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Topic

Study of Elastic and Inelastic Buckling Behaviour of Plates Made with High-Strength Steel Under Shearing Stresses with Different Support Conditions

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بسم الله الرحمن الرحيم

شكر وعرفان

سبحانك اللهم لا علم لنا إلا ما علمتنا، نشكر الله ونحمده فضل نعمه علينا، نعمة العقل التي أنار بها دربنا وفكرنا ونعمة الذاكرة التي حفظنا بها سرنا وجهرنا.

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إن من تمام شكر الله، شكر أهل الفضل والبر، وعملا بقول نبيه محمد صلى الله عليه وسلم: « من لم يشكر القليل لم يشكر الكثير ومن لم يشكر الناس لم يشكر الله » رواه " أحمد و الترمذي " .

و نخص بالشكر الجزيل لأساتذة الخير الذين علموا بلا شك أن العلم من أجمل العبادات وأفضلها،

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ولكم منا فائق التقدير و الاحترام.

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τcr: critical shearing stresses

σ :Normal (perpendicular) stress

τ:Shear (parallel) stress

t: Thickness of plate

a: length of plate

H: width of plate

$k\tau$:Elastic or theoretical equivalent stress concentration factor

E: Modulus of elasticity

ε: Strain

ν: Poisson's ratio

P: the dead load

Pref: reference load

ζ: Load Proportionality Factor

b_{eff} : the effective width of the plate

σc: the critical stress of a flat plate under uniform compressive stress

ρ: the reduction factor

r: the radius of gyration

Ac: the gross cross-sectional area

$[K_{\sigma}]_{ref}$: the stress stiffness matrix

[K]: the stiffness matrix

Abstract

The rising interest in the use of high strength steel (HSS) materials is justified by several advantages that they provide since their applications make the design with slender steel possible. In the civil engineering field, high-rise buildings and large span bridges can be designed with smaller foundations and less structural weight. Few researches work on buckling under pure shear are available in literature specially when it comes to the inelastic buckling behaviour. This dissertation investigates the analytical and the numerical simulation to predict the shear strength

of the plates with different steel grades. The elastic local buckling stress τ_{cr} can be estimated using the plate stability theory. Critical load is usually calculated in eigenvalue linear buckling analysis and to find out the loads for which the model stiffness matrix becomes singular. The boundary conditions and the aspect ratio affect the local shear buckling coefficient. There are two steps of analysis carried out in this work. For the sake of comparison, the first step consisted of the linear elastic shear buckling analysis with three means: analytical procedure, EBPlATE software and ABAQUS package. Broadly speaking, results of the linear elastic buckling were as expected. The parametric study has shown once again the importance of the grade of the steel in the evaluation of the critical stress. In fact, as the steel grades grows, the smaller stress is given, showing that the mild steel has a better resistance against the elastic shear buckling than HSS cases with figures as twice as much for both the case of square or even rectangular plates simply supported. The aspect ratio plays an important role in elastic buckling under pure shear as it modifies the value of τ_{cr} by decreasing them as the ratio become larger. Changing the support conditions from simply supported to clamped one's has shown its importance namely by increasing the value of τ_{cr} for the particular case of square plate roughly twice as much for each studied case. The square plate shows better behaviour compared to the rectangular ones. And then, the numerical model by ABAQUS has been successfully validated for nonlinear buckling analysis. The second step consisted of the 2^{nd} order non-linear analysis with the initial geometric imperfection obtained from the final deformed shape from the first step. Furthermore, nonlinear analysis was possible by a step realistic behaviour of the structure, which is programmed in ABAQUS/Standard. Incremental procedure based on RIKS algorithm is used to solve system of nonlinear equations is considered when a post-buckling behaviour is of interest this was possible by a step-by-step loading process. Also, the results show that the steel grades of the plate are of prime importance as it influences the elastic shear buckling of all studied plates by increasing the elastic buckling stress. The decrease of V_{cr} depends on the size of the geometry of the plates as well as the steel grade and boundary condition. The non-linear finite element method with initial geometric imperfection is compulsory to capture the shear buckling behaviour of the steel thin plates. Then, a comprehensive trial and error technique for the amplitude value of the initial geometric imperfection under shear was carried out to obtain the correct buckling load. The numerical simulation observed that the scaled magnitude of the imperfection required to match the buckling load was affected by the mesh aspect ratio, which eventually is affected by the mesh size. Finally, it can be asserted that the initial objectives

of this study, which were to focus on the investigation of the elastic and inelastic behaviour of steel plates were generally achieved. Modelling the inelastic shear buckling has not been very successful because of the geometric imperfection for simply supported plate as it does not show the post-buckling behaviour which needs more effort to find out the true value of the geometric imperfection. While for clamped plate a more realistic model has been built-up giving much more valuables results.

Keywords: elastic and inelastic buckling; Eigen analysis, FEA modelling; shear, P-Δ; imperfection; Von-Mises contours;

Résumé

L'intérêt croissant pour l'utilisation de l'acier à haute résistance (HSS) est justifié par plusieurs avantages qu'ils offrent puisque leurs applications rendent possible la conception avec de l'acier HSS. En effet, dans le domaine du génie civil, les immeubles de grande hauteur et les ponts à grande portée peuvent être conçus avec des fondations plus petites et un poids structurel moindre. Peu de travaux de recherche sur le voilement sous cisaillement pur sont disponibles dans la littérature notamment en ce qui concerne le comportement au voilement inélastique. Ce mémoire

étudie la simulation analytique et numérique pour prédire la résistance au cisaillement des plaques avec différentes qualités d'acier. La contrainte de voilement local élastique τ_{cr} peut être estimée à l'aide de la théorie de la stabilité des plaques. La charge critique est généralement calculée dans l'analyse de voilement linéaire aux valeurs propres et pour connaître les charges pour lesquelles la matrice de rigidité du modèle devient singulière. Les conditions aux limites et le rapport d'aspect affectent le coefficient de déversement par cisaillement local. Deux étapes d'analyse sont réalisées dans ce travail. Afin de comparer les résultats, la première étape a consisté en l'analyse du voilement par cisaillement élastique linéaire avec trois moyens : procédure analytique, logiciel EBPLATE et progiciel ABAQUS. D'une manière générale, les résultats du voilement élastique linéaire étaient comme prévu. L'étude paramétrique a montré une fois de plus l'importance de la nuance de l'acier dans l'évaluation de la contrainte critique. En fait, à mesure que les nuances d'acier augmentent, la contrainte est plus faible, ce qui montre que l'acier doux a une meilleure résistance au voilement élastique que les cas HSS avec des chiffres jusqu'à deux fois plus élevés pour le cas de plaques carrées ou même rectangulaires simplement supportées. Le rapport d'aspect joue un rôle important dans le voilement élastique sous cisaillement pur car il modifie la valeur de τcr en les diminuant à mesure que le rapport augmente. Changer les conditions de support de simplement supporté à l'encastrement a montré son importance notamment en augmentant la valeur de τcr pour le cas particulier de la plaque carrée environ deux fois plus pour chaque cas étudié. La plaque carrée présente un meilleur comportement que les plaques rectangulaires. Ensuite, le modèle numérique d'ABAQUS a été validé avec succès pour l'analyse de voilement non linéaire. De plus, une analyse non linéaire a été possible grâce à un comportement réaliste de la structure, programmé dans ABAQUS/Standard. Une procédure incrémentale basée sur l'algorithme RIKS est utilisée pour résoudre un système d'équations non linéaires. Elle est prise en compte lorsqu'un comportement post-voilement est intéressant, ce qui a été possible grâce à un processus de chargement étape par étape. En outre, les résultats montrent que les nuances d'acier de la plaque sont d'une importance primordiale car elles influencent le voilement élastique par cisaillement de toutes les plaques étudiées en augmentant la contrainte de voilement élastique. La diminution de Vcr dépend de la taille de la géométrie des plaques ainsi que de la nuance d'acier et des conditions aux limites. La charge de voilement et le comportement post-flambage peuvent être étudiés à l'aide de la FEA non linéaire du 2ème ordre avec l'imperfection géométrique initiale obtenue à l'étape précédente. Lors du processus d'assemblage, plusieurs causes peuvent déclencher

la formation de l’imperfection géométrique initiale. Ensuite, une technique complète d’essais et d’erreurs pour la valeur d’amplitude de l’imperfection géométrique initiale sous cisaillement a été réalisée afin d’obtenir la charge de voilement correcte. La simulation numérique a observé que l'ampleur de l'imperfection requise pour correspondre à la charge de voilement était affectée par le rapport d'aspect du maillage, qui est finalement affecté par la taille du maillage. La modélisation du voilement par cisaillement inélastique n'a pas été très réussie en raison de l'imperfection géométrique pour les plaques simplement appuyées, car elle ne montre pas le comportement postcritique qui nécessite plus d'efforts pour déterminer la vraie valeur de l'imperfection géométrique. Alors que pour les plaques encastrées, un modèle plus réaliste a été construit, donnant des résultats beaucoup plus précieux.

Mots clés : voilement élastique et inélastique ; Modélisation FEA ; cisaillement, P-Δ ; imperfection ; contours de Von-Mises ;

ملخص :

الاهتمام المتزايد باستخدام مواد الصلب عالي القوة (HSS) مبرر بالعديد من المزايا التي توفرها، حيث تتيح تصاميم رشيقة وهياكل أقل وزنًا. في مجال الهندسة المدنية، يمكن تصميم المباني الشاهقة والجسور ذات الفترات الطويلة بأساسات أصغر ووزن هيكلي أقل. هناك أبحاث قليلة تناولت الانبعاج تحت القص النقي في الأدبيات، خاصة عندما يتعلق الأمر بالسلوك اللاديناميكي للانبعاج. تتأثر قوة القص للألواح الصلبة عالية القوة بأنماط الانبعاج. تهدف هذه الرسالة إلى التحقيق في التحليل والمحاكاة العددية للتنبؤ بقوة القص للألواح بمختلف درجات الصلب. يمكن تقدير إجهاد الانبعاج المحلي المرن τcr باستخدام

نظرية استقرار اللوحة. عادةً ما يتم حساب الحمل الحرج في تحليل الانبعاج الخطي القيمي الخاص بإيجين للحصول على الأحمال التي تجعل مصفوفة صلابة النموذج تصبح مفردة. تؤثر شروط الحدود ونسبة الأبعاد على معامل الانبعاج القص المحلي. هناك مرحلتان من التحليل أجريت في هذا العمل. من أجل مقارنة النتائج، تمثلت الخطوة الأولى في تحليل انبعاج القص المرن الخطي بثلاث وسائل: الإجراء التحليلي، برنامج EBPlATE وحزمة ABAQUS. بشكل عام، كانت نتائج الانبعاج المرن الخطي كما هو متوقع. أظهرت الدراسة البارامترية مرة أخرى أهمية درجة الصلب في تقييم الإجهاد الحرج. في الواقع، كلما زادت درجة الصلب، قل الإجهاد الممنوح، مما يُظهر أن الفولاذ الطري يتمتع بمقاومة أفضل ضد الانبعاج المرن بالقص مقارنة بحالات الصلب عالي القوة مع أرقام تصل إلى ضعف الحالات لكل من اللوحات المربعة أو حتى المستطيلة المدعومة ببساطة. تلعب نسبة الأبعاد دورًا هامًا في الانبعاج المرن تحت القص النقي حيث تعدل قيمة τcr بتقليلها كلما زادت النسبة. تغيير شروط الدعم من الدعم البسيط إلى التثبيت أظهر أهميته من خلال زيادة قيمة τcr في حالة اللوحة المربعة بنحو ضعف لكل حالة مدروسة. اللوحة المربعة تظهر سلوكًا أفضل مقارنة بالمستطيلة. ومن ثم، تم التحقق من صحة النموذج العددي بواسطة ABAQUS لتحليل الانبعاج غير الخطي. تضمنت الخطوة الثانية تحليلًا غير خطي من الدرجة الثانية مع وجود العيب الهندسي الأولي الذي تم الحصول عليه من الشكل المشوه النهائي من الخطوة الأولى. بالإضافة إلى ذلك، كان التحليل غير الخطي ممكنًا من خلال سلوك واقعي للهيكل، والذي تمت برمجته في ABAQUS/Standard. تم استخدام إجراء تزايدي يعتمد على خوارزمية RIKS لحل نظام المعادلات غير الخطية عند الاهتمام بسلوك ما بعد الانبعاج وهذا كان ممكنًا من خلال عملية تحميل تدريجية. كما أظهرت النتائج أن درجات الصلب للألواح ذات أهمية قصوى حيث تؤثر على الانبعاج المرن بالقص لجميع الألواح المدروسة بزيادة إجهاد الانبعاج المرن. يعتمد انخفاض Vcr على حجم هندسة الألواح وكذلك درجة الصلب وشروط الحدود. يمكن التحقيق في حمل الانبعاج وسلوك ما بعد الانبعاج باستخدام تحليل العناصر المحدودة غير الخطية من الدرجة الثانية مع وجود العيب الهندسي الأولي الذي تم الحصول عليه من الخطوة السابقة. طريقة العناصر المحدودة غير الخطية مع العيب الهندسي الأولي ضرورية لالتقاط سلوك الانبعاج بالقص للألواح الفولاذية الرقيقة. أثناء عملية التجميع، يمكن أن تؤدي عدة أسباب إلى تشكيل العيب الهندسي الأولي. ثم، تم إجراء تقنية شاملة للتجربة والخطأ لقيمة سعة العيب الهندسي الأولي تحت القص للحصول على قيمة حمل الانبعاج الصحيحة. لاحظت المحاكاة العددية أن مقدار العيب المطلوب لمطابقة حمل الانبعاج تأثر بنسبة أبعاد الشبكة، والتي تتأثر في النهاية بحجم الشبكة. أخيرًا، يمكن التأكيد على أن الأهداف الأولية لهذه الدراسة، التي كانت تركز على التحقيق في السلوك المرن واللاديناميكي للألواح الفولاذية، قد تحققت بشكل عام. لم يكن نمذجة الانبعاج بالقص اللاديناميكي ناجحًا جدًا بسبب العيب الهندسي للوحة المدعومة ببساطة حيث لم يظهر سلوك ما بعد الانبعاج مما يتطلب المزيد من الجهد لمعرفة القيمة الحقيقية للعيب الهندسي. بينما تم بناء نموذج أكثر واقعية للوحة المثبتة، مما يعطي نتائج أكثر قيمة.

