad (II.40)$$

By replacing u with its expressions (II.31), (II.34) and using the adaptation law (II.37), the relation (II.40) becomes:

$$\dot{V} = S\big(c_1 e_2 + f(x) + g(x)\big(u_{eq} + u_n\big) - \dot{x}_{2d}\big) - \big(K - \widehat{K}\big)|Sg(x)| \qquad (II.41)$$

$$\dot{V} = S\big(c_1 e_2 + f(x) + g(x)\big(u_{eq} - u_{eq}^* + u_{eq}^* + u_n\big) - \dot{x}_{2d}\big) - \big(K - \widehat{K}\big)|Sg(x)| \qquad (II.42)$$

From (II.33), (II.34) and (II.36), comes:

$$\dot{V} = S\left(g(x)\big(u_{eq} - u_{eq}^* + u_n\big)\right) - \big(K - \widehat{K}\big)|Sg(x)| \qquad (II.43)$$

$$\dot{V} = S\left(g(x)\left(K - \widehat{K}sign\big(Sg(x)\big)\right)\right) - \big(K - \widehat{K}\big)|Sg(x)| \qquad (II.44)$$

$$\dot{V} = Sg(x)K - Sg(x)\widehat{K}sign\big(Sg(x)\big) - K|Sg(x)| + \widehat{K}|Sg(x)| \qquad (II.45)$$

$$\dot{V} = Sg(x)K - K|Sg(x)| \le 0 \qquad (II.46)$$

II.9. Example: control of an inverted pendulum

II.9.1. Discontinued control

Consider the inverted pendulum represented in the following figure:

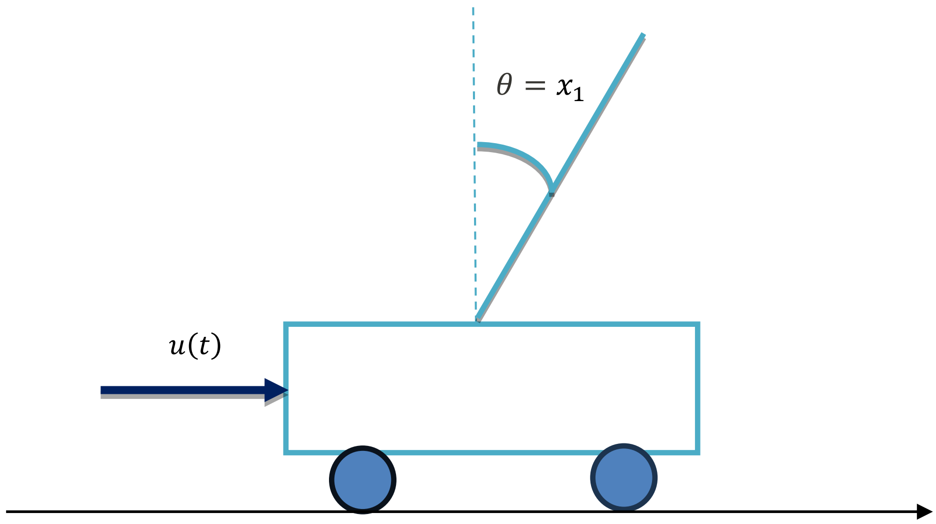

Figure II.11. inverted pendulum

The dynamics of the system can be described by the following system of differential equations [19]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x) + g(x) \cdot u + d(t) \\ \dot{x}_2 = \dfrac{g\,sin(x_1) - \dfrac{mLx_2^2\,sin(x_1)}{M+m}}{L\left(\dfrac{4}{3} - \dfrac{m\,cos(x_1^2)}{M+m}\right)} + \dfrac{\dfrac{cos(x_1)}{M+m}}{L\left(\dfrac{4}{3} - \dfrac{mcos(x_1^2)}{M+m}\right)} u + sin(t) \\ y = x_1 \end{cases} \quad (II.47)$$

With:

$\theta = x_1$: is the angle of the pendulum.

$m = 0.1kg$: is the mass of the pendulum.

$M = 1\,kg$: is the mass of the trolley.

$L = 0{,}5\,m$: is half length of pendulum.

$g = 9.81m\,/\,S^2$: Gravity.

the discontinuous control is:

$$u_n = -\widehat{K}Sign(Sg) \quad (II.48)$$

With:$\widehat{K}$ is adaptative gain.

With $\widehat{K} = 10$ and the initial $x(0) = [0.5{,}0.5]^T$;.

simulation results represent with an external disturbance: $d(t) = 0.8\,sin(t)$

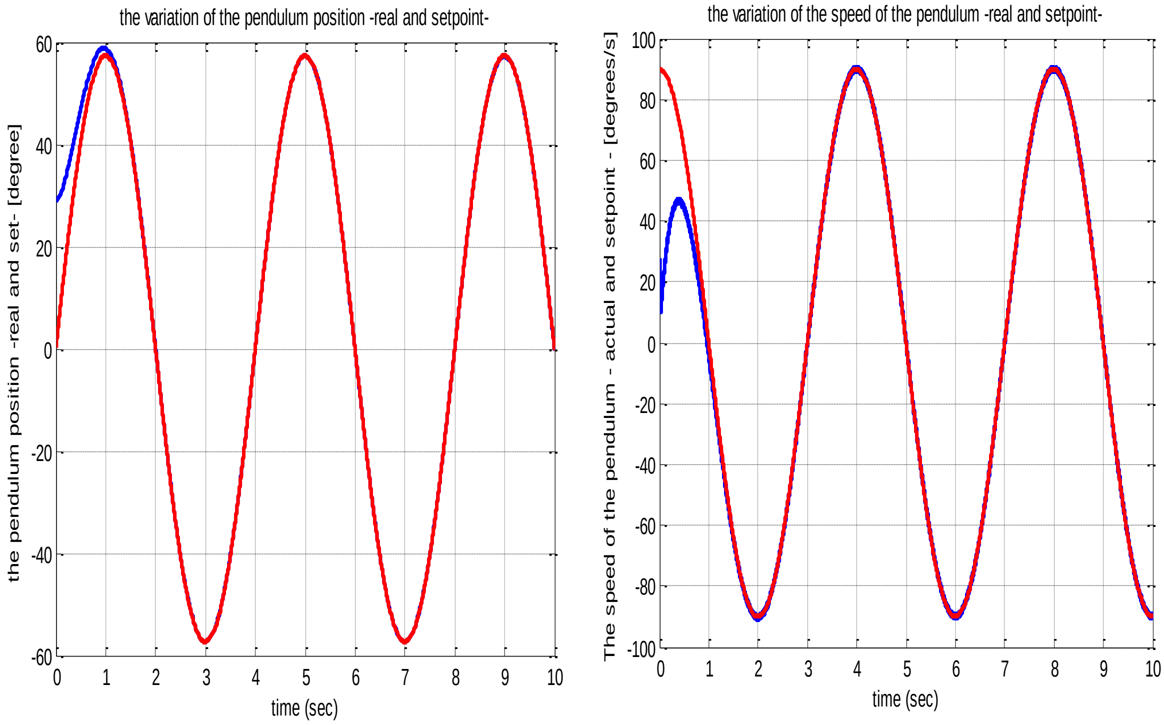

Figure II.12. The variation of the position and the speed of the pendulum -real and setpoint-

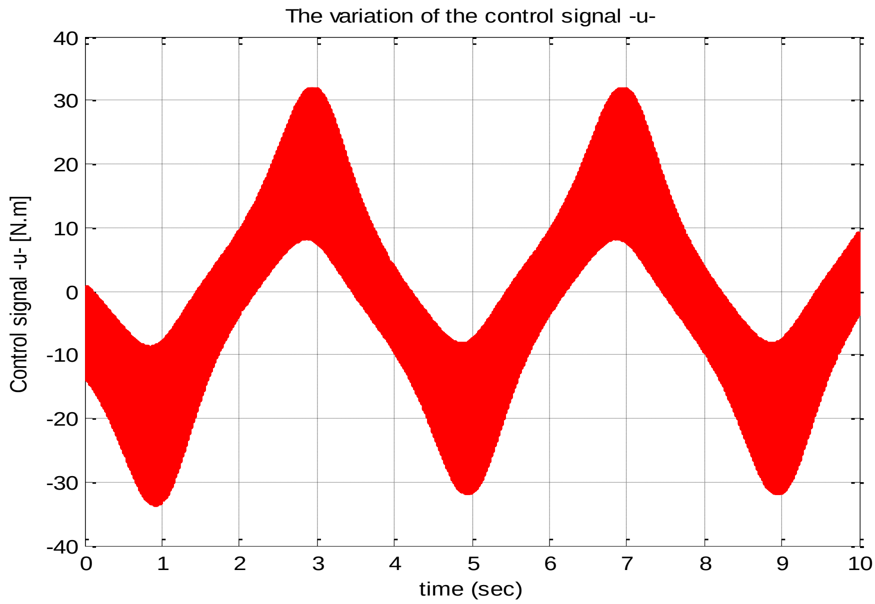

Figure II.13. The variation of the control signal-u-

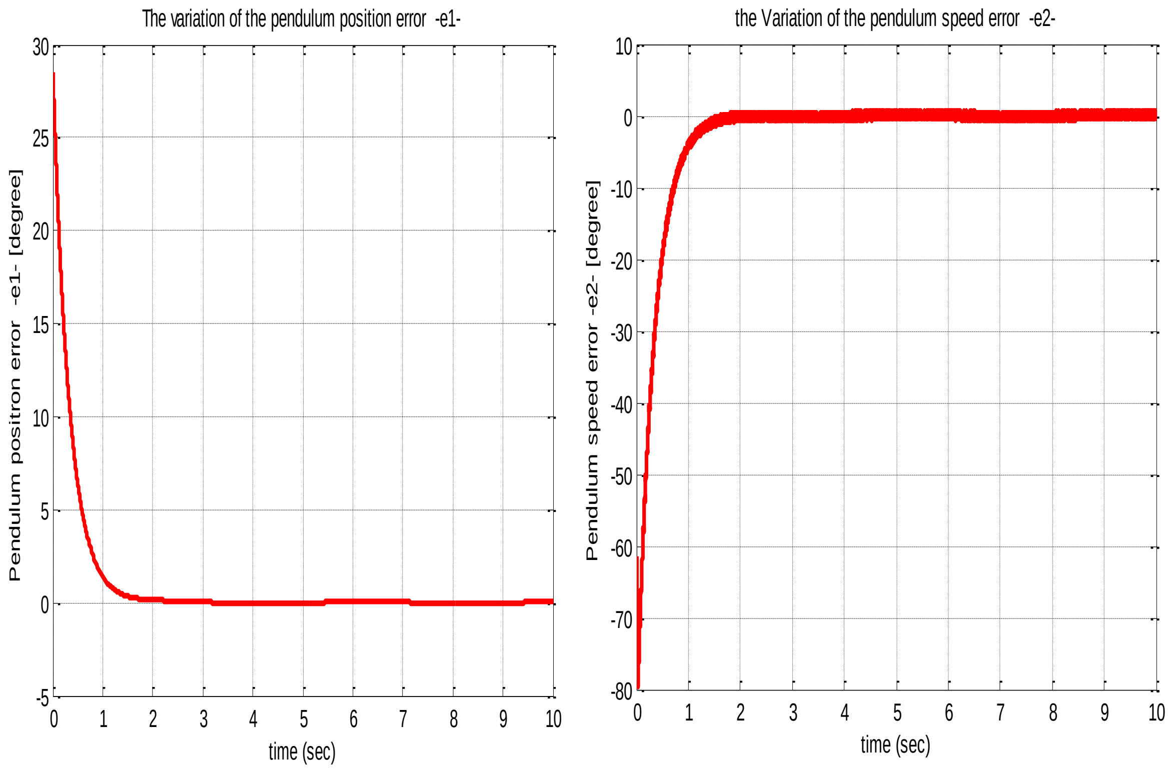

Figure II.14. The error variation of the pendulum position-e1- and speed-e2-

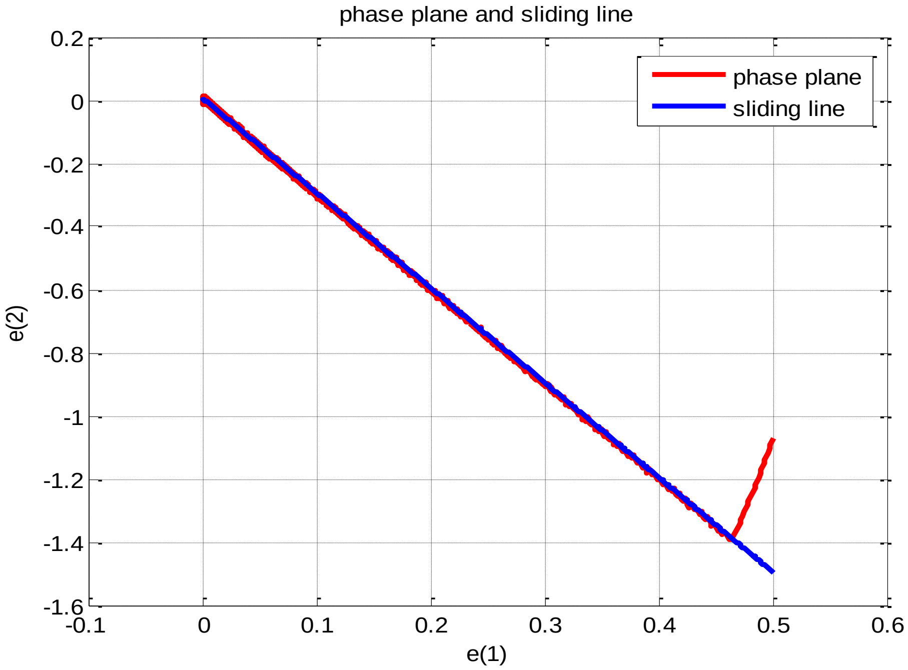

Figure II.15. Phase plan and sliding line

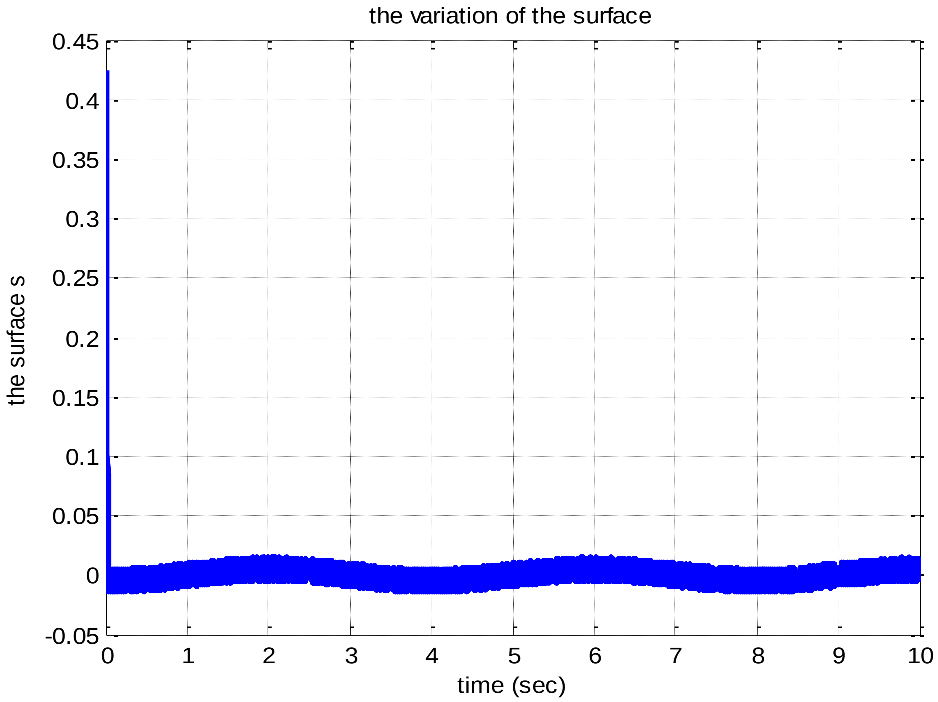

Figure II.16. The variation of the surface

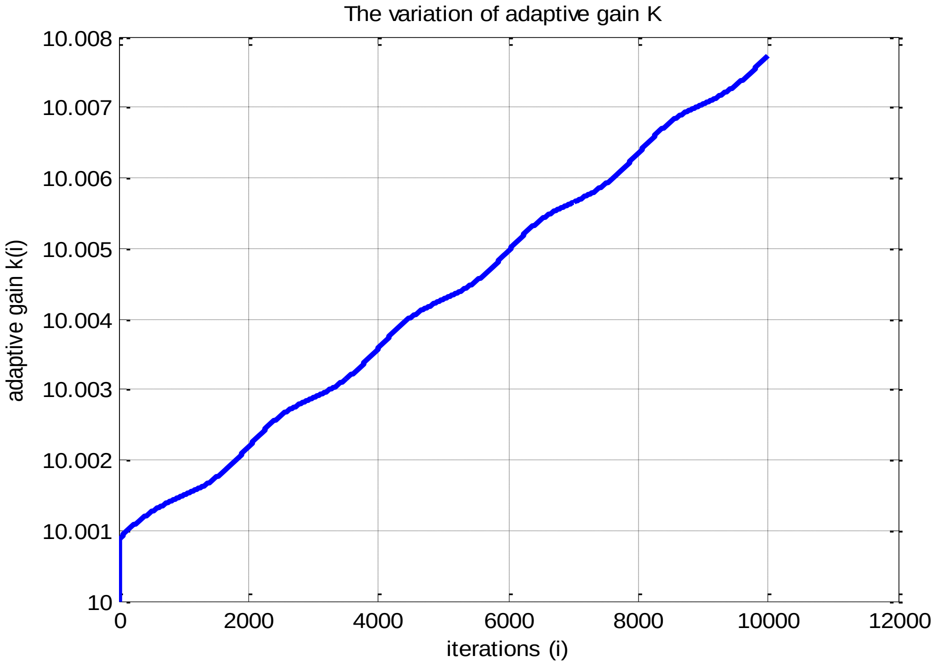

Figure II.17. The variation of adaptive gain K

Figure (II.12) shows that the variation of the position and speed of the real and set-point pendulum converges to the desired state. Figure (II.13): As shown, the variation of the control signal indicates that the system is subjected to a high control signal at each instant to ensure its convergence to the desired state. Figure (II.14) shows the variation of the position and speed error of the pendulum. Figure (II.15) shows that the phase plane is originally in the permanent regime but generates a chattering phenomenon in the sliding mode. The figure (II.16) shows the variation of the sliding surface. Figure (II.17) illustrates that the adaptive gain variation is not constant. Instead, it varies according to the system. This variation allows for optimizing the system's quality.

This type of control law achieves the desired objective but generates oscillations that lead to undesirable "chattering" in practice for certain variables.

II.9.2. continued control

By replacing the $sign(s)$ function in (II.43) with the $sat\left(\frac{Sg(x)}{\phi}\right)$ function, it results in the control Law the expression:

$$u = -\hat{K}sat\left(\frac{Sg(x)}{\phi}\right) \quad (II.49)$$

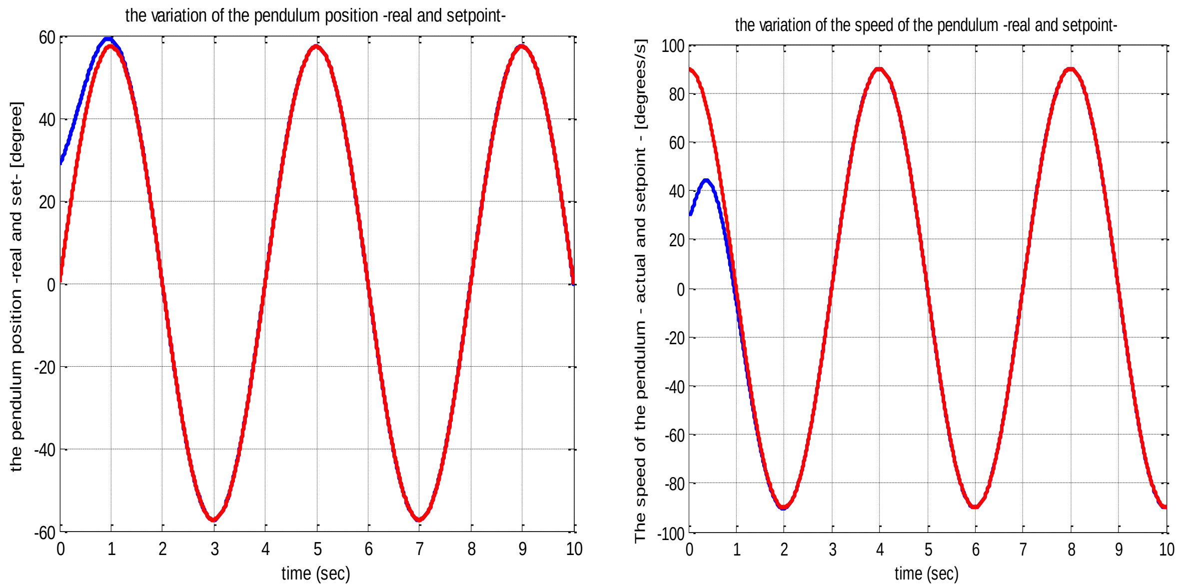

Figure II.18. The variation of the position and the speed of the pendulum -real and setpoint-