CHAPTER ONE

GENERAL INTRODUCTION

The work presented in this dissertation is my final assignment towards the fulfilments of degree Academic Master of Science in Structural Engineering degree. The work has been initiated and carried out at the Department of Civil Engineering at the University of Tébessa (Algeria) during the period of February 2024 to June 2024. It was supervised by Dr. Abderrahim. Labed. The advanced subject of this dissertation is confined to the study of the elastic and inelastic shear buckling behaviour of thin plates made from high strength steel (HSS) and that trigger initial geometrical nonlinearities only. As few researches work on shear buckling are available, several difficulties have risen during the achievement of this work especially the 3D modelling of boundary conditions supports for clamped plates and mainly the built-up of the inelastic FEA models which has been principally very hard task and time consuming.

1.1. General :

Steel is a mixture of iron and carbon, with trace amounts of other elements, including principally manganese, phosphorus, sulphuric, and silicon. Structural steel is an ideal material for the design of resistant structures. It is strong, light weight, ductile, and tough, capable of dissipating extensive energy through yielding when stressed into the inelastic range. In addition, as with all structural materials, it is very important to assure that the structures are actually constructed as designed, that quality is maintained in fabrication and field welding operations, and that the structure is maintained over its life.

The rising interest in the use of high strength steel (HSS) materials is justified by several advantages that they provide. In the civil engineering field, high-rise buildings and large span bridges can be designed with smaller foundations and less structural weight, since the application of HSS makes the design with slender steel sections possible.

Stability of a structure can be analysed by computing its critical load, i.e., the load corresponding to the situation in which a perturbation of the deformation state does not disturb the equilibrium between the external and internal forces.

Critical load is usually calculated in Eigenvalue linear buckling analysis. The objective of the eigenvalue buckling problem is to find out the loads for which the model stiffness matrix becomes singular. Also, this analysis is generally used to estimate the critical buckling loads of ideal structures. Their response usually involves very little deformation prior to buckling

1.2 High-Strength Steel :

Steel is one of the most useful materials in the world. In the simplest context, steel is composed of iron and carbon, but in actuality, steel isn't so simple.

High-strength steels are becoming increasingly popular in a wide range of industries. And for a good reason – they offer several advantages over traditional steel.

For one, their higher strength-to-weight ratio allows for lighter, more energy-efficient products.

They're also highly resistant to corrosion and wear, making them ideal for applications where durability is vital. In addition, high-strength steels can be customized to meet the specific needs of each project, whether it's increased strength, ductility, or weldability. As a result, they offer designers and engineers greater flexibility in creating innovative products that meet the challenges of the 21st century.

High strength steel (HSS) with the high nominal yield strength $f_y \geq 460$ MPa has been applied in numerous modern building and bridge structures all over the world. Steel structures using high strength steel have obvious advantages in structural, architectural, economical, environment protection and energy saving aspects. The advantages that may be derived from the use of high strength steels are strongly dependent on the project context and the type and function of the structural component considered.

1.3 Buckling in steel components:

Most steel constructions such as cranes, ships, bridges and building frames experience compressive and/or shear loads and are thus susceptible to buckling. Buckling of construction members is usually a very sudden process without much warning in advance. Therefore, it is important to account for the effect of buckling in the design of a construction members.

Slender and thin-walled structures constitute main structural components in various engineering areas since they feature a high strength-to-weight and stiffness-to-weight ratio. These structures are prone to function at high displacement levels when subjected to operational loads, while staying in the material linear elastic range.

The steel material is characterized by a symmetrical mono- axial stress-strain (σ–ε) constitutive law, which can be determined by monotonic tensile tests on samples taken from the base material before the working process or from the products in correspondence of appropriate locations.

The response of steel members can, however, be significantly different in tension or compression, owing to the relevant influence of the buckling phenomena. The resistance can significantly be limited by instability phenomenon in the range of standard products.

Buckling is caused by in-plane stresses (compression and/or shear) exceeding the buckling capacity of the structure, causing local yield and permanent deformation of the structure.

The buckling capacity is a property of the plate, depending on many factors. Two cases of instability may occur, that is overall or global column instability and plate element local instability. Local buckling of plate elements in a steel column can cause premature failure of the entire section and reduce the overall strength. The mode of failure and load carrying capacity of steel columns are affected by the behaviour of plate elements and the interaction between local and overall buckling.

Buckling can be divided upon three states:

1. Elastic buckling: occurs only in the elastic regime of the stress -strain material behaviour.
2. Elastic - plastic buckling: occurs when a local region inside the plate experiences a plastic deformation.
3. Plastic buckling: happens after the plate has yielded over large regions.

Many factors affect the buckling behaviour of panels, including geometric or material properties, loading characteristics, boundary conditions, initial geometrical imperfections, and material nonlinearity.

1.4. Buckling of steel thin plates :

The behaviour of a plate under applied loads may be classified into five regimes: pre-buckling, buckling, post-buckling, ultimate strength, and post-ultimate strength. In the pre-buckling regime, the structural response between loads and displacements is usually linear, and the structural component is stable. As the predominantly compressive stress reaches a critical value, buckling occurs. Unlike columns, in which buckling is meant to cause collapse, plates that buckle in the elastic regime may have sufficient redundancy to remain stable in the sense that further loading can be sustained until the ultimate strength is reached, even if the in-plane stiff-ness significantly decreases after the inception of buckling. In this regard, the elastic buckling of a plate between stiffeners may in some design cases be allowed to reduce the structural weight associated with efficient and economic design.

An eigenvalue buckling analysis calculates the linear buckling load factors i.e. the load factors that if applied on the loading the structure will buckle with a specific deformed shape (eigenmode). It should be noted that as eigenvalue buckling analysis is a linear analysis, thus it doesn't give any information on the post-buckling behaviour (if any) of the structure. It won't give any information on whether the structure has a post-buckling resistance, if this is stable or unstable etc, although in certain cases it could calculate with a good approximation the actual buckling load factor of the structure.

From the detailed structural point of view, it is more accurate to use nonlinear analysis which includes post buckling behaviour with elasto-plastic behaviour of material. However, the unavoidable imperfections of the structures may influence their stability behaviour considerably, with respect to the value of the critical load, and even in terms of the characteristics of the deformation.

In order to find out about these influences, a parametric linear and nonlinear study was carried out highlighting the influence of some key parameters: steel grades; aspect ratios and the support conditions of a thin plate made with HSSS of an eigenvalue linear buckling analysis, nonlinear buckling analysis using the modified RIKS method with elasto-plastic behaviour of material.

1.5 Failure modes of steel members :

At basic level, predicting material failure may be accomplished using linear finite element analysis. The strains and corresponding stresses obtained from this analysis may be compared to design stress (or strain) allowable anywhere within the structure. If the finite element solution indicates regions where these allowable are exceeded, it is assumed that material failure has occurred. Design allowable are based on experimentally-derived material strengths and usually include a safety margin. This type of analysis will give an adequate prediction for statically determinate metallic structures undergoing small deformations.

The technical term 'collapse' is generally used to refer to the sudden loss of structural integrity and does not distinguish between the two major categories leading to structural failure: (a) material failure or (b) structural instability due to loss of structural stiffness within the elastic limit of the material. While the technical term 'buckling' is often reserved specifically for the latter category.

According to [Chen and Duan 2014] there are four fundamental failure modes for steel members; yielding, rupture, buckling, and fatigue. Buckling failures can be characterized by an instability of a member as a whole (global buckling) or as an instability of one or more of the elements of a cross section (local buckling). In this context, the word "element" is meant to describe a plate component that makes up part of a cross section.

1.6 Organisation of the project :

- **Motivation and aim of the present work**

Exploring the impact of different buckling phenomenon on the elastic and inelastic behaviour of slender st2eel thin plates under shear static loadings is quite interesting. The consideration of several parameters that are believed to greatly influence the elastic and inelastic buckling of thin plates made with different grades of steel. These parameters are: variation of the plate slender ratio from 1.0 to 1.2 and 1.5, the support conditions: simply supported to clamped plates. The numerical modelling of the beams has been developed through well-known software: **EBPLATE** for elastic buckling analysis and **ABAQUS** for both elastic and inelastic buckling behaviours of slender steel beams. By performing this task, the author of this dissertation has learnt how to deal with complex analyses in 3D models.

- **Methodology**

To be able to meet the aim of the thesis the following steps were performed with tasks that need to be executed:

- Understanding the different instability phenomenon susceptible to occur in plates under shear static loadings.
- Understanding the theoretical background of the buckling analyses both classical way or the FEA method.
- Hand-calculations of the critical shear stress using the classical approach.
- Explore the methods and assumptions used in EBPLATE and ABAQUS software
- Understand the elastic and inelastic instability behaviour of steel thin plates subjected to buckling.

- Built-up 3D models of thin plates for linear and nonlinear buckling analyses in ABAQUS software which was particularly tough task..
- Extract and discuss the obtain results.
- Compare the performance of different beams plates with different steel grades.
- Draw some conclusions and suggestions for future wok.

- **Organisation of the dissertation**

The present thesis has been partitioned in seven chapters as follows:

- **Chapter 1**: A general introduction on steel thin plates with HSS steel summarises some concepts used in this work along with important definitions. Also, a description of the scope and objective of the present research work and the way to attain the planned objectives is given.
- **Chapter 2**: A concisely presentation of the high-strength steel (HSS) particularities in the civil and other fields of applications. Emphasis has been made for their higher strength-to-weight ratio allows for lighter, more energy-efficient products.
- **Chapter 3**: The classical theory of plates or shells, which formulates and solves problems from the point of view of rigorous mathematical analysis, is an important application of the theory of elasticity. A concisely presentation of the theory of thin plates is given. Introduces a theoretical insight of the theoretical classic formulation of the behaviour of elastic thin plates.
- **Chapter 4**: A Historical review of the developments in buckling and Post- buckling analysis of plates is given. A Historical review of the developments in buckling and Post buckling analysis of plates. Then the objective of the buckling analysis is explained. The theoretical development of the buckling and post-buckling is summarised.
- **Chapter 5**: This chapter is devoted the FEA approach to the buckling problem. It gives the essential features of FEA for the buckling analysis. Displays an overview of the use in this work the finite element method in structural analysis.
- **Chapter 6**: Firstly, the results of will be presented and fully discussed of .linear elastic buckling analysis of a parametric study in terms of some key parameters believed to have an influence on the buckling of a thin plate, namely the steel grades, the ratio aspect and the support conditions It is worth to recall that three means have been used: Classical analytical approach, EBPLATE software and ABAQUS package. EBPLATE and ABAQUS. Demonstrations were provided for the use of these two packages. Secondly, the presentation of the outcomes of 3D

ABAQUS of inelastic shear buckling of plates are given and discussed with some general conclusions.

- **Conclusions and recommendations for future works:** provides the essential the conclusions coming up from this research work on some of the key parameters governing the elastic and inelastic behaviours of thin plates, and followed by some recommendations for future works.

CHAPTER TWO

High- Strength Steel

(HSS)

2.2. Structural steel :

2.2.1 General :

Steel is one of the most useful materials in the world. In the simplest context, steel is composed of iron and carbon, but in actuality, steel isn't so simple. The concentration of carbon and iron or the addition of other elements affects steel's properties and strength, which aids in steel's usefulness for an endless variety of end uses.

Structural steel is one of the most popular and useful metals known to mankind and has been widely used in various structures such as buildings, bridges, industrial complexes, etc. Steel is also used in other applications such as vehicles and aircraft. The wide variety of applications of steel is because it possesses more desirable material properties in comparison with other construction materials, such as concrete and timber.[1]

The use of steel for structural applications goes back to the second part of the 19th century when steel became one of the cornerstones of the world's economy as steel framing quickly replaced iron in buildings. The use of steel revolutionized the building industry, allowing construction to reach new heights – the 10-storey Home Insurance Building built in 1895 in Chicago, USA, was the first tall building to be supported be a steel skeleton of vertical columns and horizontal beams. It rose to a height of just over 42 metres. [2]

Classification of steel materials is necessary to determine whether these materials should be allowed for structural use in the construction industry with or without any restrictions. The adequacy and reliability of steel materials should be verified against the material performance requirements as well as the quality assurance requirements, respectively, in the entire process of classification.

Knowledge of the material properties such as its yield strength is very important when designing components, since it usually represents the upper limit of the load that can be applied. Additionally, the yield strength is very important for controlling many materials' production techniques, such as forging, rolling or pressing. [3]

2.2.2 Historical context :

Steel construction combines a number of unique features that make it an ideal solution for many applications in the construction industry. Steel provides unbeatable speed of construction and off-site fabrication, thereby reducing the financial risks associated with site-dependent delays. The inherent properties of steel allow much greater freedom at the conceptual design phase, thereby helping to achieve greater flexibility and quality. In particular, steel construction, with its high strength to weight ratio, maximizes the useable area of a structure and minimizes self-weight, again resulting in cost savings.

The use of steel for structural applications goes back to the second part of the 19th century when steel became one of the cornerstones of the world's economy as steel framing quickly replaced iron in buildings. The use of steel revolutionized the building industry, allowing construction to reach new heights – the 10-storeys of Home Insurance Building built in 1895 in Chicago, USA, was the first tall building to be supported using a steel skeleton of vertical columns and horizontal beams. It rose to a height of just over 42 metres.

Following the initial discovery and refinement of general steel alloys by Robert Hadfield in 1882 the industry continued to find ways to improve key characteristics and properties. Development of steel accelerated as a result of World War II where the requirement for higher strength and higher hardness steels came as a result of the ever-increasing requirements of tanks and other armored vehicles. Through research, and trial and error, it was found that heating and cooling certain alloys of steel resulted in significant and beneficial changes in key mechanical properties including hardness, toughness, tensile strength and weldability.