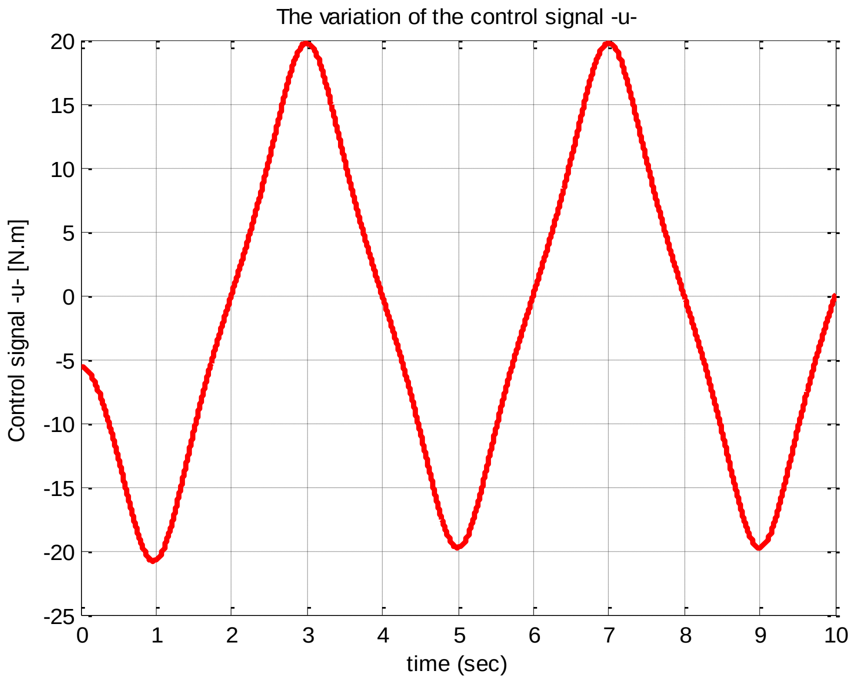

Figure II.19. The variation of the control signal-u-

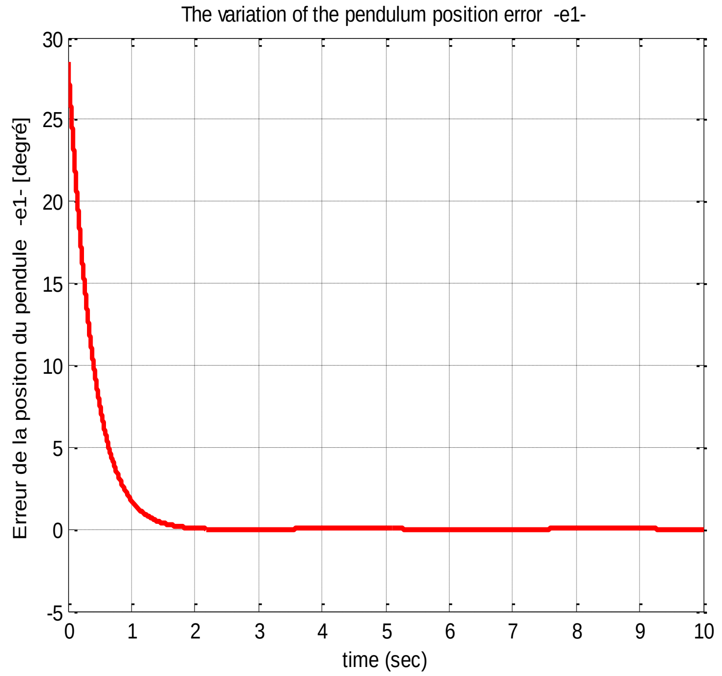

Figure II.20. The error variation of the pendulum position-e1-

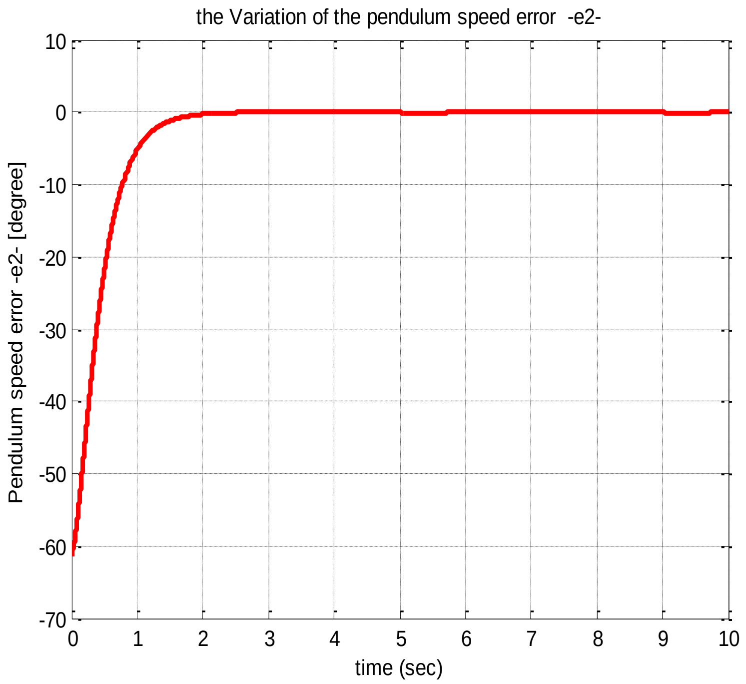

Figure II.21. The error variation of the pendulum speed-e2-

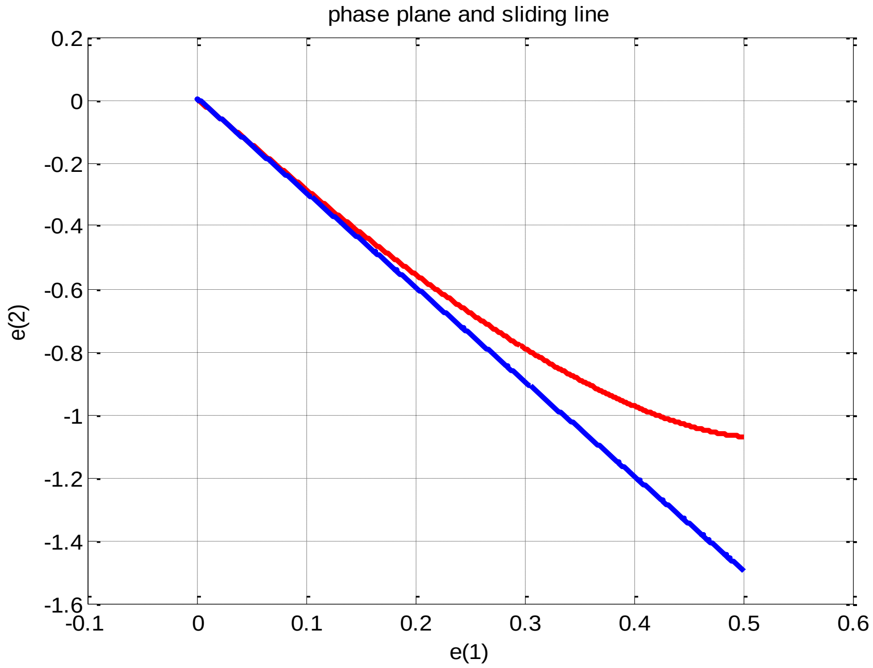

Figure II.22. Phase plan and sliding line

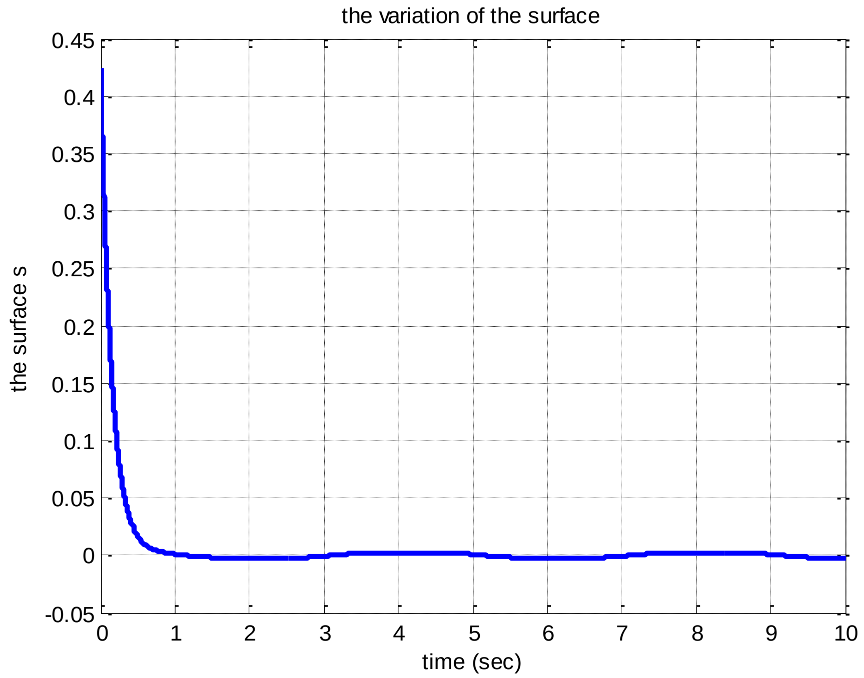

Figure II.23. The variation of the surface

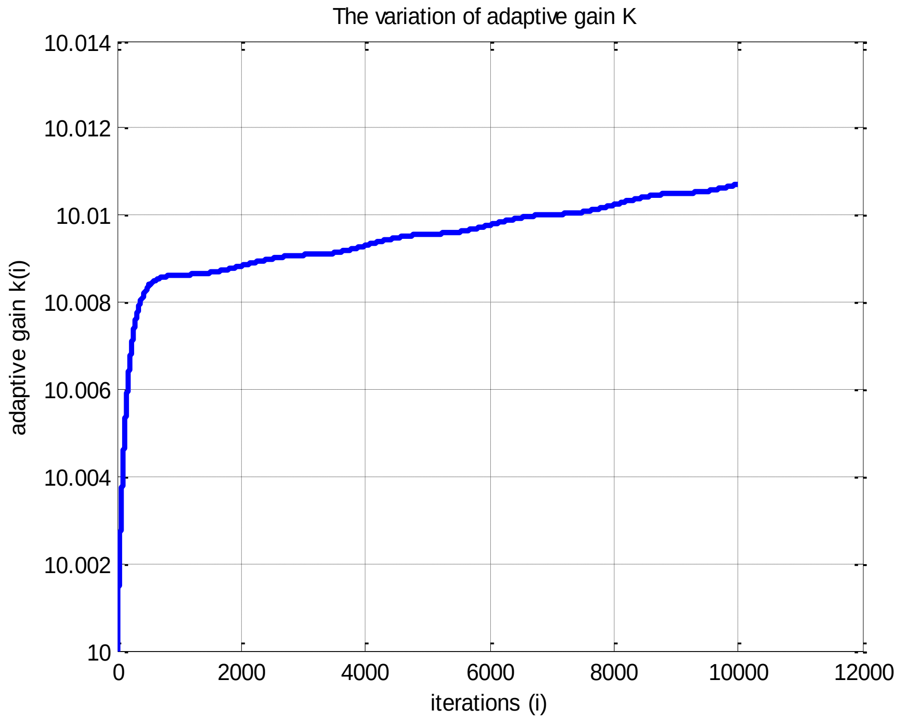

Figure II.24. The variation of adaptive gain K

Figures (II.18) show that the variation in position and speed of the pendulum real and setpoint. The control u is shown in Figure (II.19) it is a continuous and real control, Figures (II.20; II.21) show the error variation of the pendulum position-e1- and speed-e2, and the phase plane and the sliding line in Figure (II.22), and the variation of the surface is shown in Figure (II.23). Figures (II.24) show that the variation of adaptive gain 'K'.

It is observed that the actual trajectories converge to their equilibrium point, the phase plane at the origin in steady state, and the sliding surface tends to zero.

The results of simulations show that the continuous control allows mitigation of the effects of external disturbances and uncertainties, as well as eliminating phenomenon of "chattering" introduced by the discontinuous control.

II.10. Conclusion

Sliding mode control is that it is a robust and computationally efficient control technique suitable for nonlinear systems. Sliding mode control alters the dynamics of a system by applying a discontinuous control signal, forcing the system to "slide" along a cross-section of its normal behavior. This method is a variable structure control approach where the control law switches between different continuous structures based on the system's current state.

Sliding mode control offers robustness, simplicity, and insensitivity to parameter variations, making it ideal for systems with uncertainties. It can lead to optimal control for a wide range of dynamic systems and has applications in areas like electric drives and robotics. Chattering, a common issue in sliding mode control, can be mitigated through techniques like dead bands or adaptive control methods. Overall, sliding mode control provides a powerful tool for controlling nonlinear systems effectively.

Chapter III:

Sliding mode control for nonlinear MIMO systems

III.1. Introduction

Sliding mode control (SMC) is a robust design methodology developed using a systematic scheme based on a sliding surface and the Lyapunov stability theorem. The main advantage of SMC is that system uncertainties can be handled under the invariance characteristics of the system's sliding state with guaranteed system stability. However, the discontinuity of the control signal is its drawback. An approach to avoid discontinuous control signals in SMC is to replace the sign function with a saturation function.

In this chapter, a sliding mode control with adaptive gain for a class of multivariable nonlinear systems is presented. The coupling system can be divided into two subsystems, and two sliding surfaces are constructed using the state variables of the decoupled system. An intermediate variable is introduced to incorporate these two slipping surfaces.

We will study a sliding mode controller with adaptive gain to control a multivariable system to ensure stability and robustness to parametric variations of the system and external disturbances.

III.2. Synthesis controller of robust sliding mode

Let be the nonlinear system of order four described by the following state representation:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = f_1(x) + g_1(x)u(t) + d_1(t) \\ \dot{x}_3(t) = x_4(t) \\ \dot{x}_4(t) = f_2(X) + g_2(X)u(t) + d_2(t) \end{cases} \tag{III.1}$$

With:

$x = [x_1, x_2, x_3, x_4]^T$ is the state vector, $f_1(x), f_2(x), g_1(x)$ and $g_2(x)$ are non-linear functions, $\forall X$, $u(t)$ is the control, $d_1(t)$ and $d_2(t)$ are disturbances assumed to be bounded: $|d_i(t)| \le D_i$, $i = 1,2$.

Two sliding surfaces are defined S_1 and S_2

$$S_1 = c_1 x_1 + x_2 \tag{III.2}$$

$$S_2 = c_2 x_3 + x_4 \tag{III.3}$$

From the theory of sliding mode presented in the previous section, one can choose a control law of the form [28]:

$$u_1 = u_{1eq} - K_1 Sat(S_1 g_1(x)/\Phi_1), \text{ with } K_1 > \frac{D_1}{g_1(x)} \tag{III.4}$$

With:

$$u_{1eq} = \frac{-c_1 x_2 - f_1(x)}{g_1(x)} \tag{III.5}$$

and :

$$u_2 = u_{2eq} - K_2 Sat(S_2 g_2(x)/\Phi_2), \text{ with } \quad K_2 > \frac{D_2}{g_2(x)} \tag{III.6}$$

It is evident that if we set $u = u_1$, this control allows to bring back states x_1 and x_2 to surface S_1 as quickly as possible, and then to slide them to the equilibrium point. The same will be true for states x_2 and x_3 with S_2 if we take $u = u_2$, in other words, this controller can only control one of the two subsystems.

For the system given by equation (III.01), we seek to determine an output control law $u = u(x)$ so that the closed-loop system is globally stable, in the sense that all state variables are uniformly bounded and converge asymptotically to their equilibrium point.

The main idea of this decoupled regulator is to decompose the system into two subsystems A and B, subsystem A consists of x_1 and x_2 its corresponding sliding surface is S_1, the subsystem B consists of x_3 and x_3 its corresponding sliding surface is S_2.Assuming that the primary goal is to stabilize subsystem A, it is reasonable to consider information from subsystem B as secondary, and this secondary information must be taken into account by subsystem A, an intermediate variable z representing this secondary information is incorporated into S_1 [29].

The surface S_1 takes shape $c_1(x_1 - z) + x_2$, meaning that the main objective is changed to $x_1 = z$, $x_2 = 0$, or z is a function of S_2.

The expression S_1 and S_2 can be chosen as:

$$S_1 = c_1(x_1 - z) + x_2 \tag{III.7}$$

$$S_2 = c_2 x_3 + x_4 \tag{III.8}$$

So, the control law becomes:

$$u = u_1 = u_{1eq} - K_1 Sat\left(S_1 g_1(x) / \Phi_1\right) \tag{III.9}$$

With:

$$u_{1eq} = \frac{-c_1 x_2 - f_1(x)}{g_1(x)} \tag{III.10}$$

The value of the state z can be limited by setting

$$|z| \leq z_U, 0 < z_U < 1 \tag{III.11}$$

where z_U is the maximum value of $|z|$.

The variable z can be defined as:

$$z = Sat\left(\frac{S_2}{\Phi_Z}\right) z_U \quad \text{with:} \quad 0 < z_U < 1 \tag{III.12}$$

with Φ_Z is the boundary band of the slipping surface S_2 who ensures the smoothing of the control and maintains the system state in this range. Starting from the equation if $S_2 \neq 0$ where $Z \neq 0$, if $S_2 \to 0$ so $z \to 0$, $x_1 \to 0$ and $S_1 \to 0$ The objective of the order can be completed[30].

III.3. Elestrative examples

III.3.1. Inverse Pendulum

The system consists of a mobile translation cart supporting a freely rotating pendulum as shown in Figure III.1:

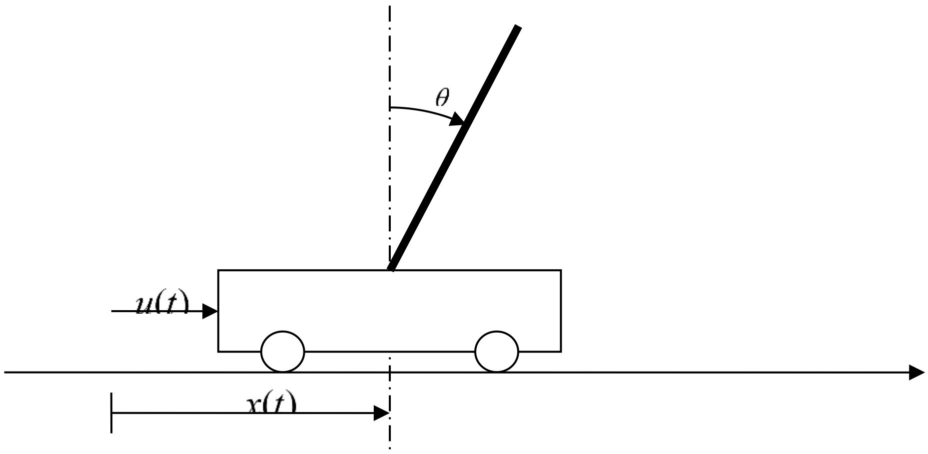

Figure III.1. Schematic diagram of the simple inverted

By exerting a horizontal force $u(t)$ On the cart, the cart moves to the position x causing the pendulum to rotate an angle θ. The inverted pendulum is an unstable system in an open loop, non-linear and multivariable.

The control of this system must achieve:

• Stabilization of the pendulum around its equilibrium position, starting from an initial condition $\theta(0)$ Comprised within the interval [$-\pi/2, +\pi/2$].

• Stabilization of the carriage in position $x=0$, starting from an initial condition comprised within the interval [$-1m, +1m$].

Motion can be described by the following differential equations [31]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{m_t g \sin x_1 - m_p L \sin x_1 \cos x_1 x_2^2 + \cos x_1 . u}{L.\left(\dfrac{4}{3} m_t - m_p \cos^2 x_1\right)} + d(t) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{\dfrac{4}{3} m_p L x_2^2 \sin x_1 + m_p g \sin x_1 \cos x_1}{\dfrac{4}{3} m_t - m_p \cos^2 x_1} + \dfrac{4}{3.\left(\dfrac{4}{3} m_t - m_p \cos^2 x_1\right)} u + d(t) \end{cases} \quad \text{(III.13)}$$

With:

$$x_1 = \theta, x_2 = \dot{\theta}, x_3 = x, x_4 = \dot{x}$$

$u(t)$: the control applied on the trolley.

x : the position of the trolley.

θ: the angle of the pendulum.

$m_p = 0.1kg$: is the mass of the pendulum.

$m_c = 1kg$: the weight of the trolley.

$m_t = m_c + m_p$: the total mass of the pendulum trolley.

$L = 0.5m$: half length of the pendulum.

$g = 9.81m/s^2$: gravity.

The simulation results are given in the following figures for a condition

initial $x(0) = [-0.5,0,0.5,0]^T$; for a regulator by sliding mode whose

parameters are: $\mathrm{C}_1 = 5$; $\mathrm{C}_2 = 0.5$; $\mathrm{z_u} = 0.9425$; $k = 10$

simulation results represent with an external disturbance: $d(t) = 0.4\, sin(\,t)$

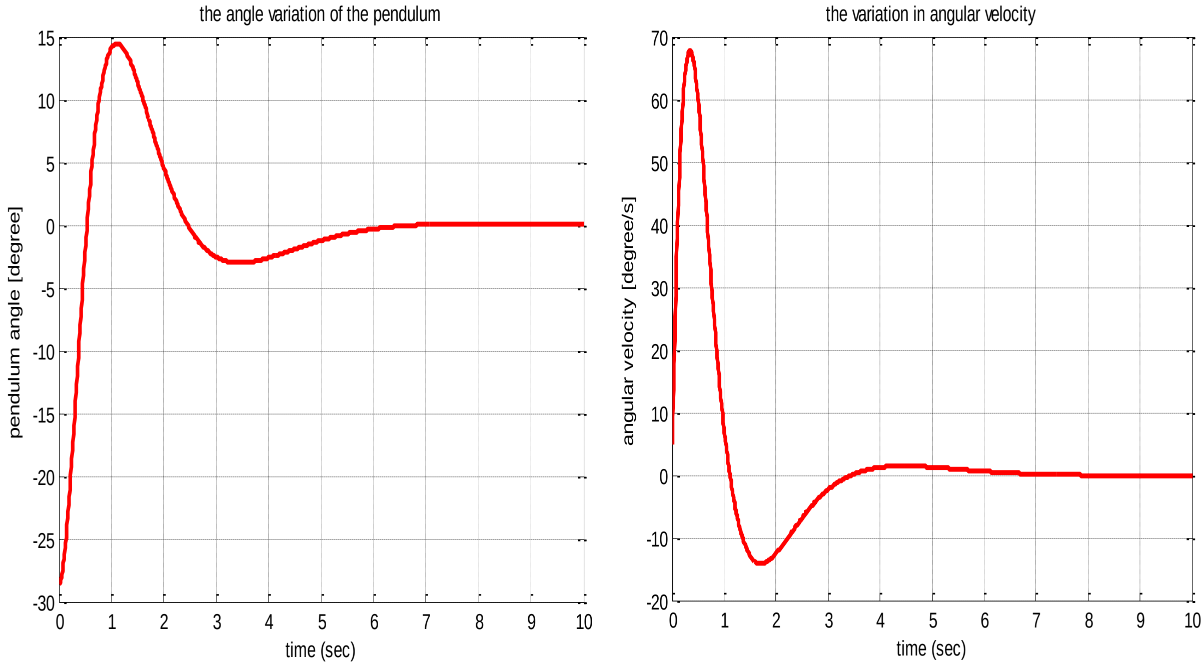

Figure III.2. The Variation of state variables $\theta(t), \dot{\theta}(t)$

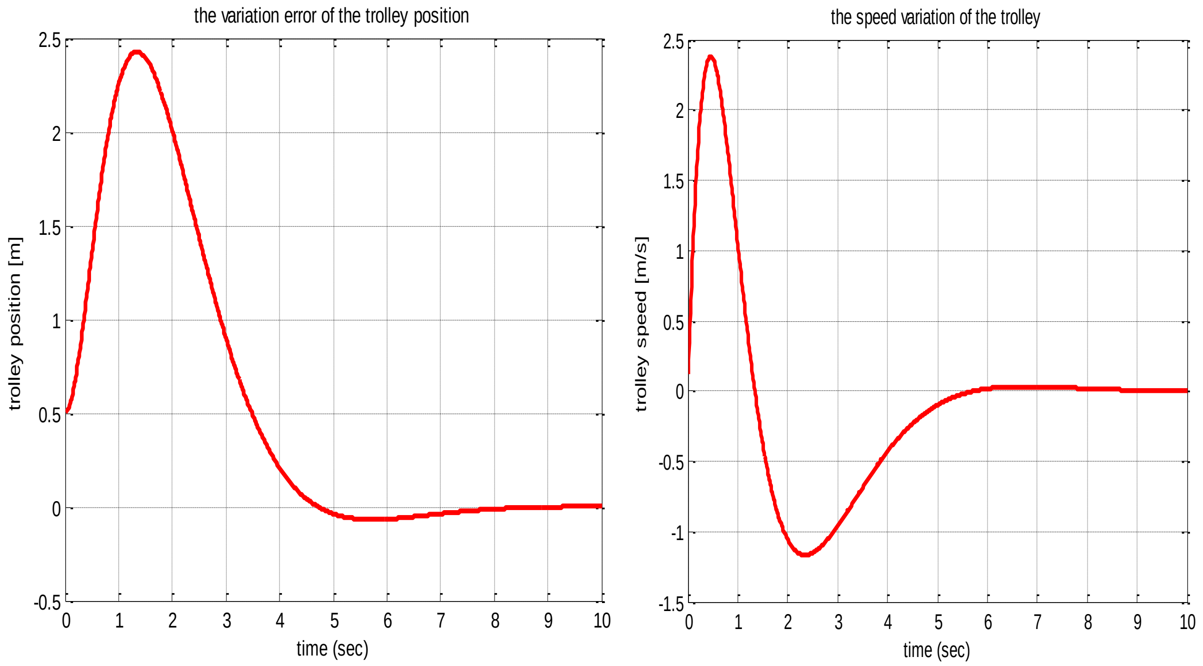

Figure III.3. The Variation of state variables $x(t) et\ \dot{x}(t)$

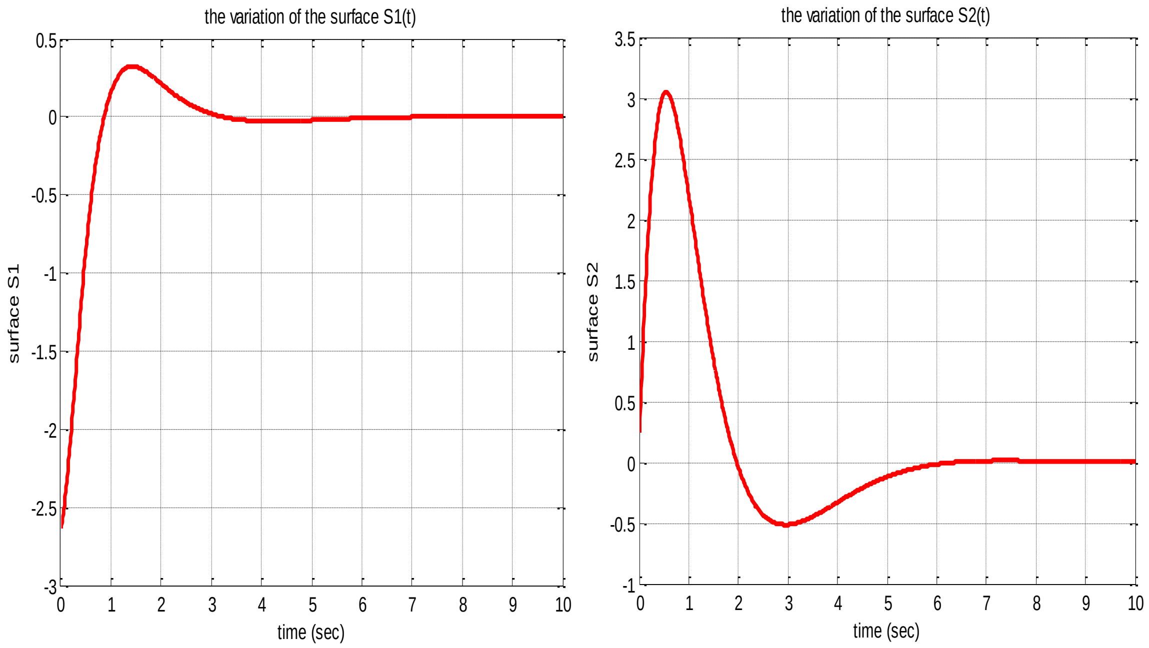

Figure III.4. The variation of the surface S1 and S2.

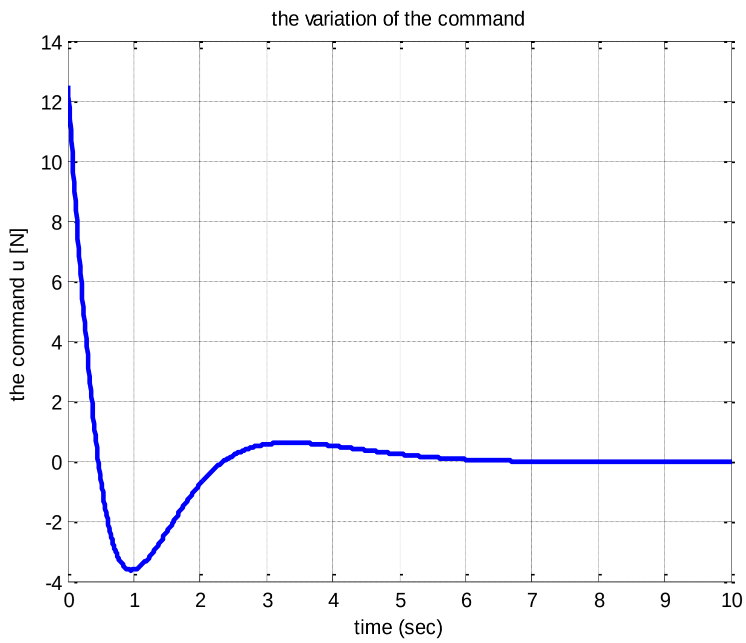

Figure III.5. The variation of the control U.