2.2.3 Structural steel frames :

Using the steel as construction material, the engineer will calculate the loadings, stresses and reactions, then select a standard steel member whose section properties meet the design requirements. Additionally, structural steel frames are available as standard: Hot-rolled section or as Cold-rolled section [4]

- **Hot-rolled steels**

A rolling process at temperatures over 1000 degrees Fahrenheit is use to create hot-rolled steel. Steel products processed in this manner will have a blue-gray finish that feels rough to the touch Hot-rolled steel reconfigures itself during the cooling process, giving the finished product looser tolerances than the original material. This makes hot-rolled steel a good choice for the manufacturing of structural components

Hot-rolled steel sections, also known as Standard Structural Steel Sections, are standardised in shape and dimensions e.g. Universal Columns. Hot-rolled steel is more malleable, allowing it to be forced into a variety of different shapes. These sections are produced direct from material to form in intense heat condition and later cooling. In the process the material properties/'grains' are changed thus giving it its strength.

- **Cold-rolled steels**

Cold-formed steel sections are pressed or rolled to shape at close to normal room temperature from thin steel strip and are known respectively as pressed-steel or cold rolled steel sections. This increases the strength of the finished product through the use of strain hardening by as much as 20%. The cold-rolled process creates a finished product that is more precise dimensionally than a hot-rolled product. Finished products created by the cold-rolled steel process include bars, strips, rods and sheets which are usually smaller than the same products available through hot rolled methods.

N.B. Generally, Hot-rolled sections are stronger than cold formed thus suggesting their use in the industry. Hot rolled for structural used. [4]

2.2. High strength steel:

2.2.1 General:

High-strength steels are becoming increasingly popular in a wide range of industries. And for a good reason – they offer several advantages over traditional steel. For one, their higher strength-to-weight ratio allows for lighter, more energy-efficient products. They're also highly resistant to corrosion and wear, making them ideal for applications where durability is vital. In addition, high-strength steels can be customized to meet the specific needs of each project, whether it's increased strength, ductility, or weldability. As a result, they offer designers and engineers greater flexibility in creating innovative products that meet the challenges of the 21st century.

Contemporary high strength Q&T steels are all generally considered low to medium alloy content steels, exhibiting high strength and hardness coupled with good formability and weldability. They also maintain exceptionally low temperature notch toughness and crack propagation resistance despite the high strength levels involved. This unique combination of properties is the result of careful selection of both chemical composition and heat treatment.

Comparatively higher alloy contents are required in thicker plates to offset the retardation in cooling rate which inevitably accompanies quenching of the heavier plate sections. Comparatively higher alloy contents are required in thicker plates to offset the retardation in cooling rate which inevitably accompanies quenching of the heavier plate sections.

High-tensile steel plates are utilized in numerous high-stress applications due to their superior strength and durability. They are used extensively in buildings, bridges, automobiles, manufacturing heavy equipment, pipelines for transporting liquids and gas, and ships. High-tensile steel plates have been used for decades as a cost-effective solution to increase the lifespan of these structures by improving their resistance to breakage or corrosion. Additionally, they provide superior shock absorption ability with great elasticity and uniform thickness throughout the plate, making them an ideal choice for vibration-damping applications such as car bumpers.

2.3 Advantages of High-Strength Steels HSS:

High strength steel (HSS) with the high nominal yield strength $f_y \geq 460$ MPa has been applied in numerous modern building and bridge structures all over the world. Steel structures using high strength steel have obvious advantages in structural, architectural, economical, environment protection and energy saving aspects. The advantages that may be derived from the use of high strength steels are strongly dependent on the project context and the type and function of the structural component considered.

Potential advantages include:

• Design stresses can be increased by taking advantage of high yield stress and tensile strength. This may result in a reduction of required plate thickness which can also save in terms of dead weight.

• If plate thickness reductions are possible, volumes of deposited weld material, and therefore weld consumables and weld times, can be significantly reduced.

• Simplified structural components and construction techniques are possible, particularly in the case of larger structures or heavily loaded sections. Therefore, not only is it possible to save on materials, but also on fabrication, transportation, handling and construction.

• Savings can be made in foundation costs and space requirements due to the reduced dead weight of a structure and the reduced physical size of its elements. [2]

2.4 Mechanical properties of different HSS types

There are a variety of different high-strength steel types, each with its unique mechanical properties. For example, some steels are particularly resistant to abrasion, while others are more resistant to impact. [3]

● Abrasion Resistance

This property is found in steels that have been designed for use in applications where there is a lot of wear and tear. Structural steels experience a lot of abrasion in their lifetime, so it is vital to choose a steel that can withstand this type of wear.

• Impact Resistance

Steel structures experience a lot of impact during their lifetime. This is especially true for bridges and buildings constantly bombarded by high winds and heavy rain. Choosing steel that can withstand these types of impacts is essential.

● Hardness

Hardness is a measure of a steel's resistance to deformation. The harder the steel, the more resistant it is to changes in shape. This is important for applications where the steel will be subject to a lot of stress, such as in bridges and buildings.

● Ductility:

High-strength steel is required to have moderate ductility so it can withstand the stresses of construction and still retain its shape. High ductility can cause the steel to become brittle, so it is important to strike a balance between hardness and ductility.

2.4 Basic Properties of HSS and Design Codes:

The true stress-strain curves for all steel grades look similar and only differ by a parallel shift controlled by the yield strength fy, Figure 2.2.

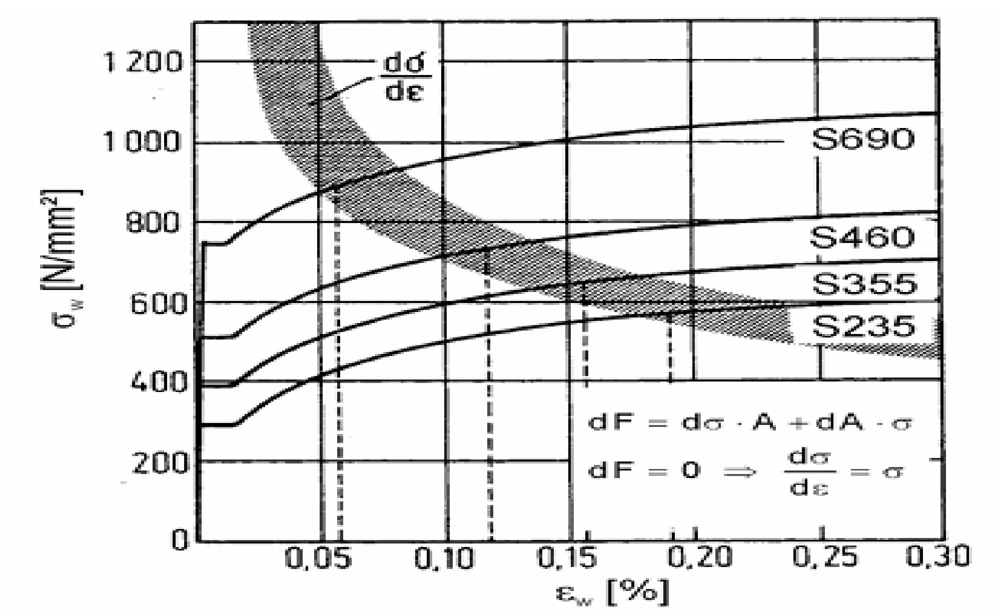


Figure2. 1 True stress strain lines for different steel grades

The ultimate tensile strengths f_u obtained from tensile coupon tests, Figure 2.2 result from the stability criterion

$$\delta A \, . \, \sigma_w - \delta\sigma . A = 0 \qquad (1\text{-}1)$$

Ffor the load deflection curve. Therefore, the true strain eu associated with the tensile strength fu is the smaller the higher the strength fu is, Figure 2.3.

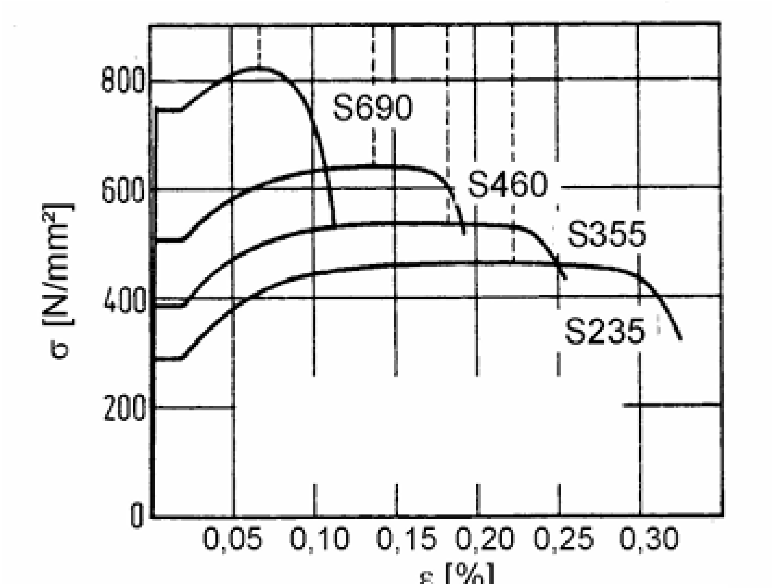


Figure2. 2 Load-deflection lines for different steel grades

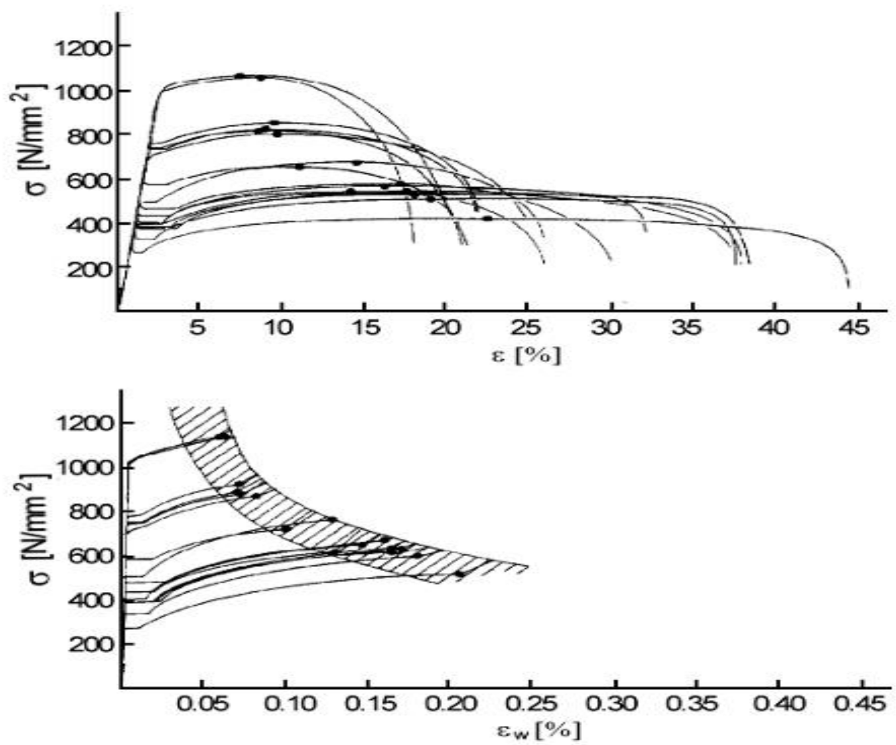


Figure2. 3 Stress strain lines and true stress strain lines for different steel grades.

This is the reason why high strength steels are considered to be more sensitive than ordinary steels, when high local ductility demands from the structural detailing have to be fulfilled from the material. A certain minimum ductility of material is assumed to be available in the design rules for ultimate limit state verifications for steel grades up to S460, where, Figure 2.4. That is :

– secondary moments at connections and joints of welded trusses or lattice girders that result from restraints to deformations are disregarded and hinged connections are assumed instead as models for the calculations,

– residual stresses from the production or welding are neglected,

– the notch effects on the strain field from geometric notches, welds, holes, etc. are disregarded and nominal stresses (linear mean stress distributions) are used,

– simplified assumptions are made for plastic distributions of stresses in cross-sections (stress bloc distributions) or forces in connections (equal distributions of forces to bolts in bolt groups with hole clearances or constant shear distributions in fillet welds),

– full unlimited plastic rotation capacity is assumed in “plastic hinges” to achieve economical moment redistributions in frames by plastic hinge actions.

For steel grades S235 to S460 it is assumed that because of yielding resulting from a sufficiently large yielding plateau and strain hardening on the stress-strain-curve these phenomena have no effects on the ultimate limit state behaviour of structural components.