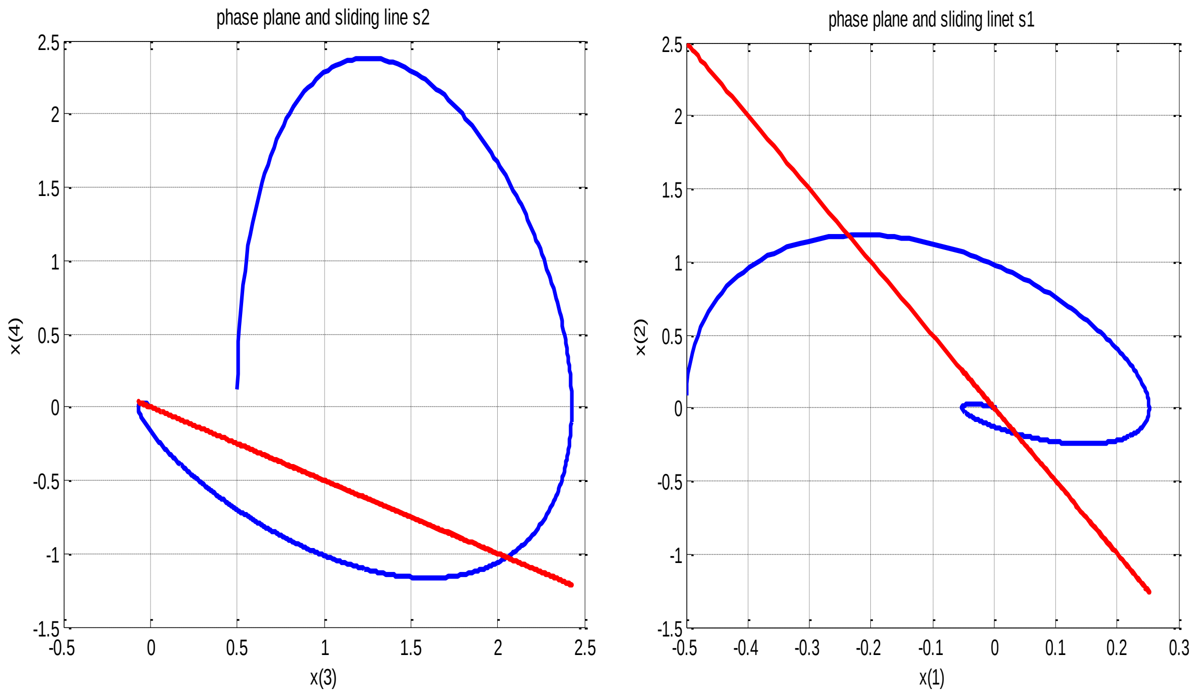

Figure III.6. phase plan and sliding lines of S1 and S2.

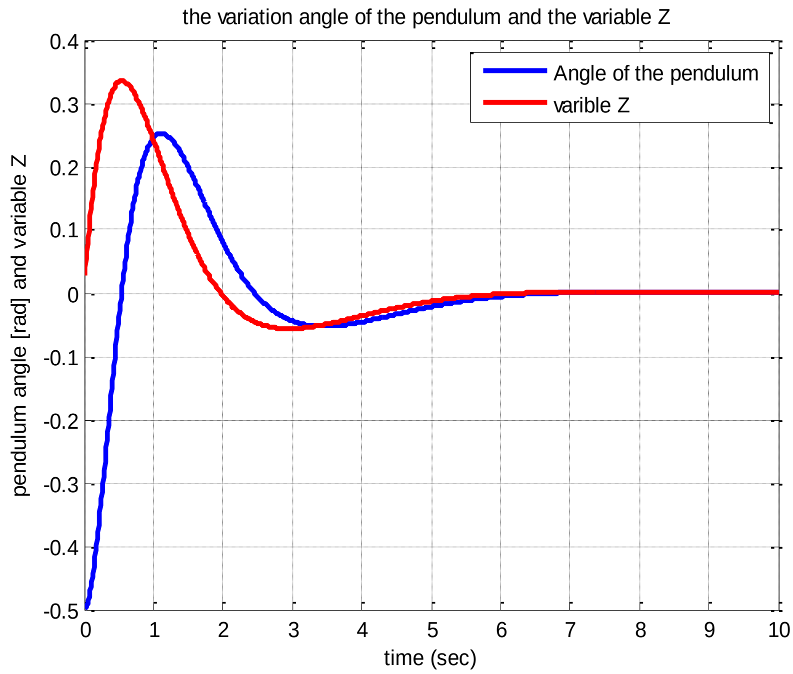

Figure III.7. The variation angle of the pendulum and variable Z.

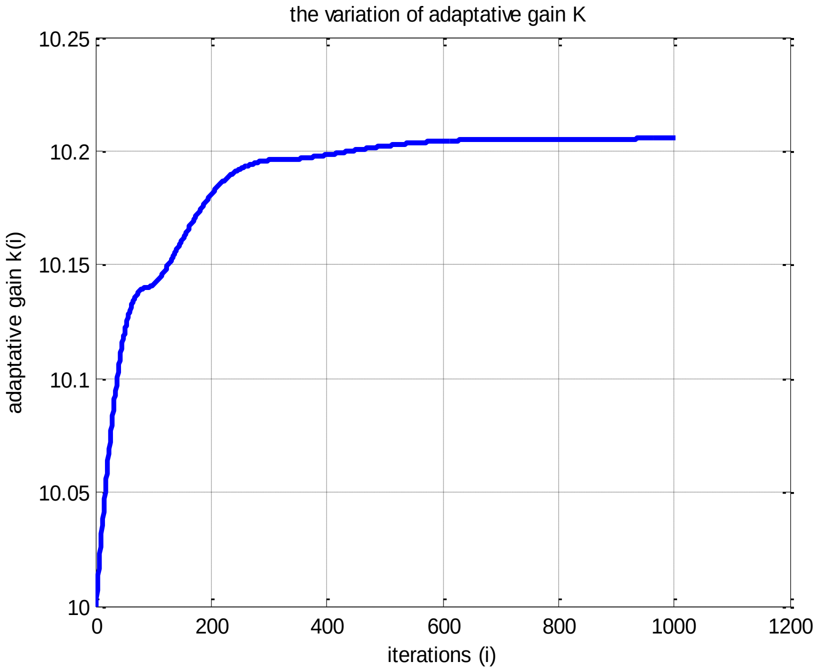

Figure III.8. The variation of the adaptative gain.

Figures (III.2 and III.3) shows the states of the system $\theta(t), \dot{\theta}(t), x(t) and\ \dot{x}(t)$converge towards the equilibrium points despite the presence of disturbances. Figure (III.4) shows that the two sliding surfaces S1 and S2 tend towards zero. Figure (III.5) shows That the control converges to zero. Figure (III.6) represents phase plan is in the permanent regime.

Figure (III.7) shows that the variable z is a fuzzy value $(0 \leq Z \leq 1)$; Note that the angle following the variable z is convergent to zero; and note that the variation in adaptive gain is not constant in Figure (III.8).

It is found that the control is reliable and robust despite external disturbances.

III.4. Tracking signal reference

Consider the single-input single-output system described by the following differential equations:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = x_3(t) \\ \vdots \\ \dot{x}_n(t) = f(x) + g(x)u(t) + d(t) \\ y(t) = x_1(t) \end{cases} \tag{III.14}$$

with:

$x(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^T=[x(t),\dot{x}(t),\ldots,x^{(n-1)}(t)]^T$ the state vector.

$x_d(t)=[x_{1d}(t),x_{2d}(t),\ldots,x_{nd}(t)]^T=[x_d(t),\dot{x}_d(t),\ldots,x_d{}^{(n-1)}(t)]^T$ the desired state vector.

$f(x)$ and $g(x)$ are non-linear functions of the state vector, $u(t)$ is the control and $d(t)$ the disturbance considered to be bounded: $|d(t)|\leq D$.

The sliding mode control approach for trajectory tracking is to find a control law such that, given a desired trajectory the tracking error tends to zero despite the presence of external disturbances.

The tracking error vector is defined as:

$$E(t)=x(t)-x_d(t)=[e(t),\dot{e}(t),\ldots,e^{(n-1)}(t)]^T \quad \text{(III.15)}$$

$$E(t)=[e_1(t),\ldots,e_{n-1}(t),e_n(t)]^T \quad \text{(III.16)}$$

And the pursuit error:

$$e(t)=x(t)-x_d(t) \quad \text{(III.17)}$$

whose derivative is:

$$\dot{e}(t)=\dot{x}(t)-\dot{x}_d(t) \quad \text{(III.18)}$$

A linear function is defined S which represents the sliding surface [32]:

$$S(x,t)=c_1e_1+c_2e_2+\cdots+c_{n-1}e_{n-1}+e_n \quad \text{(III.19)}$$

$$S(x,t)=\sum_{i=1}^{n-1}c_ie_i+e_n \quad \text{with: } c_i>0,\ i=1,\ldots,n-1 \quad \text{(III.20)}$$

whose derivative is:

$$\dot{S}=\sum_{i=1}^{n-1}c_ie_{i+1}+x^{(n)}-x_d^{(n)} \quad \text{(III.21)}$$

$$\dot{S} = \sum_{i=1}^{n-1} c_i e_{i+1} + f(x) + g(x)u + d(t) - x_d^{(n)} \tag{III.22}$$

If you choose a Lyapunov function of the form:

$$V = \frac{1}{2} S^2 \tag{III .23}$$

then the derivative of the Lyapunov function:

$$\dot{V} = S\dot{S} = S\sum_{i=1}^{n-1} c_i e_{i+1} + Sf(x) + Sg(x)u + Sd(t) - Sx_d^{(n)} \tag{III.24}$$

if we assume that $g(x) > 0, \; \forall \; x$ then:

u increases with the increase of $\dot{S}$ and vice versa.

if $S > 0$ the decrease of u will decrease $S\dot{S}$ so that $\dot{V} < 0$ and $S < 0$ the increase in U will decrease $S\dot{S}$ also so that $\dot{V} < 0$.

According to Lyapunov's theorem if $\dot{V}$ is negative the trajectory $x(t)$ will be drawn towards the desired trajectory $x_d(t)$, tracking error $x(t) - x_d(t)$ tends to zero and will be attracted to the sliding surface $S(e) = 0$ for all $t \geq 0$.

III.4.1. The sliding mode for trajectory tracking

First, the sliding surface is defined S_1 as follows:

$$S_1 = c_1(e_1 - z) + e_2 \tag{III.25}$$

With: $e_1(t) = x_1(t) - x_{1d}(t)$ and $e_2(t) = x_2(t) - x_{2d}(t)$

in the same way another sliding surface is defined S_2 :

$$S_2 = c_2 e_3 + e_4 \tag{III.26}$$

With:

$$e_3(t) = x_3(t) - x_{3d}(t) \quad \text{and} \quad e_4(t) = x_4(t) - x_{4d}(t)$$

The expression of the control becomes:

$$u = u_1 = u_{1eq} - K_1 Sat\left(S_1 g_1(x) / \Phi_1\right) \tag{III.27}$$

With:

$$u_{1eq} = \frac{-c_1 e_2 - f_1(x) + \dot{x}_{2d}}{g_1(x)} \tag{III.28}$$

The value z may be limited by posing:

$$|z| \le z_U, 0 < z_U < 1 \tag{III.29}$$

Or z_U is the maximum value of $|z|$

The Variable z can be defined as:

$$z = Sat\left(\frac{S_2}{\Phi_Z}\right) z_U \quad \text{with} \quad 0 \prec z_U \prec 1 \tag{III.30}$$

III.5. Sliding mode control with adaptive gain

in the previous part, it was assumed that the gain K of the control by sliding mode can be determined. however, in practice, there is no method for calculating this gain. To solve this problem, we use in this section, an adaptive gain control.

III.5.1. position of the problem

Now we are looking at the following control law:

$$u(t) = u_{eq} + u_n \tag{III.31}$$

The equivalent control u_{eq} can be obtained from the temporal surface derivative $\dot{S}_1 = 0$

$$\dot{S}_1 = c_1(\dot{e}_1 - \dot{z}) + \dot{e}_2 \tag{III.32}$$

$$\dot{S}_1 = c_1(e_2 - \dot{z}) + f_1(x) + g_1(x)u - \dot{x}_{2d} \tag{III.33}$$

$\dot{S}_1 = 0$

$$u_{eq}^{*} = \frac{-c_1 e_1 + c_1 \dot{z} - f_1(x) + \dot{x}_{2d}}{g_1(x)} \tag{III.34}$$

The variable z cannot be derived, $\dot{z}$ cannot be obtained, for this we approach the optimal equivalent control law u_{eq}^{*} by the equivalent control u_{eq} given by:

$$u_{eq} = \frac{-c_1 e_1 - f_1(x) + x_{2d}}{g_1(x)} \tag{III.35}$$

and either $k(t) = u_{eq} - u_{eq}^{*}$ given by:

$$k(t) = u_{eq} - u_{eq}^{*} \qquad 0 \le |k(t)| \le K \tag{III.36}$$

The limit of uncertainty K is a positive constant. However, this uncertainty limit cannot be measured in practice [29].

Let $\hat{K}$ be the estimated value of K , we consider the estimation error:

$$\tilde{K}(t) = K - \hat{K}(t) \tag{III.37}$$

u_n : the discontinuous order whose purpose is to check the attractiveness conditions, an adaptive sliding mode control term is introduced to compensate for the difference between the optimal equivalent control u_{eq}^{*} and the equivalent control u_{eq} .

$$u_n = -\hat{K} Sign(S_1 g_1) \tag{III.38}$$

To ensure the objectives of the order, the following adaptation law is adopted:

$$\dot{\hat{K}} = -\dot{\tilde{K}} = n|S_1 g_1| \tag{III.39}$$

With $n > 0$.

III.5.2. Stability analysis

The aim is to ensure the stability of the control structures in the sense that all input and output signals remain bounded and the tracking error tends asymptotically to zero.

In general, the synthesis of Lyapunov consists of selecting a candidate function of Lyapunov and then choosing laws of control or adaptation ensuring its decay.

To demonstrate the stability of the system, we consider the following Lyapunov candidate function:

$$V = \frac{1}{2} S_1^2 + \frac{1}{2n} \tilde{K}^2 \tag{III.40}$$

With:

$$\tilde{K}(t) = K - \hat{K}(t)$$

The temporal derivative of (III.59) is:

$$\dot{V} = S_1 \dot{S}_1 + \frac{1}{n} \tilde{K} \dot{\tilde{K}} \tag{III.41}$$

from (III.36) and (III.40), it comes:

$$\dot{V} = S_1(c_1 e_2 - c_1 \dot{z} + f_1(x) + g_1(x)u - \dot{x}_{2d}) + \frac{1}{n}(K - \hat{K})\dot{\tilde{K}} \tag{III.42}$$

by replacing u by its expression (III.31) and using the law of adaptation (III.39), the relation (III.42) becomes:

$$\dot{V} = S_1\left(c_1 e_2 - c_1 \dot{z} + f_1(x) + g_1(x)(u_{eq} + u_n) - \dot{x}_{2d}\right) - (K - \hat{K})\left|S_1 g_1(x)\right| \tag{III.43}$$

$$\dot{V} = S_1\left(c_1 e_2 - c_1 \dot{z} + f_1(x) + g_1(x)(u_{eq} - u_{eq}^* + u_{eq}^* + u_n) - \dot{x}_{2d}\right) - (K - \hat{K})\left|S_1 g_1(x)\right| \tag{III.44}$$

$$\dot{V} = S_1\left(g_1(X)(k - \hat{K} Sign(S_1 g_1(x)))\right) - (K - \hat{K})\left|S_1 g_1(x)\right| \tag{III.45}$$

from (III.36) and (III.38), it comes:

$$\dot{V} = S_1\left(g_1(x)(k - \hat{K} Sign(S_1 g_1(x)))\right) - (K - \hat{K})\left|S_1 g_1(x)\right| \tag{III.46}$$

$$\dot{V} = S_1 g_1(x)k - S_1 g_1(x)\hat{K}Sign(S_1 g_1(x)) - K|S_1 g_1(x)| + \hat{K}|S_1 g_1(x)| \quad \text{(III.47)}$$

$$\dot{V} = S_1 g_1(x)k - K|S_1 g_1(x)| \quad \text{(III.48)}$$

$$\dot{V} \leq 0$$

III.6. Elestrative example

III.6.1. Simulation results for Continuation of trajectory

The inverted pendulum is represented by (Figure III.1), the movement of which can be described by equation (III.13), with the following parameters:

$$m_p = 0.1kg, m_c = 1kg, m_t = 1.1kg, L = 0.5m, g = 9.81m/s^2$$

The figures (III.10 III.13 III.14 III.15) represent the results obtained for the continuation of the desired state vector as follows:

$$\theta_d(t) = 0, \dot{\theta}_d(t) = 0, x_d(t) = t \text{ and } \dot{x}_d(t) = 1$$

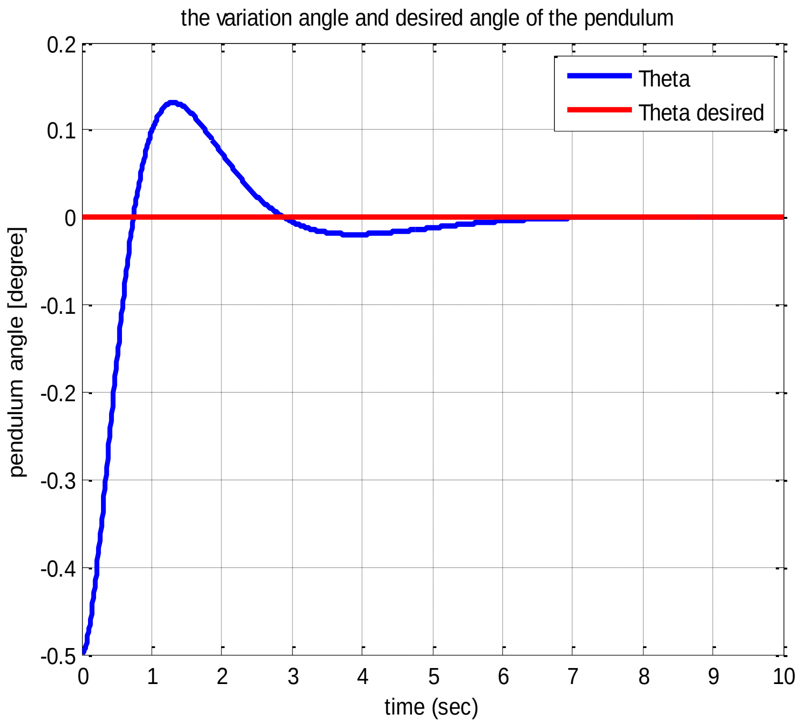

Figure III.9. The variation angle and desired angle of the pendulum.

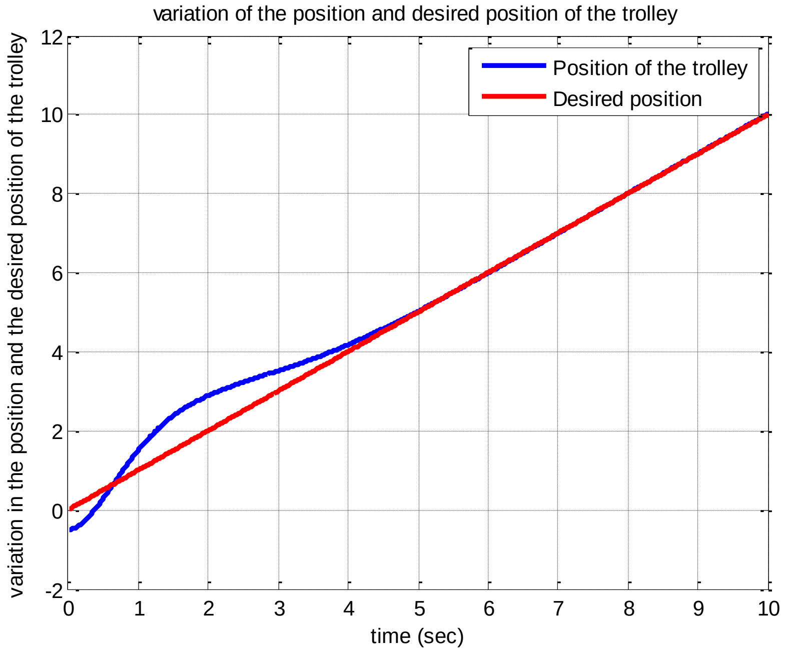

Figure III.10. The variation position x and desired position x_d of the trolley.

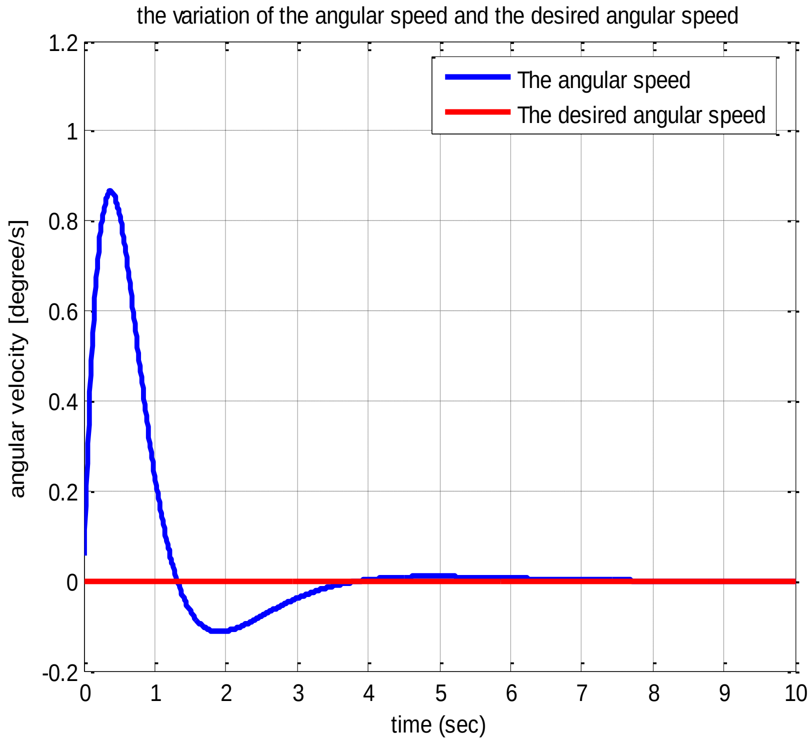

Figure III.11. The variation angular speed and desired angular speed.

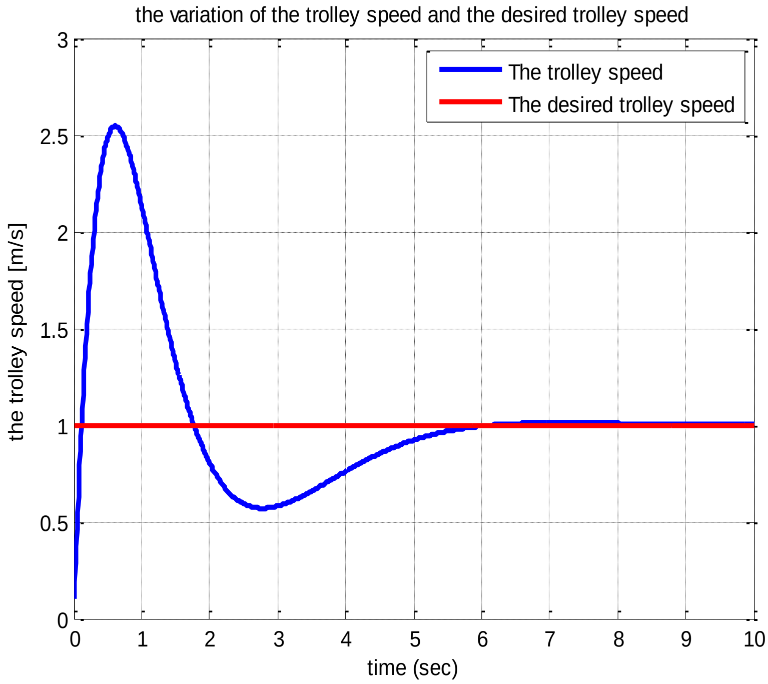

Figure III.12. The variation of the trolley speed and desired trolley speed.

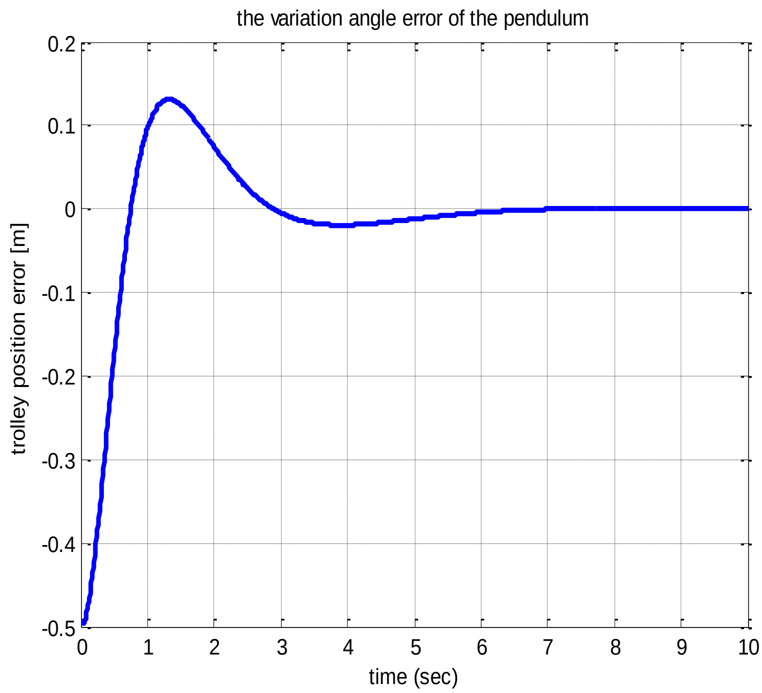

Figure III.13. The variation angle error and desired angle error of the pendulum.

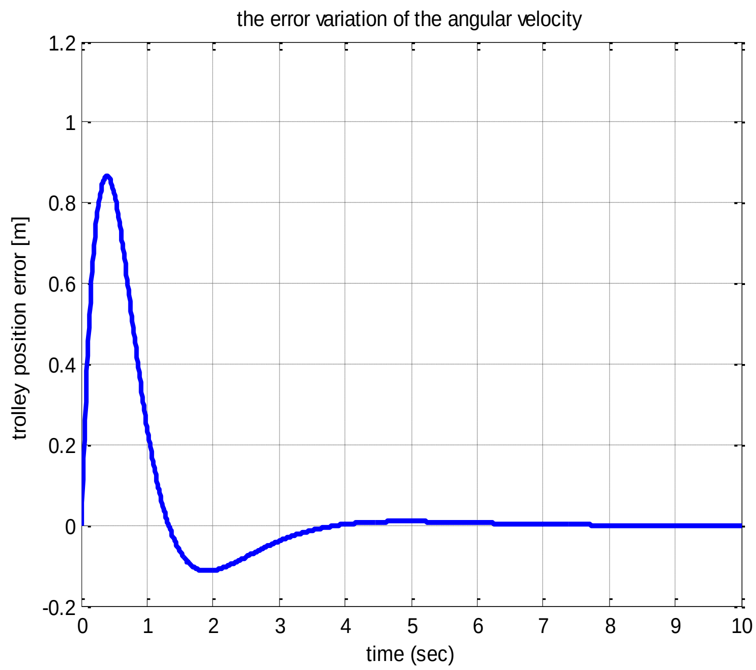

Figure III.14. The variation of the angular error velocity.