High strength steels with grades higher than S460 are therefore excluded from the general design rules in Eurocode 3 and also indirectly limitations were given by specifying the yield strength ratio f_u/f_y to fu/fy>=2.1

There is however a working group preparing “Recommendations for the design of structures made of high strength steels” that in a later stage shall be implemented into the Eurocodes.[5]

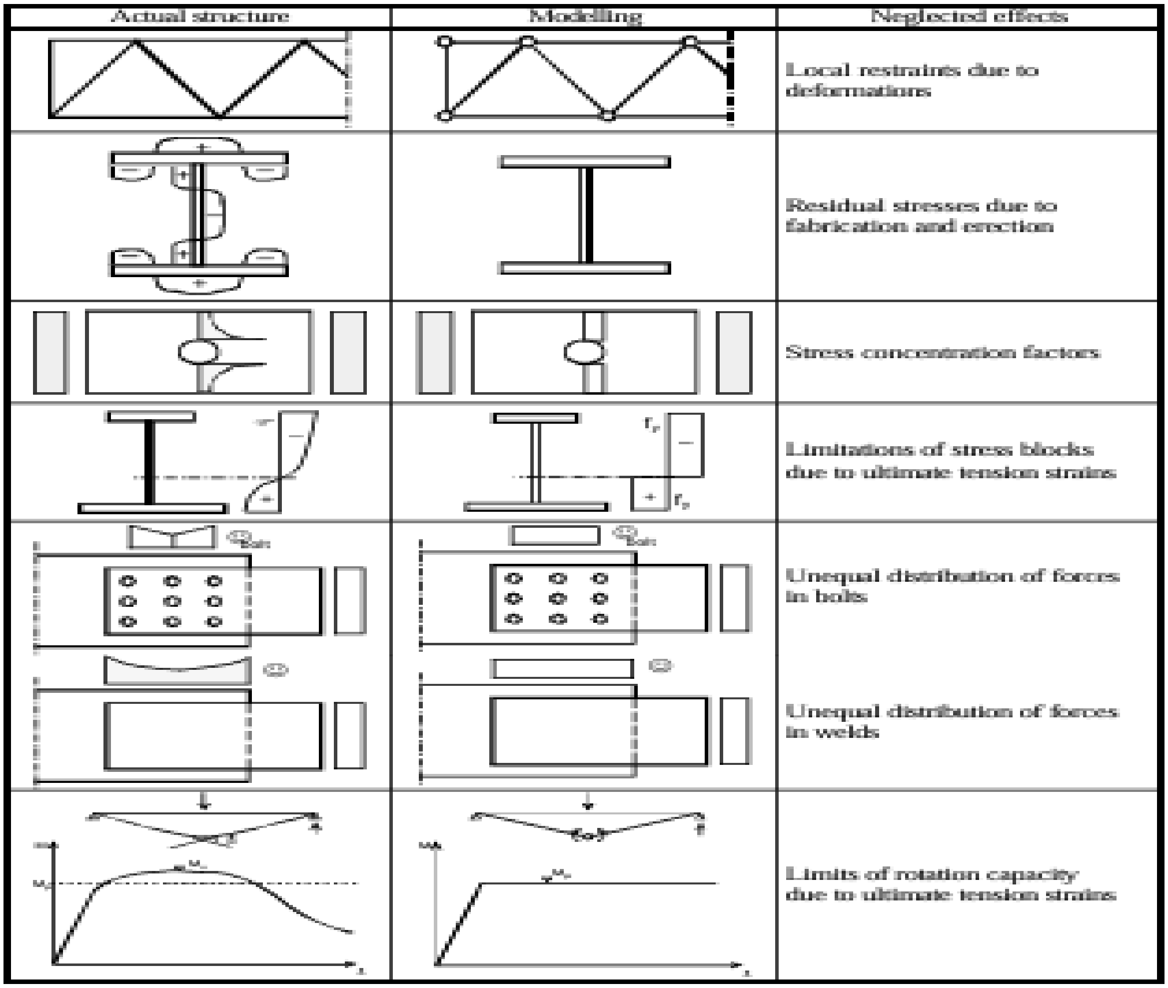


Figure 2. 4 Behaviour of steel members

There is however a working group preparing “Recommendations for the design of structures made of high strength steels” that in a later stage shall be implemented into the Eurocodes.[5]

2.5 Material property of HSS:

High strength steels are mainly used for structural applications and are hence also called structural steels. High strength steels are extensively used in automotive body structure where good durability is a requirement. Langenberg (2008) investigated the material properties of European structural steel S355, S460, S690 and S890 from many tensile coupon test data, and compared the yield-to tensile strength ratio fy/fu, percentage elongation after fracture A and percentage reduction of area Z of these steel in Figure 2.5. It can be seen that the ratio of fy/fu increases as the yield strength increases. The yield-to-tensile strength ratios of S460 steels are around 0.8, while for the steels having yield strengths over 690 MPa, the ratios are between 0.90 and 0.95 and tend to 2.0

with the increase in the yield strength. In the Australian standard AS4100 and the Eurocode 3, the value of the fy/fu ratio is limited to 0.83 and 0.95, respectively. It is demonstrated that high strength steel has lower ductility than normal structural steel, and might not meet the requirement of seismic design in the standards (Shi et al., 2014b). In Figure 2.1, the elongation after fracture A, another important index for ductility, decreases with the increasing of the yield strength, which also indicates that the ductility of HSS is relatively low. But Langenberg (2008) contended that the ductility of HSS cannot be assessed by these parameters alone. Sivakumaran (2008) and Ban et al. (2011) also overviewed the material property characteristics of high strength steels with various yield strengths reported in the literature and provided a similar result.

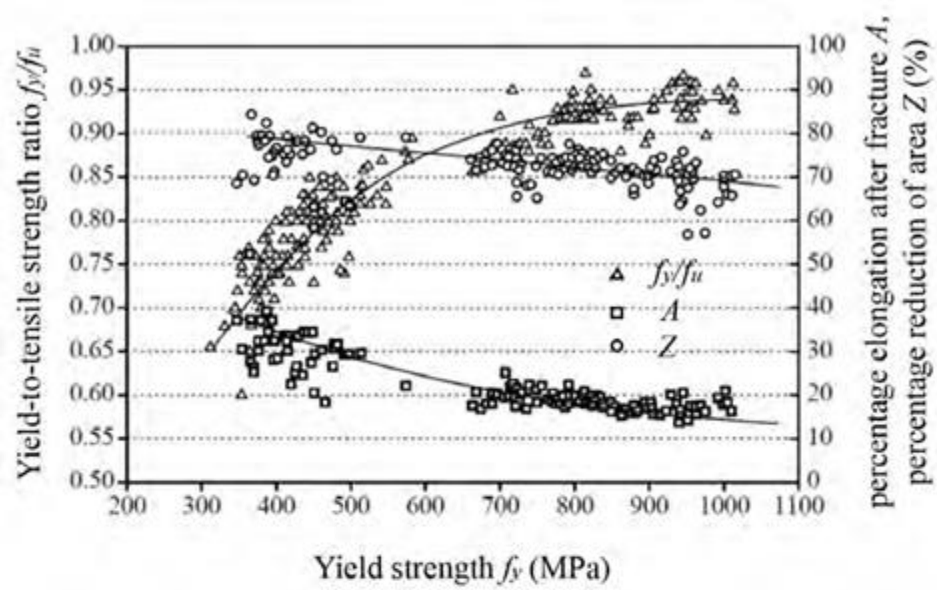


Figure2. 5 Material properties of European steel (Langenberg, 2008)

Figure 2.7 shows the stress-strain curves of Q460 steel produced in China, and S690, S960 steel in Europe, obtained from tension coupon tests by Shi et al. (2009) and Ban (2012). It can be seen that the Q460 steel has a clear region of the yield plateau, ranging approximately from the strain $\varepsilon = 0.2\%$ to 2.0%, while the yield plateaus of S690 and S960 steels almost disappear, and the strains at the ultimate tensile strength reduce with an increase in the yield strength. According to these tension test results and others, Ban et al. (2011) simulated the material properties of Q460, S690 and S960 steels by two multi-linear stress-strain relationship models with and without the yield plateaus in Figure 2.8. The corresponding characteristic values used for describing the stress-strain curve include the yield strength fy, the ultimate strength fu, the yield strain εy, the strain at the onset of strain hardening εt, and the ultimate strain εu listed in Table 2.2.

Table 2. 1Material properties of steel (Ban et al., 2011)

Steel Grade	f_y (MPa)	f_u (MPa)	ε_y (%)	ε_t (%)	ε_u (%)
460 MPa	460	550	0.22	2.00	14.0
690 MPa	690	770	0.33	0.33	8.0
960 MPa	960	980	0.46	0.46	5.5

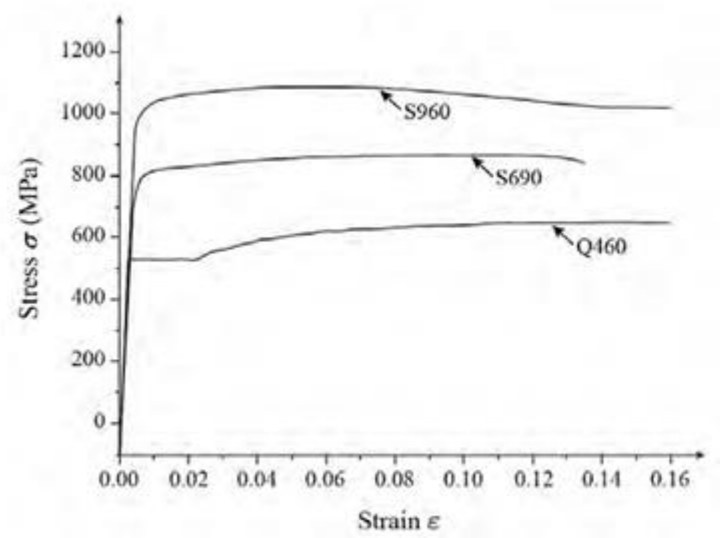
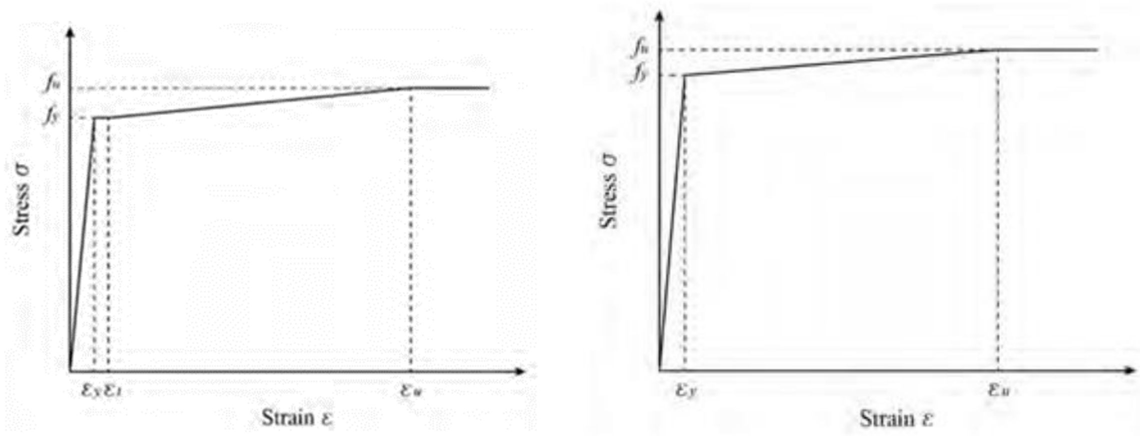


Figure2. 6 Stress-strain curves of different high strength steel grades (Shi et al., 2009; Ban, 2012)



(a) with yield plateau **(b) without yield plateau**

Figure2. 7 Stress-strain relationship model

2.6 International development of the HSS:

High-tensile steel plates are utilized in numerous high-stress applications due to their superior strength and durability. They are used extensively in buildings, bridges, automobiles, manufacturing heavy equipment, pipelines for transporting liquids and gas, and ships. High-tensile steel plates have been used for decades as a cost-effective solution to increase the lifespan of these structures by improving their resistance to breakage or corrosion. Additionally, they provide superior shock absorption ability with great elasticity and uniform thickness throughout the plate, making them an ideal choice for vibration-damping applications such as car bumpers.

2.6.1 Japan:

In fact, Japan has a long history of iron and steel manufacturing for a range of industry and applications. The Japan Iron and Steel Federation (JISF) was established in November 1948 and has been restructured several times over subsequent decades.

The JISF has a strong focus on ongoing technical development, and the Japanese steel industry is constantly developing technologies to supply high-grade steel that can meet the diversifying and exacting requirements of companies that use steel products. In Japan, common high strength steel grades start at around 490 MPa however research continues around the use of steel with tensile strengths up to 1,000 MPa.

Landmark Tower, Yokohama, shown in Fig. 2.9, was the first Japanese structure to use high strength steel elements, where the I-section columns were fabrication from thermo-mechanically controlled process (TMCP) plates with minimum tensile strength of 600 MPa. This building was completed in 1989.

One advantage of using high strength Q&T steel is the reduction in column section sizes which becomes especially beneficial in the lower levels of tall buildings. In Japan, aside from the Landmark Tower, other high-rise buildings have very successfully utilised the properties that high strength Q&T steel can offer, including the JR East Japan Headquarters Building and the NTV Tower, Tokyo, shown in Fig. 3 and Fig. 4 respectively.

2.6.2 USA

In the USA, the key high strength Q&T steel grades are 50W, 70W and 100W which have minimum yield strengths equivalent to 345 MPa, 485 MPa and 690 MPa respectively – these are commonly referred to as high performance steels (HPS). The 70W grade was created first in 1996 with the other grades following a few years later [6]. More commonly the grades are now referred to by their ASTM designation, with ASTM A514 (700 MPa) perhaps being the most well-known

The development of high strength Q&T steel for structural applications came about from an American research program that was commenced around 1994 with the aim of developing a 'better performing steel' in terms of weldability, increased toughness and improved weathering resistance without altering the strength of the steel.

In 1997 the first test of a HPS, 70W, was undertaken on a single-span road bridge in Nebraska, shown in Fig. 5. The original design utilized conventional Grade 345 steel. To investigate the fabrication process utilizing High Performance Steel the original Grade 345 steel was replaced with HPS-485W without modifying the design. The fabricators concluded there were no significant changes required in the HPS fabrication process.

A range of different bridges and buildings across the continental USA use various HPS and high strength Q&T steels, but the most well-known may have been One World Trade Centre, New York. It was a 104-storey building and, when including the spire, reached to 1,776 metres high which made it the tallest building in America.

The tower's structure was designed around a strong steel frame made of beams and columns. Steelmaker ArcelorMittal Diffidence, Luxembourg, supplied approximately 14,000 tonnes of Histar grade structural steel, while approximately 30,000 tonnes of steel plate was sourced from ArcelorMittal Coatesville in Pennsylvania. It was estimated that by using high strength steel, the weight of the columns was reduced by around 32% and beams by around 19% versus normal steel grades.

Figure2. 8(left) Landmark tower, yokohama (japan) rises above surrounding buildings. [Sunwater2015. Reproduced under CC BY-SA 4.0 International].(Centre) JR EAST japan head- quarters in shibuya-ku, tokyo. [kure. reproduced under cc by-sa 3.0]].(right) Nippon television tower head- quarters in Minato, tokyo. [Kure. Reproduced under CC BY-SA 3.0].

Figure2. 9snyder bridge south in nebraska. source: (azizinamini, van ooyen, jabar, & fallaha, 2002). reproduced with kind permission of prof. a. azizinamini.