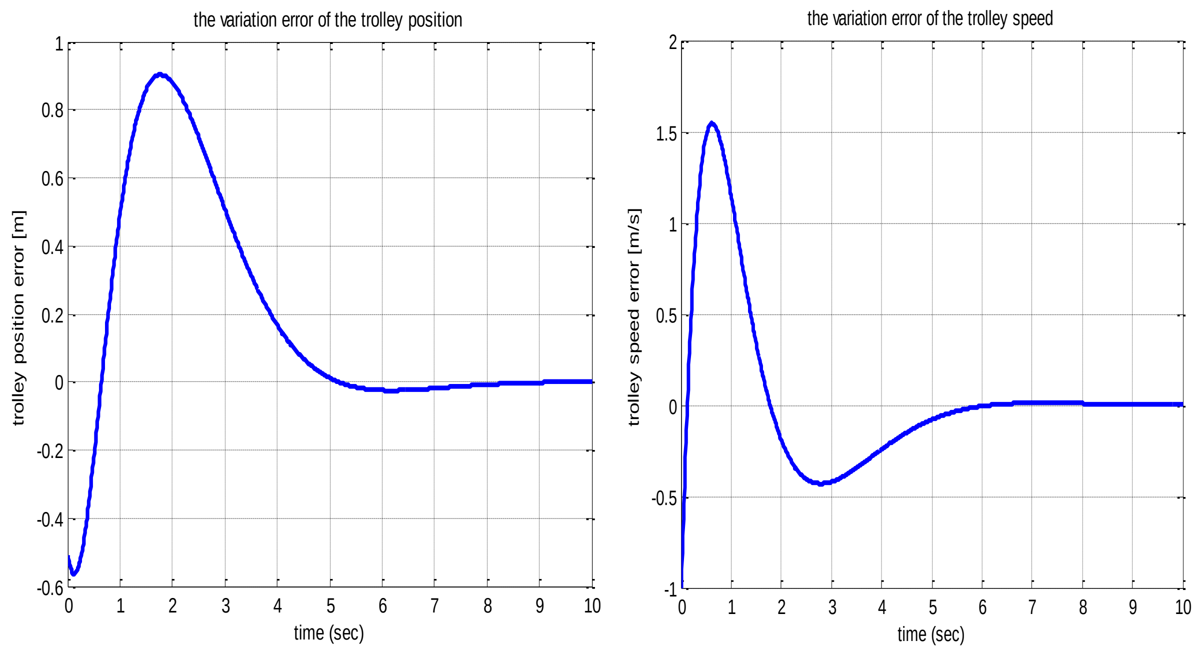

Figure III.15. The variation error of the trolley position and speed.

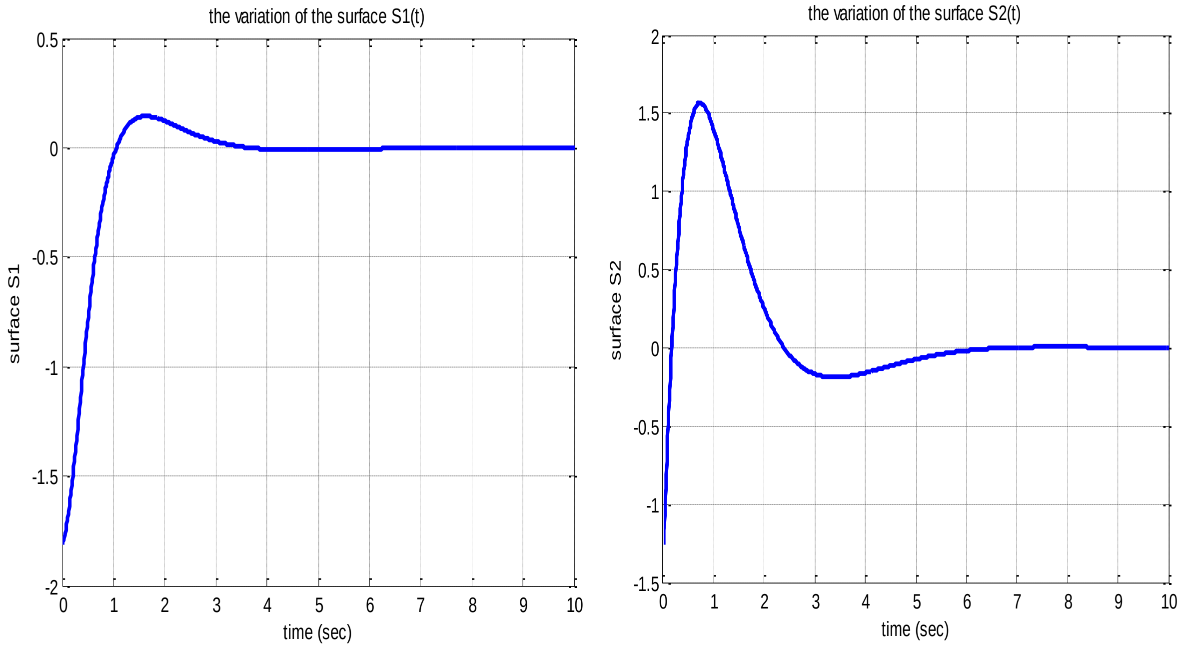

Figure III.16. The variation of the surface S1 and S2.

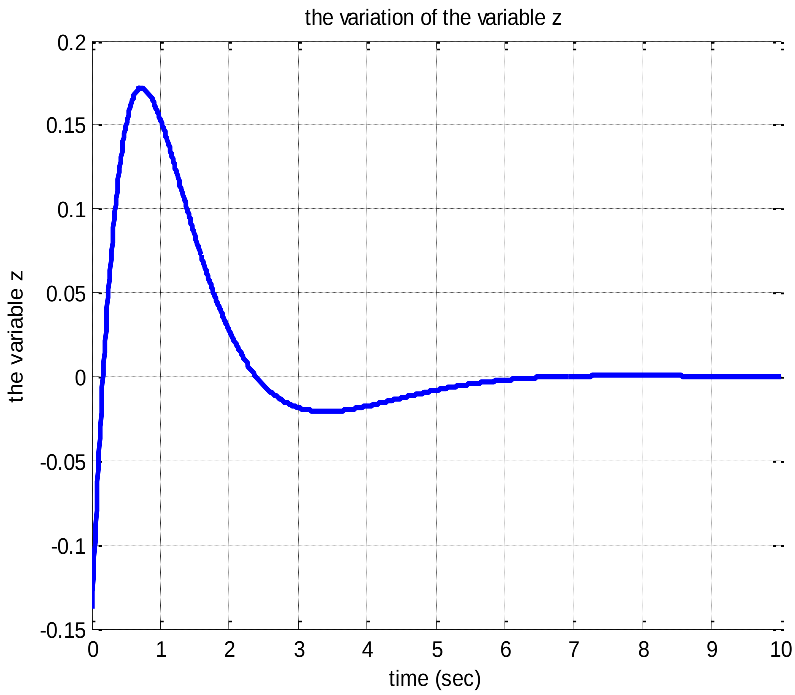

Figure III.17. The variation the variable Z.

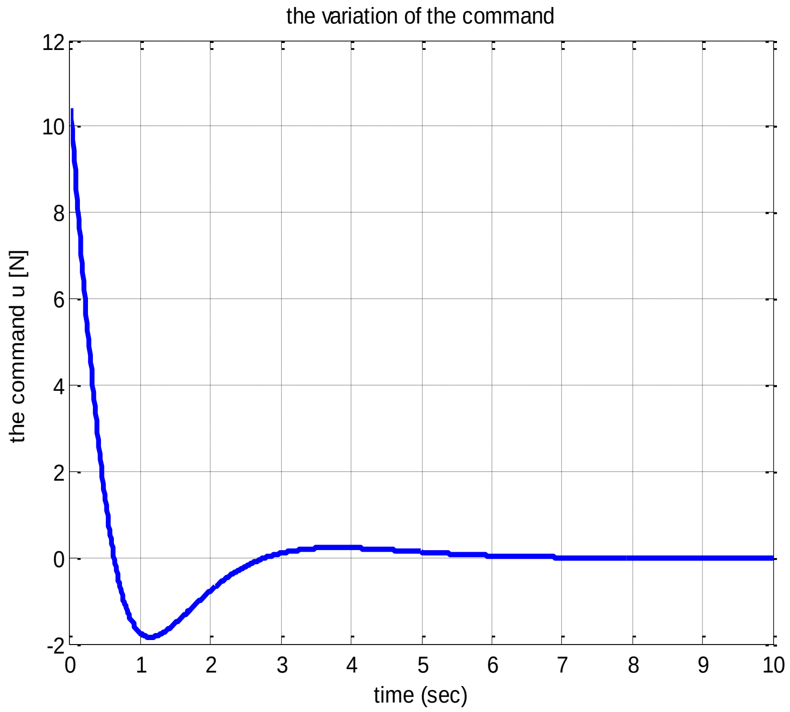

Figure III.18. The variation of the control.

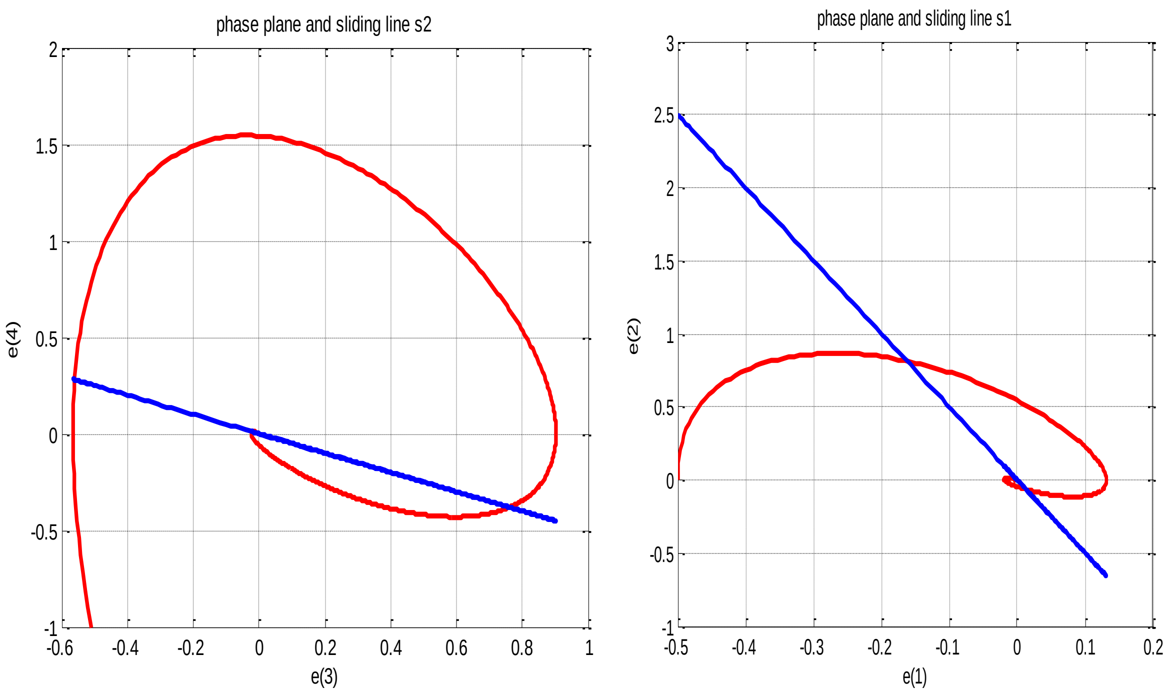

Figure III.19. The variation of the phase plane and sliding line S1 and S2.

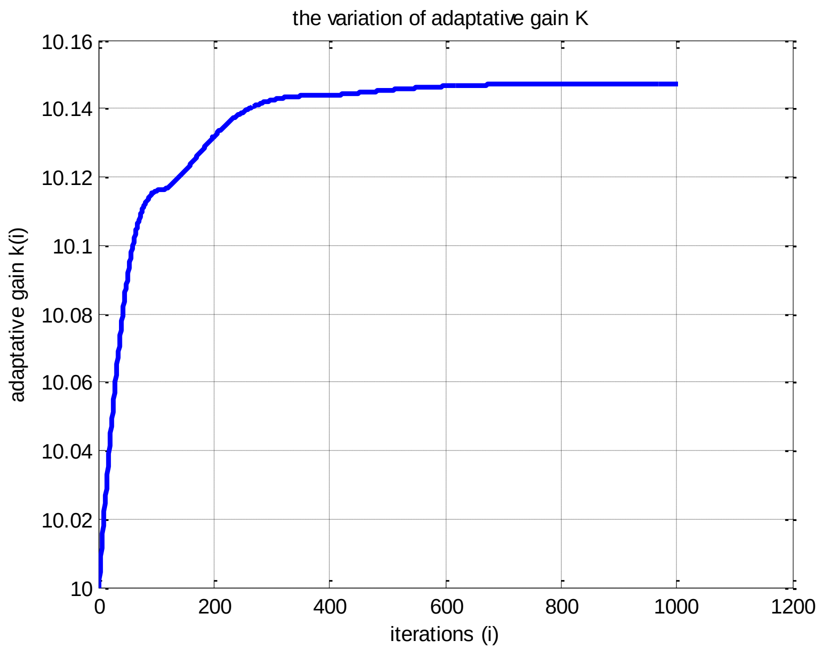

Figure III.20. The variation of the adaptative gain.

It is noted that the pursuit error of the system converges to the desired state in the figures (III.13; III.14; III.15), and the figures (III.9; III.10; III.11; III.12) observe that they converge to the desired state, while the sliding surfaces S1 and S2 tend towards zero (Figure III.16). And note that the intermediate variable z and the control applied converge to zero in the figures (III.17; III.18). The representation of phase plans is originally in the permanent regime (figure III.19). and Figure (III.20) shows that adaptive gain variation is not constant; this variation allows for optimizing the system's quality.

It is found that this pursuit is carried out by minimizing the error of continuation on the one hand and by ensuring the stability of the system on the other.

III.7. Comparative Study between Sliding mode control with adaptive gain and fixed gain

III.7.1. The position of the tracking rectangular of signals references

Figures(III.21-III.23)illustrate the results obtained for tracking the following desired state vector: $\theta_d(t) = 0$, $\dot{\theta}_d(t) = 0$, $x_d(t) = \begin{cases} -2 & if \quad 20\,sec \le t \le 40sec \\ 2 & if \quad else \end{cases}$ and $\dot{x}_d(t) = 0$

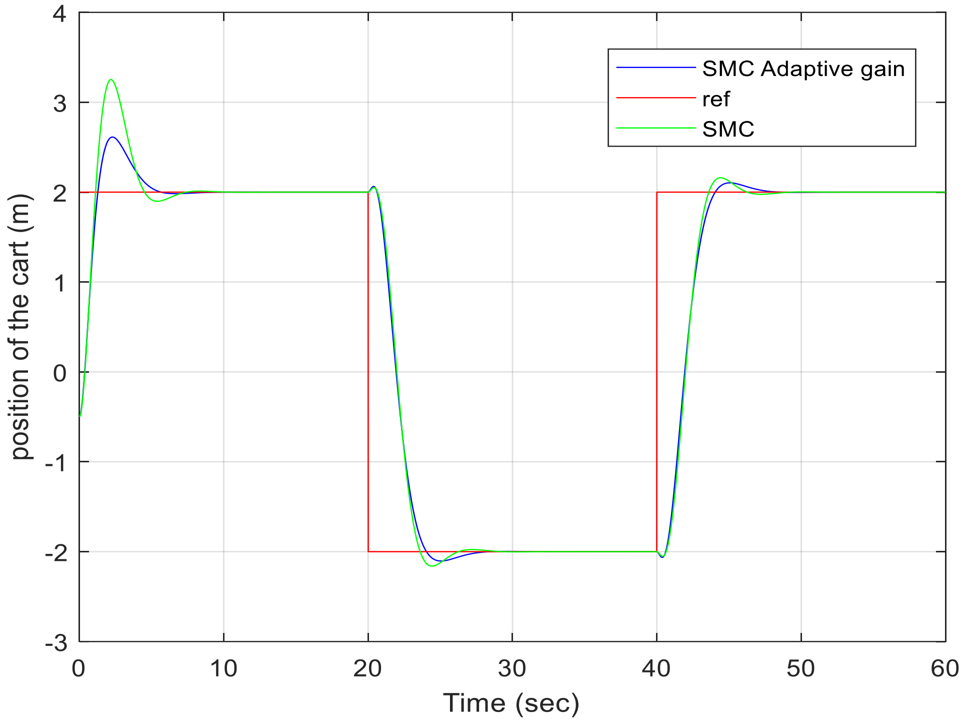

Figure III.21. Position evolution of the cart $x(t)$

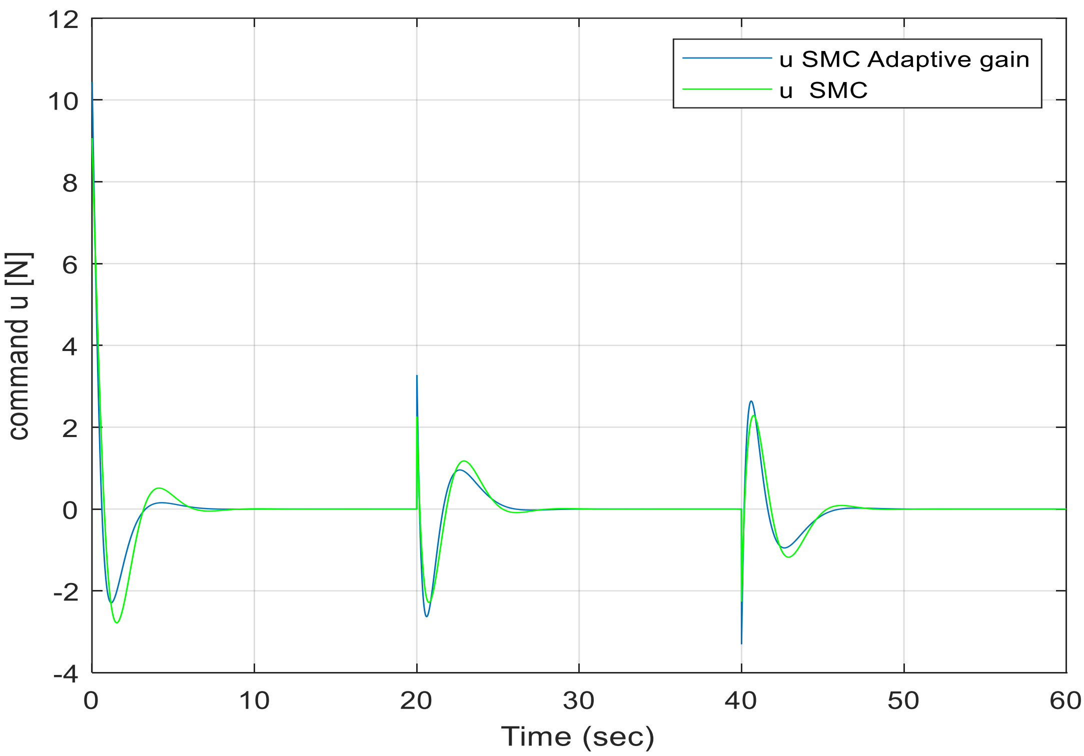

Figure III.22. Command signals by $u(t)$ SMC and $u(t)$ SMC adaptive gain

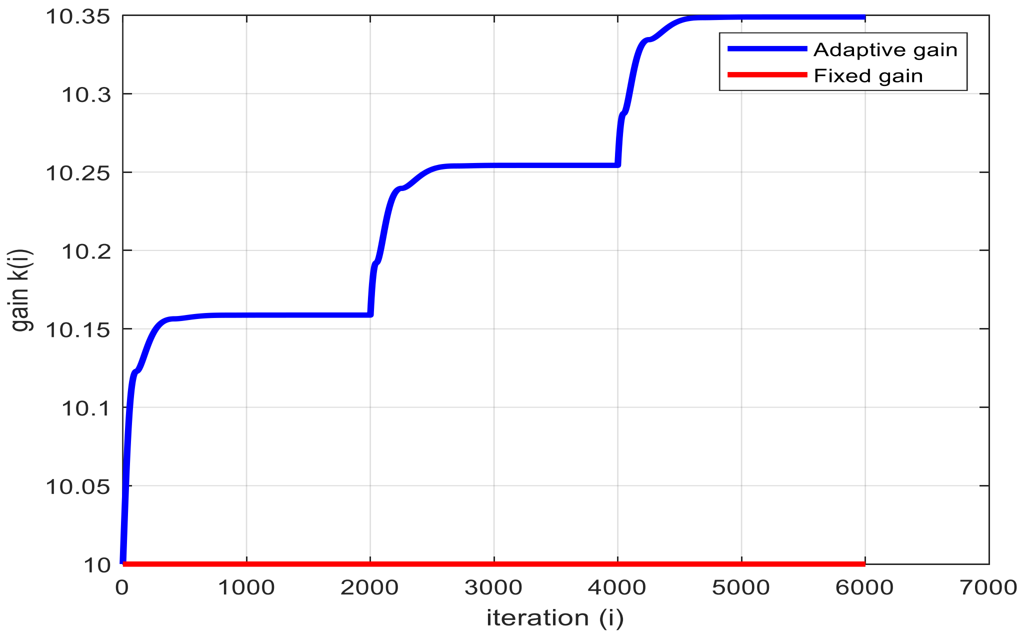

Figure III.23. Evolution of the adaptive gain and fixed gain

The simulation results show the efficiency and performance of the sliding mode control with adaptive gain. It can be seen that this controller has eliminated the chattering and ensured the smoothing of the control, the stabilization of the system, and the tracking of the trajectory.

III.8. Conclusion

In this chapter, we have studied a sliding mode control approach with adaptative gain that can be used from a large class of nonlinear systems.

The application of this control on an example of simulation (the inverted pendulum) gave very satisfactory results for the stabilization and the continuation of trajectory and the robustness compared to the external disturbances while overcoming the chattering problems of sliding mode control.

Conclusion and perspective

In this thesis, a very important property in the study of systems has been presented, which is robustness against external and parametric disturbances. Sliding mode control has significant advantages in the study of linear and nonlinear systems. The nature of sliding mode control is non-linear and its control law changes in a discontinuous manner.

The work presented in this thesis concerns the implementation of the sliding mode technique for the control of single-input and multi-input non-linear systems with adaptive gain.

In the first chapter, we presented some reminders about the general properties of nonlinear systems, and the different methods most commonly used for the control of nonlinear systems.

In the second chapter, we presented the operating principle of sliding mode control for single-input single-output (SISO) systems with adaptive gain, accompanied by an application of it to a non-linear system, the inverted pendulum system.
This command has good performance with a good choice of sliding surface and command parameters.

This control conducted at good performance, and gets a better quality of adjustment compared to the non-linear control, it will be necessary to choose the sliding surface as well as the parameters of the control to be used.

In the third chapter, we apply the command mode sliding with adaptative gain on the inverted pendulum which is a multivariable system (MIMO), using the simulation under MATLAB which gives satisfactory results to the stability, performance and robustness of our system.

In this work we studied a technique that combines the advantages of both techniques. The studied method allows the mitigation of the effects of external disturbances and eliminate the phenomenon of "chattering" introduced by the adaptive sliding mode. The pursuit of the desired trajectory is done in two phases: the approach and the sliding. Thus, the control used in this case consists of two parts: the first allowing the approach to the arrival at the surface, and the second maintaining the sliding along this surface.

At the end of this work, we can say that sliding mode control with adaptative gain offers certain advantages:

1. Robustness with respect to variations in system parameters.

2. A highly efficient dynamic "acceptable response time and practically zero steady-state error".

Finally, as a result of this work, we propose the implementation of the technique presented in this work in order to experimentally verify the results found, in practice. Since the measurement of all states is generally impossible because of the physical constraints and/or the high cost of the sensors. It would then be interesting to develop the command by sliding mode with adaptative gain using observers by return of state or by return of out-put.

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Résumé :

On présente dans ce mémoire, une étude sur la commande des systèmes non linéaires monovariables et multivariables par mode glissant avec gain adaptatif dont l'objectif est la stabilisation où la poursuite d'une trajectoire désirée. Dans la première partie, nous avons étudié la stabilité et la commande des systèmes non linéaires, Dans la deuxième partie, nous avons présenté le principe de fonctionnement du contrôle en mode glissant pour les systèmes monovariables avec gain adaptatif, accompagné d'une application à un système non linéaire (le pendule inversé). L'application de la commande par mode glissant avec gain adaptatif pour les systèmes multivariables sur le pendule inversé est l'objet de la dernière partie, les résultats de simulation ont bien montré la robustesse et la stabilisation de la commande.

Mots clés : commande, mode glissant, système non linéaire, Stabilité du système.

Abstract:

We present in this thesis, a study on the control of nonlinear monovariable and multivariable systems by sliding mode with adaptive gain whose objective is stabilization or the tracking of a desired trajectory. In the first part, we studied the stability and control of non-linear systems, In the second part, we presented the operating principle of sliding mode control for monovariables systems with adaptive gain, accompanied by an application to a non-linear system (the inverted pendulum). The application of sliding mode control with adaptive gain for multivariables systems on the inverted pendulum is the subject of the last part, the simulation results showed the robustness and stabilization of the control.

Keywords: control, sliding mode, nonlinear system, system stability.

ملخص:

نحن نقدم في هذه المذكرة، دراسة حول التحكم في الأنظمة الأحادية والمتعددة المتغيرات غير الخطية عن طريق وضع الانزلاق مع معامل تكيفي هدفه الاستقرار أو السعي لتحقيق المسار المطلوب. في الجزء الأول، درسنا استقرار الأنظمة غير الخطية والتحكم فيها، في الجزء الثاني، قدمنا مبدأ التشغيل للتحكم في الوضع المنزلق لأنظمة أحادية متغيرة ذات معامل تكيفي، مصحوبة بتطبيق على نظام غير خطي (البندول المقلوب). تطبيق التحكم في الوضع المنزلق مع معامل تكيفي للأنظمة متعددة المتغيرات على البندول المقلوب هو موضوع الجزء الأخير، أظهرت نتائج المحاكاة متانة واستقرار السيطرة.

الكلمات المفتاحية: التحكم، الوضع المنزلق، النظام غير الخطي، استقرار النظام.