HPS was developed in the USA specifically for bridge construction, costs savings and protection against weathering making it possible for structures to be redesigned without compromising strength or ease of fabrication.

2.6.3 Europe and the UK

The first use of high strength Q&T steel in Europe was a floating structure used in the Hutton Oil Field, 200 km off the coast of Norway. The tension-leg platform used steel with a minimum yield strength of 795 MPa.

Generally, high strength Q&T use in the UK has been mainly focused around offshore applications where designers generally specify steel with a strength of around 460 MPa. While higher strength steels (500 – 700 MPa) are becoming more common , the advantage for offshore applications is the potential for reducing sectional weight. Not only can this aid design, but fabrication can become faster and more economical provided the high strength steel has good weldability and formability.

Other examples across Europe include the bridge across the canal in Zuid Beveland, in the southern part of the Netherlands. Here, a girder construction made of 460 MPa steel was chosen in order to reduce the girder depth and to maximize the clearance height for the canal under the bridge.

A further example of the use of high strength steel in bridge applications comes from Rémoulins in the south of France. For this continuous twin-girder construction with span lengths of 47, 66 and 51 metres, a combination of standard 355 MPa steel was used with a 460 MPa high strength steel. The high strength 450 MPa was especially applied in the highly stressed pier region of the girders to reduce the maximum thickness, resulting in a reduction of thickness from 120.0 mm using standard 355 MPa steel, to 80.0 mm using the hybrid combination (Ref. 21). Overall, it is estimated that a weight reduction of more than 8% was achieved using this combination of standard and high strength steels.

In terms of higher strength Q&T steel applications, a pier and pier-girder bridge near Ingolstadt, Germany, used 690 MPa steel in a medium span application, with span lengths of 24 + 3*30 + 24 meters [7]. In this instance, 690 MPa material was applied for the connection between the girder and the piers formed by concrete filled steel tubes of 600 mm diameter where the 70 mm thick 690 MPa material was welded to the girder to form a bending-stiff connection [7].

Across Europe, 460 MPa high strength Q&T steel remains the material of choice for the construction of bridges. This includes the Rhine Bridge in the north of Dusseldorf, Germany, which was opened for traffic in mid-2002 [8].

The Shard, at 306 metres high, is the tallest building in Western Europe and arguably one of London's most ambitious and distinctive buildings. It took over a decade of planning, design and execution to overcome many design and engineering challenges, all without compromising the architect's original vision. Whilst early designs envisaged an all-steel framed building, stiffened by steel outriggers that connected sizeable internal columns with the core, a 'mixed structure' was finally settled upon, with steel in the lower office levels, and concrete for the hotel and residences at the top

The steel used in The Shard provided significant benefits – not only did it become possible to build The Shard in line with the architect's vision, but high strength steel also delivered significant other benefits (see Fig. 6), clearly highlighting the versatility and value in using high strength Q&T steel for this type of application.

2.6.4 Other examples structures using HSS

- **Buildings**

The use of S460 in buildings, e.g. for columns and beams of composite decks in park- ing houses is already frequent.

An example for the use of steels S690 is the roof truss of the Sony Centre in Berlin, that is used to suspend several storeys of the building to protect an old masonry building "Kaisersaal" integrated in the building from being loaded, Figure 2.12.

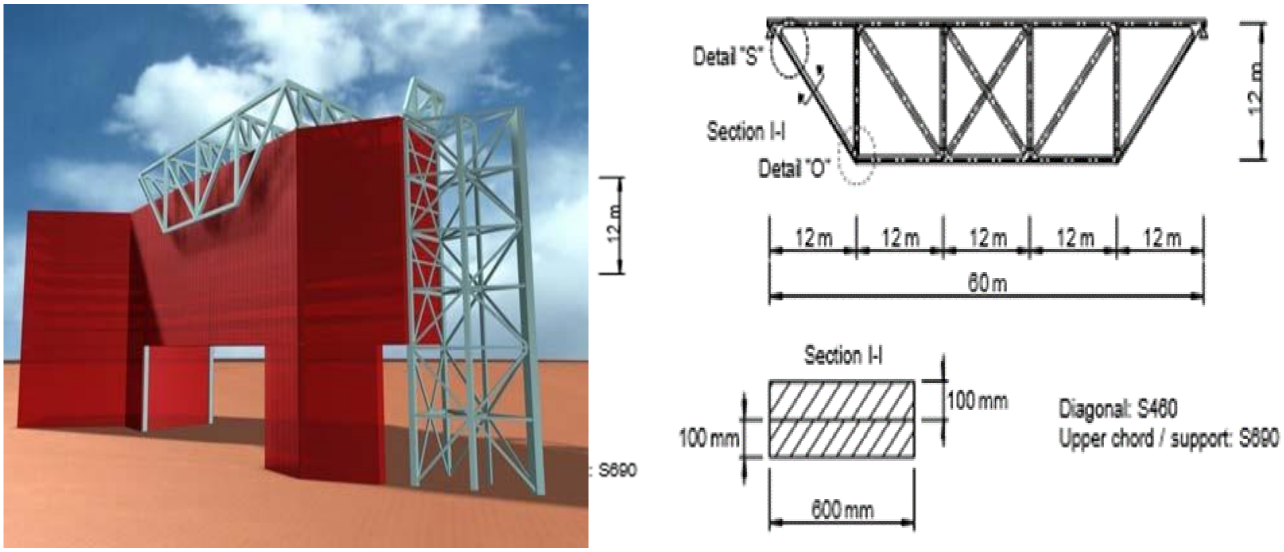


Figure2. 10 Overview on the roof structure of the Sony Centre in Berlin

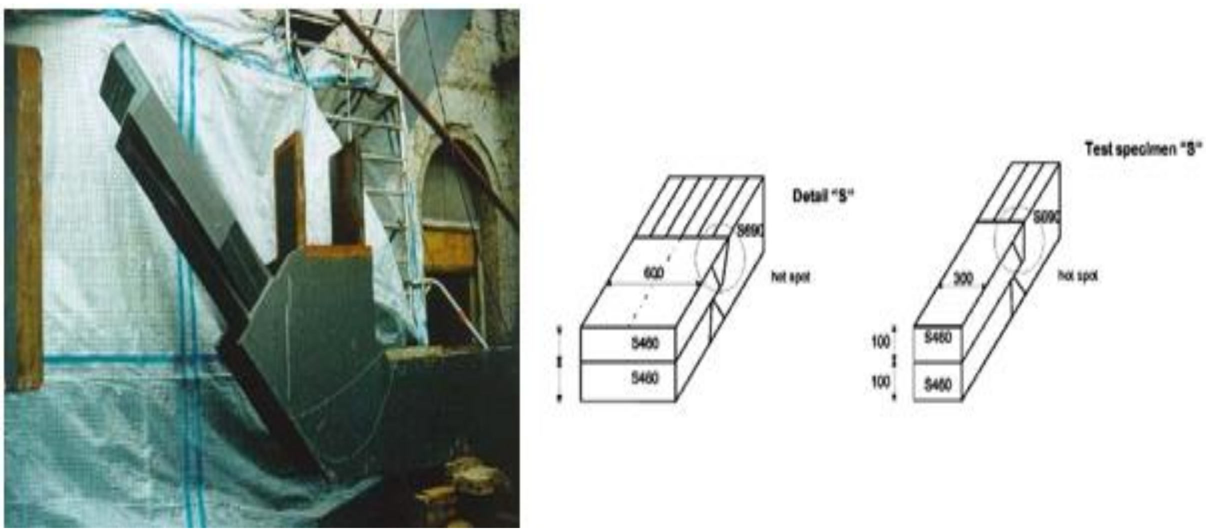


Figure2. 11Details of connection of thick plates made of S460 and S690 for testing

- **Bridges**

Examples for the use of steels S460 in bridges are the Rhine bridge Düsseldorf Ilverich, where the pylon was made of S460 to avoid preheating and keep the welding costs low, figure 2.14

Figure2. 12Rhine bridge Düsseldorf-Ilverich with tension tie in the pylons made of S460 TM

A spectacular example for the use of S460 M is the Millau-Viaduct in France, Figure 2.17_ where a total of 43000 t of steel plates have been applied with thicknesses up to 80 mm for the entire central box and some connecting elements.

Figure2. 13Millau-Viaduct and launching of pylon

In summary, there are lot of benefits to fabrication and erection costs of using high strength steel for components include:

- Higher strength = lower weight
- Lower weight = lower transport
- Lower weight = easier handling

- Higher strength = reduced thickness
- Reduced thickness = reduced welds
- Reduced welds = reduced time
- Reduced welds = reduced costs

CHAPTER THREE

THEORY OF PLATES

3.1. Introduction:

Steel plates are commonly used in buildings, bridges, hydraulic structures, containers, ships, aerospace structures, and planes, as well as instruments and machines (Giovanni et al., 2014). They may be subjected to in plane loads or lateral loads or both. Plates are generally classified as: 1) thick plates when the plate minimum dimension to thickness ratio (b/h) is less than 10. Thick plates develop internal load resultants, governed by three-dimensional elasticity to counterbalance the applied load. 2) Thin plates when b/h ranges from 10 to 100. Thin plates behave as plane stress members provided that the plate maximum deflection to plate thickness ratio (w/h) is less than 0.2. In this case, the plate develops internal load resultants, governed by two-dimensional elasticity to counterbalance the applied loading. 3) Membranes when the b/h ratio approaches 100 and w/h > 0.2. Membranes are only capable of developing internal tensile stress resultants, namely, membrane tensile force resultants acting within the plate middle plane (Yamaguchi and Wai-Fah, 1999; Ventsel and Krauthammer, 2001; Stephen et al., 2010).

In general, plates are classified as thick plates when the minimum dimension to thickness ratio (b/h) is less than 10, thin plates when the b/h ratio ranges from 10 to about 100, provided that the plate maximum deflection to thickness ratio (w/h) is less than 0.2, and membranes when the b/h ratio approaches 100 and w/h ≥ 0.2. Thick plates develop internal stress resultants governed by three-dimensional elasticity similar to that of a solid body. Thin plates behave as plane stress members governed by two-dimensional elasticity. Membranes can only develop internal tensile stress within the plate's neutral plane. Few studies have adopted b/h ratios between 90 and 110 to investigate the feasibility of the utilization of such plates in their various available forms. The current study with the b/h ratio of 100 aims to fill the gap. Steel Plates are available in different forms such as intact plates, stiffened, perforated, and stiffened perforated plates. They are used in buildings, bridges, ships, as well as aerospace structures.[9]

3.2. Plate Theory:

3.2.1. Plates

A plate is a flat structural element for which the thickness is small compared with the surface dimensions. The thickness is usually constant but may be variable and is measured normal to the middle surface of the plate, Fig3.1 [10]

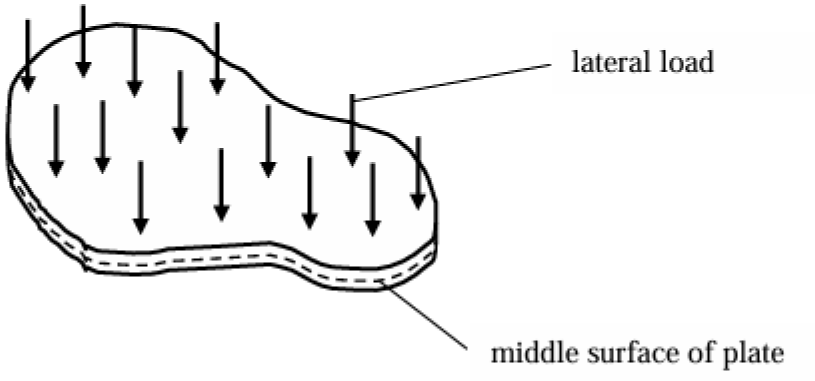


Figure 3. 1A Geometric and loading of a plate

3.2.2. Plate Theory:

Plates subjected only to in-plane loading can be solved using two-dimensional plane stress theory. On the other hand, plate theory is concerned mainly with lateral loading.

One of the differences between plane stress and plate theory is that in the plate theory the stress components are allowed to vary through the thickness of the plate, so that there can be bending moments, Fig.3.2 [10]

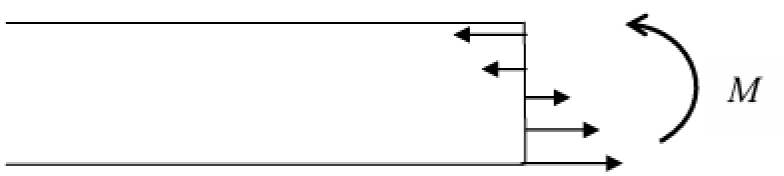


Figure 3. 2Stress distribution through the thickness of a plate and resultant bending moment

3.3. Plate Theory and Beam Theory:

Plate theory is an approximate theory; assumptions are made and the general three-dimensional equations of elasticity are reduced. It is very like the beam theory only with an extra dimension. It turns out to be an accurate theory provided the plate is relatively thin (as in the beam theory) but also that the deflections are small relative to the thickness. This last point will be discussed further

Things are more complicated for plates than for the beams. For one, the plate not only bends, but torsion may occur (it can twist), as shown in Fig3.3 [10]

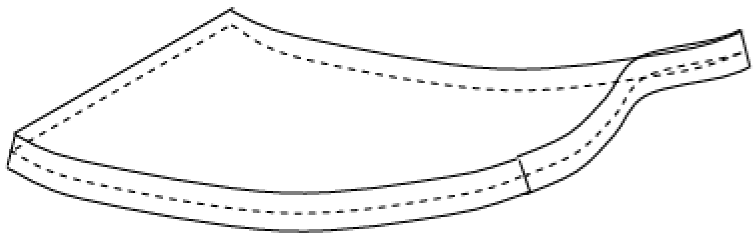


Figure 3. 3 Torsion in a plate

Assumptions of Plate Theory

Let the plate mid-surface lie in the y x – plane and the z – axis be along the thickness direction, forming a right-handed set, Fig.3.4

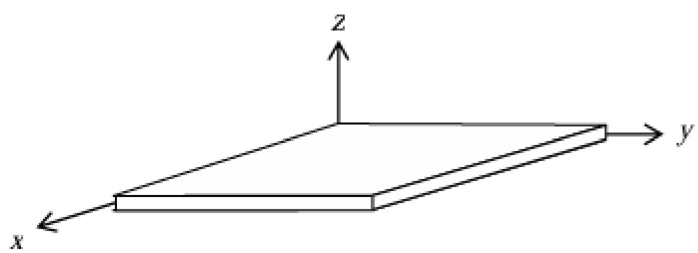


Figure 3. 4 Plate view with Cartesian axes

The stress components acting on a typical element of the plate are shown in Fig3.5

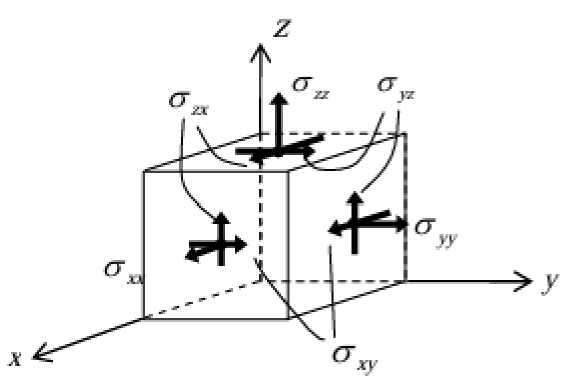


Figure 3. 5 Stresses acting on a material element

The following assumptions are made:

(i) The mid-plane is a "neutral plane"

The middle plane of the plate remains free of in-plane stress/strain. Bending of the plate will cause material above and below this mid-plane to deform in-plane. The mid-plane plays the same role in plate theory as the neutral axis does in the beam theory.

(ii) Line elements remain normal to the mid-plane

Line elements lying perpendicular to the middle surface of the plate remain perpendicular to the middle surface during deformation, Fig. 6.1.6; this is similar the "plane sections remain plane" assumption of the beam theory.

3.4. Plate Elements in Steel Members:

A wide variety of structural steel shapes are manufactured, with their cross-sectional shapes and sizes determined by several key factors: optimal structural efficiency, functional requirements, the dimensional and weight capacities of rolling mills, and material properties. According to EC3, most steel sections are composed of flat rectangular plates arranged at right angles to form the flanges and webs of their cross sections, achieving the best combination of these criteria. Figure 1.1 illustrates various cross-sectional shapes for metal compression or flexural members. With the exception of hollow cylinders, all these members consist of connected elements that, for analysis and design purposes, can be treated as flat plates. Steel sections are thus typically an assembly of flat rectangular plates joined at right angles to form the flanges and webs of their cross sections.[11]

When a plate element is subjected to direct compression, bending, shear, or a combination of these in-plane stresses, theoretical critical loads can be calculated. These critical loads indicate that the plate may buckle locally before the entire member becomes unstable or before the material reaches its yield stress. This behaviour is marked by a distortion of the member's cross section. The almost inevitable presence of initial imperfections, such as out-of-straightness, can lead to a gradual development of cross-sectional distortion, resulting in a smooth transition in real behaviour rather than a sudden change at the theoretical critical load.

Metal compression or flexural members, as illustrated in Figure 4.1 (excluding Figure 4.1a), often use cross-sectional shapes that can be idealized as combinations of flat plates. When these plate elements are subjected to direct compression, bending, shear, or a combination of these stresses within their plane, they may experience local buckling before the entire member becomes unstable or reaches the material's yield stress. This local buckling is characterized by distortion in the cross-section of the member. Unlike the sudden or abrupt nature often associated with buckling, the presence of initial out-of-plane imperfections causes a gradual increase in this cross-sectional distortion, resulting in no abrupt discontinuity in the actual behaviour at the theoretical critical load.

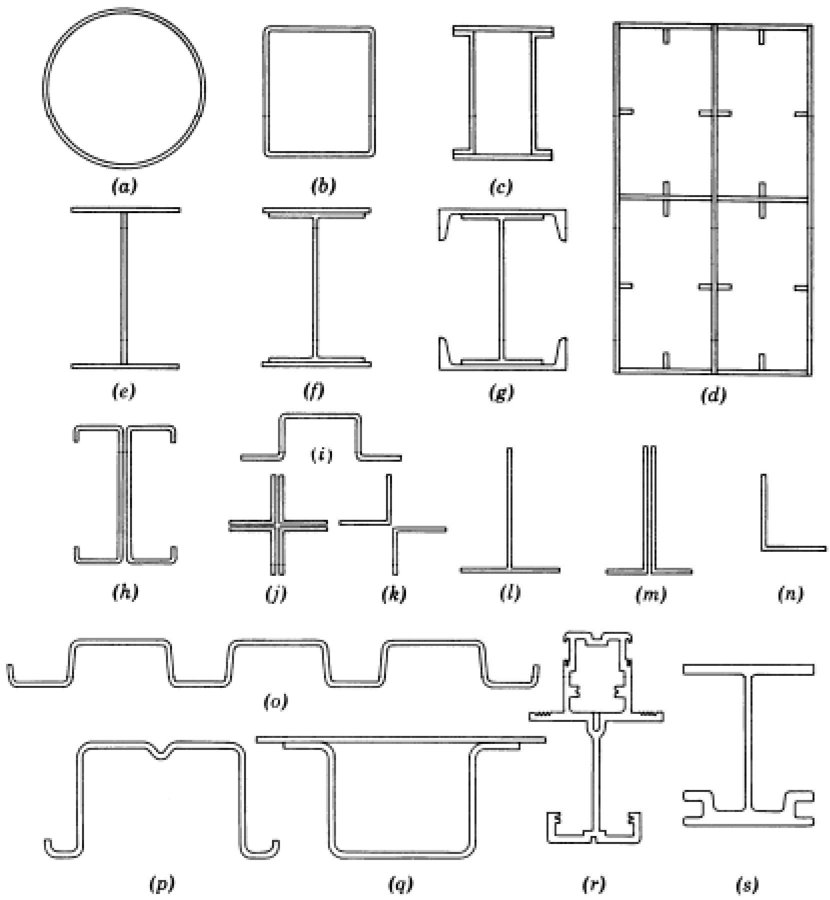


Figure 3. 6Compression or flexural steel members (Galambos, 1998)

The theoretical, or elastic critical local buckling load alone is not a sufficient basis for design. The ultimate strength of plates can be less than the critical local buckling load due to yielding or greater due to beneficial post-buckling reserves. For instance, a slender plate loaded in uniaxial compression, with both longitudinal edges supported, will undergo stress redistribution and develop transverse tensile membrane stresses after buckling, which provide a post-buckling reserve. Consequently, additional load can often be applied without causing structural damage.

Initial imperfections in such a plate may cause deformations to start below the buckling load; however, unlike an initially imperfect column, the plate can sustain loads exceeding the theoretical buckling load.[12]

3.6. Comprehensive Literature Review on the Buckling Phenomena in Thin Plates:

In numerical studies like this one, establishing a solid foundation of prior research in the field of buckling is crucial, necessitating a thorough literature survey. The synthesis of the survey's findings will provide the theoretical background for the first part of the dissertation. The objective of the first part is to conduct parametric studies to examine the impact of variables such as geometry, load cases, plate thicknesses, and the type and position of cut-outs on the buckling load.[12]

Plate in general:

Early works on the buckling problems can be found in the literature including those relative to Timoshenko (1910), Leggett (1937), Zetlin (1955), White and Cottingham (1962), Bulson (1969), Hopkins (1969), Khan and Walker (1972), Khan et al. (1977), and Young and Lui (2005). The first work in this area was perhaps reported

Shakerley and Brown (1996) investigated elastic buckling of simply supported and fully fixed plates with eccentrically placed rectangular cu-touts using conjugate method. Their results were given for uniaxial compression and shear loading. Bert and Devarakonda (2003) studied the buckling of rectangular plates subjected to non-linearly distributed in-plane loading using analytical solution method. They considered a rectangular plate subjected to parabolic loads along with the removal of some deficiencies of earlier reported work. Zhong and GU (2006) reported a study on the buckling analysis of simply supported rectangular plates subjected to linearly varying edge loads. An analytical solution to buckling problem was developed and the effect of load intensity variation on the critical load was investigated.

Recently, Mijušković et al. (2014) presented an accurate buckling analysis for thin rectangular plates under locally-distributed compressive stresses. Interestingly, Wang et al.'s (2007) and Wang's (2015) studies have also shown that the differential quadrature method can yield accurate buckling loads for rectangular plates under uniformly or non-uniformly distributed edge compressions. It is also noteworthy that some other researchers have conducted comprehensive research, most recently, on the buckling of shells and plates, e.g. McCann et al. (2016), Riahi et al. (2017 and 2018), and Vu et al. (2019).

CHAPTER 4

BUCKLING OF THIN PLATES,

THE CLASSICAL APPROCH

4.1 General about buckling phenomenon

Most steel constructions such as cranes, ships, bridges and building frames experience compressive and/or shear loads and are thus susceptible to buckling. Buckling of construction members is usually a very sudden process without much warning in advance. Therefore, it is important to account for the effect of buckling in the design of a construction members.

The use of thinner sections and high strength steels leads to design problems for structural engineers which may not normally be encountered in routine structural steel design. Structural instability of sections is most likely to occur as a result of the thickness of the sections, leading to reduced buckling loads (and stresses), and the use of higher strength steel typically makes the buckling stress and yield stress of the thin-walled sections approximately equal.

The classical theory of plates or shells, which formulates and solves problems from the point of view of rigorous mathematical analysis, is an important application of the theory of elasticity. The study of mechanics of materials and the theory of elasticity is based on an understanding of equilibrium of bodies under action of forces. While statics treats the external behaviour of bodies that are assumed to be ideally rigid and at rest, mechanics of materials and the theory of elasticity are concerned with the relationships between external loads and internal forces and deformations induced in the body.

Load-supporting action of plates resembles, to a certain extent, that of beams. However, the load-carrying mechanism of a shell differs from that of other structural forms.

When a plate is compressed in its midplane, it becomes unstable and begins to buckle at a certain critical value of the in-plane force. Buckling of plates is qualitatively similar to column buckling However, a buckling analysis of the former case is not performed as readily as for the latter. Classical buckling problems, by the so-called equilibrium method and the conditions that result in the lowest eigenvalue, or the actual buckling load, are not at all obvious in many situations.

Also, buckling analysis can be performed by Finite Element Method (FEM) packages as will be described in the following chapter, i.e. Chapter 4. However, these analyses usually cost a lot of engineering time. This because for the analysis to make any sense, it must often be done on individual plate fields. This brings a lot of uncertainties about boundary conditions into the calculation. Therefore, buckling analysis in FEM is very labour intensive and costly.

A much simpler way to check for buckling is by means of a standard, such as Eurocode 3(EC3), AISC and others. These standards are made up from mathematical models that describe buckling combined with empirical data from real life experiments. The recommendations ensure, when followed correctly, that a structure will be strong enough to resist certain applied loads.

Buckling analysis would become less time consuming when the stress results from an FEM analysis can be compared to these standards. However, often there is a discrepancy between FEM results and input parameters used in standard recommendations. [14]

4.2. Historical review of the developments in buckling and Post buckling analysis of plates:

Historic notes

According to the reference : Handbook of thin plates buckling ("Thin Plate Buckling and Post-buckling", problems of initial and post-buckling represent a particular class of bi-furcation phenomena; the long history of buckling theory for structures begins with the studies by Euler in 1744 of the stability of flexible compressed beams, an example which we present in some detail below, to illustrate the main ideas underlying the study of initial and post-buckling behaviour. Although von Karman formulated the equations for buckling of thin, linearly elastic plates which bear his name in 1910 a general theory for the post-buckling of elastic structures was not put forth until Koiter wrote his thesis in 1945; it is in Koiter's thesis that the fact that the presence of imperfections could give rise to significant reductions in the critical load required to buckle a particular structure first appears.

General theories of bifurcation and stability originated in the mathematical studies of Poincaré, Lyapunov and Schmidt and employed, as basic mathematical tools, the inverse and implicit function theorems, which can be used to provide a rigorous justification of the asymptotic and perturbation type expansions which dominate studies of buckling and post-buckling of structures. Accounts of the modern mathematical approach to bi-furcation theory, including buckling and post-buckling theory, may be found in many recent texts, most notably those of Keller and Antman, ,Sattinger, Iooss and Joseph, Chow and Hale, and Golubitsky and Schaeffer. Among the noteworthy survey articles which deal specifically with buckling and post-buckling theory are those of Potier-Ferry, Budiansky, and (in the domain of elastic-plastic response) Hutchinson Some of the more recent work in the general area of bifurcation theory is quite sophisticated and deep from a mathematical standpoint, e.g., the work of Golubitsky and Schaeffer.

4.3 Buckling of steel members

4.3.1 Introduction

Stability of a structure can be analysed by computing its critical load, i.e., the load corresponding to the situation in which a perturbation of the deformation state does not disturb the equilibrium between the external and internal forces.

Buckling is caused by in-plane stresses exceeding the buckling stability of the structure, causing local yield and permanent deformation of the structure. The buckling capacity is a property of the plate, depending on many factors. Two cases of instability may occur, that is overall or global column instability and plate element local instability.

Local buckling of plate elements in a steel column can cause premature failure of the entire section and reduce the overall strength. The mode of failure and load carrying capacity of steel columns are affected by the behaviour of plate elements and the interaction between local and overall buckling.

4.3.2 Objective of the buckling analysis

Buckling analysis is a technique used to determine buckling loads (critical loads at which a structure becomes unstable) and buckled mode shapes (the characteristic shape associated with a structure's buckled response).

Although a linear static analysis will ensure that equilibrium is fully achieved and may also predict stress levels within an acceptable range, the current structural design may still be unsuitable for the intended use.

Below the critical buckling load of a structure “stable” equilibrium will usually be achieved, whilst above this load “unstable” equilibrium may result from geometric and/or material effects.

4.3.3 Types of buckling

Plates behaviour under in-plane loading could be evaluated either in buckling regime, i.e., at the on-set of buckling or in post buckling regime, i.e., when buckling has set in. In the buckling regime, the plates are considered to be under small deflection, whereas, in the post buckling regime, the plates are considered to have executed some considerable deflection. Plates' behaviour could also be evaluated in both regimes (buckling regime and in their transition to post buckling regime).

Buckling can be classified into three states:

1. **Elastic buckling**: occurs only in the elastic regime of the stress -strain material graph.
2. **Elastic - plastic buckling**: occurs when a local region inside the plate experiences a plastic deformation.
3. **Plastic buckling**: happens after the plate has yielded over large regions.

Many factors affect the buckling behaviour of panels, including geometric or material properties, loading characteristics, boundary conditions, initial geometrical imperfections, and material nonlinearity**.**

4.4 Theory of Buckling

4.4.1 Classical case of beam buckling

Beams subjected to compressive loadings have the tendency to deflect laterally. This lateral deflection is called buckling. A beam will fail under the influence of compressive loadings when the stresses caused by lateral deflection are greater than the materials yield strength. Failure due to buckling is often a process without much warning. Therefore, it is required to give special attention to buckling in the design process of beams and columns.[14] This chapter will discuss a couple of theoretical subjects concerning buckling. First the stability of a beam under influence of compressive loadings will be discussed. The stability will dictate the load before a beam will buckle. This load is discussed in Euler buckling.

Euler buckling is only concerned with elastic buckling. Therefore, the subject of inelastic buckling is discussed next. Finally, this chapter will give a discussion about torsional buckling and warping.

4.4.2 Stability

Failure of a beam due to buckling can be a sudden process without warning. A beam transforms from a stable equilibrium to an unstable equilibrium with increasing compressive load. The point where stable will turn to unstable is called the bifurcation point and is depicted in the following Figure (3.1).

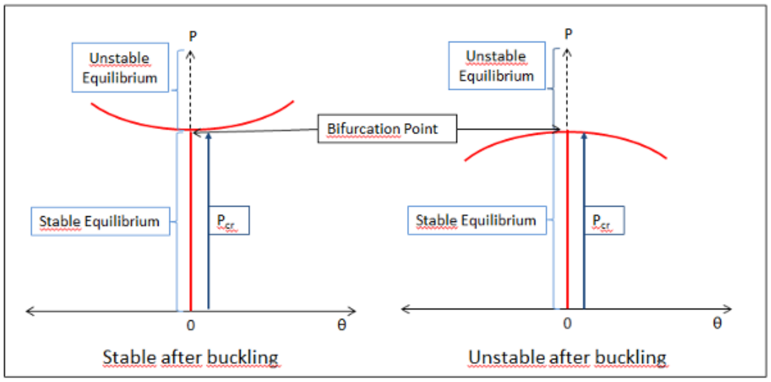


Figure 4. 17 Different Stability or instability states

After this bifurcation point, a new path is followed. This new path determines the failure of a member with respect to buckling. The left figure allows for an increase in load after buckling. This indicates that despite plastic deformation of a beam, it will not fail completely. The right figure shows an unstable path that cannot even resist the critical load. Such a beam will fail completely due to buckling.

The load that belongs to this tipping point is called the critical load. This critical load can be explained by considering the following mechanism.

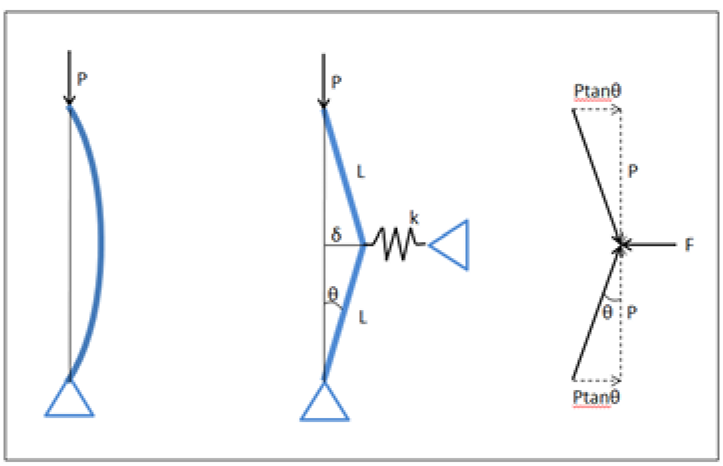


Figure 4. 2Eulerian Buckling of beam

The critical load is that load that will bring the disturbing force P in equilibrium with the restoring force F. The disturbing force P is related to the external loads acting on the beam while the restoring force F is related to the bending stiffness of the beam.[14]

4.4.3 Euler Buckling

A beam is a bar, possessing length significantly greater than the depth and width. Beams are usually loaded in a direction normal to the longitudinal axis, while bars are axially loaded or twisted. The Swiss mathematician Leonard Euler was the first to solve the linear buckling problem for ideal columns in 1757.

The ideal column considered to solve this problem has the following considerations

- Initially perfectly straight
- Made of homogeneous material
- Material behaves linear-elastic
- External load is applied exactly through the cross- section centroid
- Buckling will occur only in a single plane

As mentioned under stability, a column will remain stable when internal resistance is greater than external compressive forces.

The internal resistance is related to bending, where the external bending moment is caused by the external load and a small deflection.

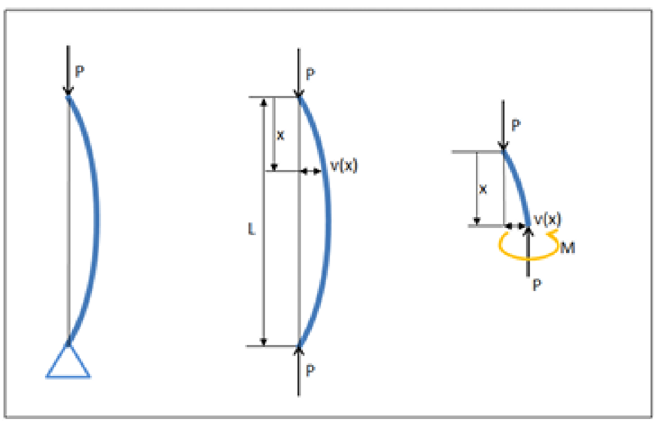


Figure 4. 3Case of a straight beam under pure compression

This leads to the following differential equation by which this buckling problem can be described.

$$\frac{d^2v(x)}{dx^2} + \frac{P}{EI}v(x) = 0 \qquad \textbf{3.1}$$

Solving this differential equation will provide a set of critical loads belonging to certain buckling shapes. The lowest value, belonging to a half sine wave buckling shape, is of interest for buckling failure, since this is the lowest value for which a column will fail due to buckling

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad \textbf{3.2}$$

The strength of columns is usually represented by stresses. Therefore, in column design, the radius of gyration is introduced, which is defined as

$$r = \sqrt{I/_A} \qquad \textbf{3.3}$$

With the radius of gyration, the critical load is transformed into the critical stress

$$\sigma_{cr} = \frac{\pi^2 E}{(L/r)^2} \qquad \textbf{3.4}$$

The boundary conditions of a column do influence the value of the buckling load. A column that is considered to be fixed at both ends is able to resist more load than a column which is pinned at both ends. This difference is indicated by the effective column length.

The effective length is defined as the column length between two points of zero moment. Therefore, every column can be considered as being pinned at both ends. Other boundary conditions are accounted for by introducing the effective length factor K. The value of this factor can be seen for different boundary conditions in the following figure.

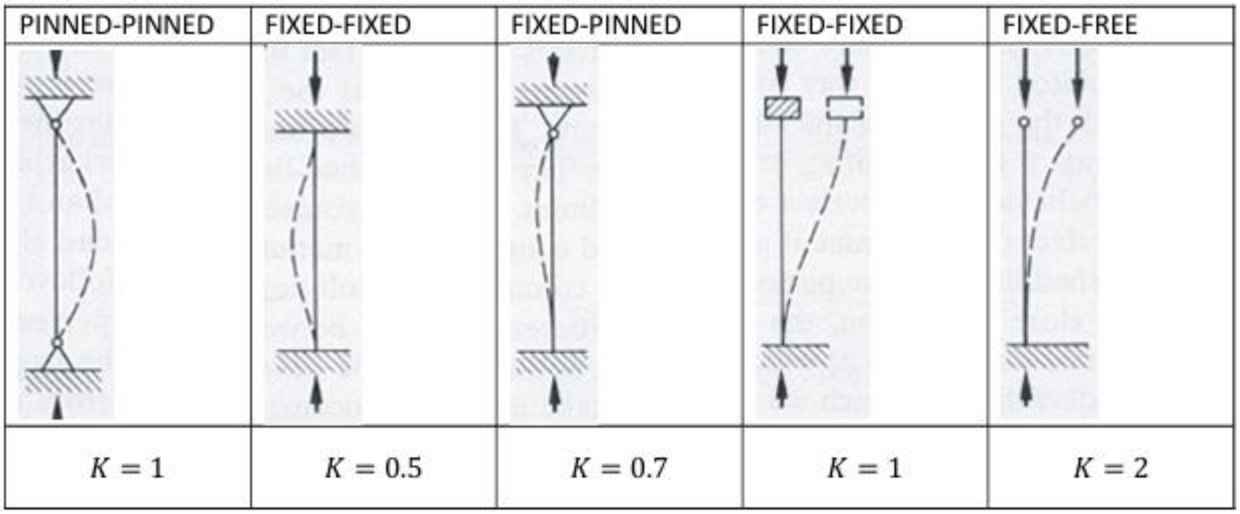


Figure 4. 4Effective length factor K for different boundary conditions

The denominator part in the critical stress formula is called the slenderness ratio. [14]

$$\lambda^2 = \left(\frac{KL}{r}\right)^2$$

4.5. Plate Buckling

Plates and shells are initially flat and curved surface structures, respectively, whose thicknesses are slight compared to their other dimensions.

When a plate is compressed in its midplane, it becomes unstable and begins to buckle at a certain critical value of the in-plane force. Buckling of plates is qualitatively similar to column buckling However, a buckling analysis of the former case is not performed as readily as for the latter. Classical buckling problems, by the so-called equilibrium method and the conditions that result in the lowest eigenvalue, or the actual buckling load, are not at all obvious in many situations

As mentioned above, plate buckling can be regarded as a special case of beam buckling. A plate can be regarded as multiple connected beams. When these beams are loaded in compression, they will show the same behaviour with regard to buckling than a single beam does. There are however some differences, as can be seen in the equation for plate buckling load.

$$N_{cr} = K\frac{\pi^2 E t^3}{12b(1-\nu^2)}$$

First of all, Poisons ratio will cause the beams to expand laterally. These lateral expansions are prevented by each neighbouring beam in the multiple beam plate model. This has a strengthening effect on the plate since more force is required in order to get the same deformations as for beams.

Another effect has to do with the plate aspect ratio. Wide plates, with relative low aspect ratios, is more resistant against buckling than long plates, with relative high aspect ratios. This effect is represented by the buckling coefficient K. This coefficient is determined by the plate's aspect ratio and the number of half-sines buckling waves.

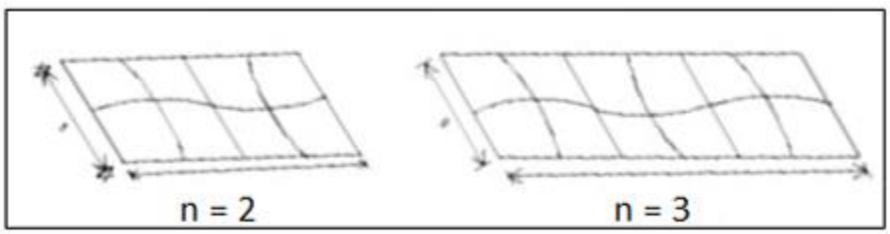


Figure 4. 5Number of half-waves in a buckled plate

Figure shows the buckling coefficient for a plate simply supported at all four edges. A special point can be seen at the aspect ratio of $\sqrt{2}$. Here the buckled plate will step from a single half sine wave buckling form, to a double half sine wave buckling form. Also, can be seen that for larger aspect ratios, the buckling coefficient will asymptote to 4, which can consequently be regarded as the minimum buckling coefficient for such plates.

The buckling coefficient depends on the boundary conditions of a plate loaded in compression. A plate which is simply supported on all four sides will have a better buckling coefficient than a plate which has one free edge as can be seen in Figure **3.6**. The buckling coefficient is further influenced by the way that load is applied. A plate loaded in compression will yield a different coefficient than a plate loaded in bending.

On more difference between plate and beam buckling is the post buckling stress. Most beams will fail completely after the critical compression load is reached. For plates this is different. Plates are commonly supported at all four edges. Due to these supports, the plate will not fail completely after reaching critical buckling load. After buckling of the middle part of the plate, the edges will be able to resist the compression forces until the materials yield strength is reached. This results in a non-uniform stress distribution as can be seen on the left in Figure 2.11. This non-uniform stress distribution can make it rather difficult to further evaluate the plate load bearing capacity. Therefore, the effective width method is often used. This method assumes that the deformed centre of the plate will no longer resist any of the compressive stresses. Instead, the stress distribution over the entire width is replaced with an equivalent continues stress over an effective width of the plate. This is shown on the right side in Figure 3.7 [14]

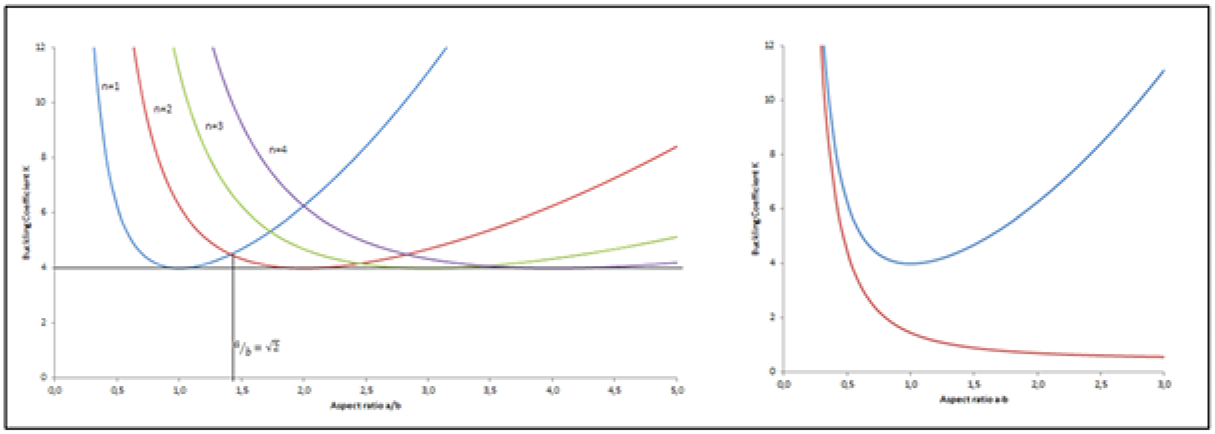


Figure 4. 6Buckling coefficients for a plate simply supported at all edges (left). Difference between buckling coefficients for a plate simply supported on all edges (right blue) and free on one side (right red)

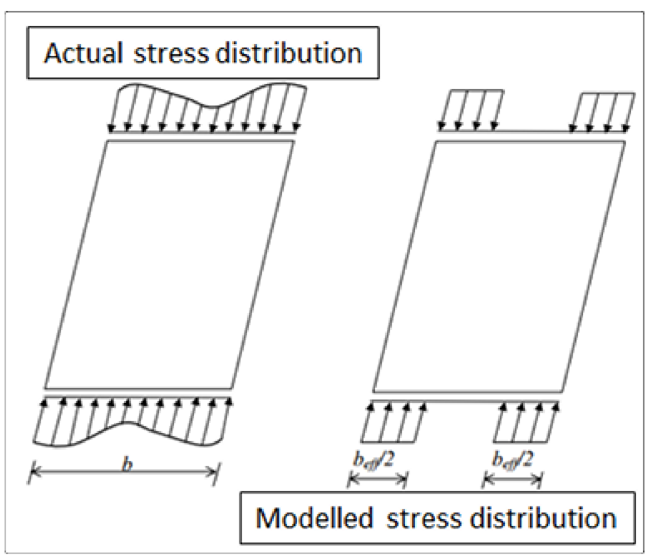


Figure 4. 7The effective width method

4.6 Plate buckling analysis according to EC3

The effective area of the compression zone for flat com pression elements is calculated using equation

Ac, eff = ρAc

where Ac is the gross cross-sectional area and ρ is the reduction factor. The reduction factor for internal compression elements, e.g. web of an I -beam, may be taken as

ρ = 1, for λp ≤ 0,673

$$\rho = \frac{\bar{\lambda}_p - 0.055(3+\psi)}{\bar{\lambda}_p^{\,2}}, \text{ for } \bar{\lambda}_p > 0{,}673.$$

where ψ is the stress ratio which depends on the stress distribution and λ is the plate modified slenderness. For plates with uniform compression stress, the stress ratio ψ = 1 and the reduction factor for class 4 plate elements, i.e., Eq. 2b, simplifies to

$$\rho = \frac{\bar{\lambda}_p - 0.22}{\bar{\lambda}_p^{\,2}}, \text{ for } \bar{\lambda}_p > 0{,}673.$$

The reduction factor for compression elements with free edges, e.g., flanges of an I -beam, may be taken as

ρ = 1, for λp ≤ 0,748

$$\rho = \frac{\bar{\lambda}_p - 0{,}188}{\bar{\lambda}_p^{\,2}}, \text{ for } \bar{\lambda}_p > 0{,}748.$$

The modified plate slenderness is defined as

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{b}/t}{28{,}4\,\varepsilon\sqrt{K_\sigma}}, \quad \varepsilon = \sqrt{\frac{235}{f_y\,[\mathrm{N/mm^2}]}},$$

where b is the appropriate width of the element depend ing on the boundary conditions, t is the thickness of the plate, fy is the yield stress, σcr is the elastic critical stress and Kσ is the buckling factor based on stress ratio and the boundary conditions. The parameter ε, which takes into account the yield stress of the material, is currently valid in EC 3 only up to value 700 MPa, whereas currently produced high strength structural steels have yield stresses up to 1200 MPa.[15]

4.7. Elastic local buckling of flat plates:

An examination of the buckling behaviour of a single plate supported along its edges is an essential preliminary step toward the understanding of local buckling behaviour of plate assemblies.

The buckling stresses are obtained from the concept of bifurcation of an initially perfect structure. In practice, the response of the structure is continuous, due to the inevitable presence of initial imperfections. Thus, the critical stress is best viewed as a useful index to the behaviour, as slender plates can continue to carry additional loads well after initial buckling.

Post buckling and strength of plates is discussed in subsequent section. When the member cross section is composed of various connected elements a lower bound of the critical stress can be determined by assuming, for each plate element, a simple support condition for each edge attached to another plate element or a free condition for any edge not so attached. The smallest value of the critical stress found for any of the elements is a lower bound of the critical stress for the cross section. This lower bound approximation may be excessively conservative for many practical design situations.

4.7.1. Shear Buckling

For the buckling instability of local rectangular plate under shear stress, analytical method and finite element method are mainly used. The calculation of the analytical method is cumbersome, and the finite element simulation can avoid the tedious calculation program of the buckling analysis, but the calculation accuracy is limited to the design accuracy of the model and boundary conditions

When a plate is subjected to edge shear stresses as shown in Fig. 4.6, it is said to be in a state of pure shear. The critical shear buckling stress can be obtained by substituting τc and ks, for σc and k in Eq. 4.1, in which ks is the buckling coefficient for shear buckling stress. Critical stress coefficients, ks, for plates subjected to pure shear have been evaluated for three conditions of edge support. In Fig. 4.9 these are plotted with the side b, as used in Eq. 4.1, always assumed to be shorter than side a. Thus, α is always greater than 1 and by plotting ks in terms of 1/α, the complete range of ks can be shown and the magnitude of ks remains manageable for small values of α. However, for application to plate-girder design it is convenient to define b (or h in plate girder applications) as the vertical dimension of the plate-girder web for a horizontal girder. Then α may be greater or less than unity and empirical formulas for ks together with source data are as follows: Plate Simply Supported on Four Edges. Solutions developed by Timoshenko (1910), Bergmann and Reissner (1932), and Seydel (1933) are approximated by Eqs. 4.3a and 4.3b, in which α = a/b

$$k_s = \begin{cases} 4.00 + \dfrac{5.34}{\alpha^2} & \text{for } \alpha \le 1 \\ 5.34 + \dfrac{4.00}{\alpha^2} & \text{for } \alpha \ge 1 \end{cases}$$

Plate Clamped on Two Opposite Edges and Simply Supported on the Other Two Edges. A solution for this problem has been given by Iguchi (1938) for the general case, and by Leggett (1941) for the case of the square plate. Cook and Rockey (1963) later obtained solutions considering the antisymmetric buckling mode which was not considered by Iguchi. The expressions below were obtained by fitting a polynomial equation to the Cook and Rockey results as shown in Fig. 2.36 of the book by Bulson (1970). For long edges clamped

$$k_s = \begin{cases} \dfrac{8.98}{\alpha^2} + 5.61 - 1.99\alpha & \text{for } \alpha \le 1 \\ 8.98 + \dfrac{5.61}{\alpha^2} - \dfrac{1.99}{\alpha^3} & \text{for } \alpha \ge 1 \end{cases}$$

and for short edges clamped,

$$k_s = \begin{cases} \dfrac{5.34}{\alpha^2} + \dfrac{2.31}{\alpha} - 3.44 + 8.39\alpha & \text{for } \alpha \le 1 \\ 5.34 + \dfrac{2.31}{\alpha} - \dfrac{3.44}{\alpha^2} + \dfrac{8.39}{\alpha^3} & \text{for } \alpha \ge 1 \end{cases}$$

Curves for α ≥ 1 are plotted in Fig. 4.6. Tension and compression stresses exist in the plate, equal in magnitude to the shear stress and inclined at 45°. The destabilizing influence of compressive stresses is resisted by tensile stresses in the perpendicular direction, often referred to as ‘tension field action' in the literature. Unlike the case of edge compression, the buckling mode is composed of a combination of several waveforms and this is part of the difficulty in the buckling analysis for shear.

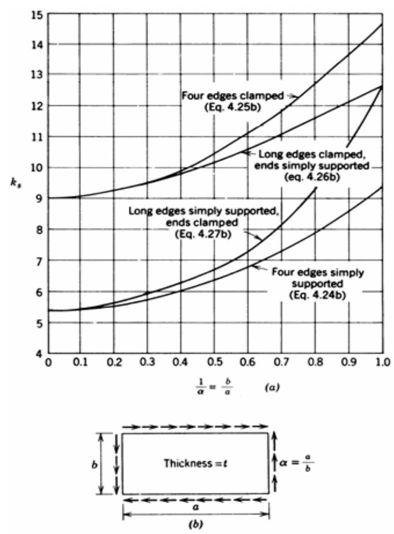


Figure 4. 8The variation of the buckling coefficient kτ

4.8 Inelastic Buckling

Columns will buckle when a critical buckling stress is reached. When this stress is in the elastic range of the material, it is called elastic buckling. Most long and slender columns will tend to buckle in this stress range. However, when a column is short and stocky, the critical buckling stress may be greater than the yield stress of the material. In this region the material no longer behaves elastically. Therefore, buckling of such regions is called inelastic buckling.[14]

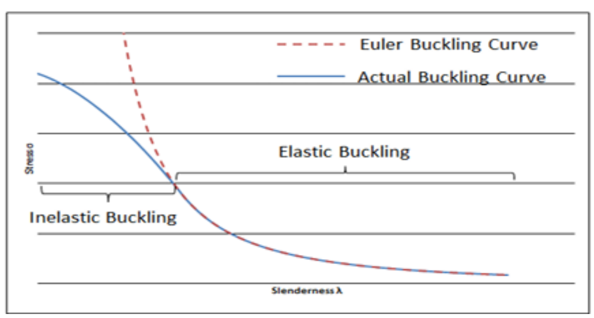


Figure 4. 9The Inelastic Buckling Curves

Inelastic buckling of columns is only due to material yielding. Therefore, it has a much less predictable shape than elastic buckling.

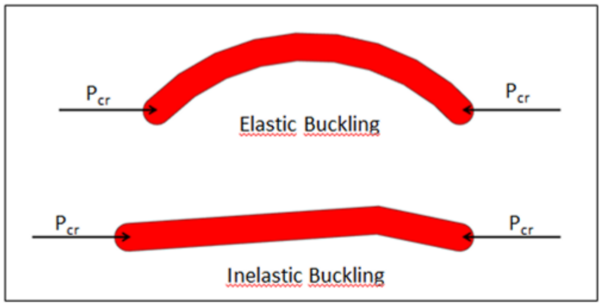


Figure 4. 10Illustration of difference in buckling shape for elastic and inelastic buckling

4.9 Inelastic buckling, post-buckling, and strength of flat plates

The elastic critical plate buckling stresses, or corresponding plate buckling coefficients (k's), provided in the previous section represent an important benchmark for understanding the behaviour of thin plates. However, such elastic critical buckling stresses do not directly indicate the actual behaviour that may occur in such a thin plate. In thin plates loaded to failure material and geometric effects complicate the response. It is common, though artificial, to use the elastic critical buckling stress as a benchmark for delineating different forms of plate buckling: if material yielding occurs prior to the elastic critical buckling stress this is known as inelastic buckling; strength at magnitudes greater than the elastic critical buckling stress, and the associated deformations that occur under such loading, are referred to as post-buckling and may be either elastic or inelastic. Finally, ultimate strength refers to the maximum load the plate may carry, typically independent of deformation, which may indeed be quite large. Actual plate response under load is more complicated than the simple notions of inelastic buckling and post-buckling, this is due in part to unavoidable imperfections. In an imperfect plate out-of-plane deformations begin immediately upon loading, such deformations lead to second order (geometrically nonlinear) forces (and strains) that must be accounted for throughout the loading/deformation, and thus the notions of buckling and post-buckling are not definitively distinct. Under load the stress field response of a thin plate is complicated and varies along the length, across the width, and through the thickness of the plate. Residual stresses that may exist in the plate further complicate the response. A plate with an applied stress well below the elastic critical plate buckling stress may still have portions of the plate yielding; thus, determining a definitive regime where a plate enters inelastic buckling is difficult. For gradual yielding metals (e.g., aluminum, stainless steel) the distinction between elastic and inelastic buckling becomes even more difficult. Currently, inelastic buckling, post-buckling, and the strength of thin plates (and plate assemblages such as Fig. 4.1) are most robustly examined through the use of numerical methods such as finite element analysis. Finite element models for stability critical structures are discussed further in Chapter 21, but key considerations for plates include: the manner in which shear in the plate is handled (namely Kirchoff vs. Mindlin plate theory), the material stress-strain relation including residual stresses and strains, the yield criterion (von Mises is by far the most common in metals), the hardening law (isotropic hardening is the most common for static loading, but is inadequate if large strain reversals are present), magnitude and distribution of geometric imperfections, inclusion of higher-order strain terms in the development of the plate stiffness, enforcement of equilibrium on the deformed geometry, details of the boundary conditions, and for finite elements the order of the elements and the discretization of the plate both in terms of element density and element aspect ratio. Finite element analysis is not the only method able to provide post-buckling and collapse analysis of plates, finite strip (Bradford and Azhari 1995; Kwon and Hancock 1991; Lau and Hancock 1986; Lau and Hancock 1989), and recently generalized beam theory (Goncalves and Camotim 2007; Silvestre and Camotim 2002) have proven to be able to provide reliable solutions. For typical design, fully nonlinear numerical collapse analysis of thin plates remains too involved of a task; in this situation one turns to classical and semi-empirical approaches. These design approximations are the focus of this section. In particular, the effective width method, has wide use as an approximate technique for determining ultimate strength of plates that accounts for inelastic buckling and post-buckling and is discussed in detail.

The notion of "inelastic buckling" is an attempt to extend the elastic critical buckling approximations of Section 4.2 to situations where material yielding has already occurred. Bleich (1952) generalized the expression for the critical stress of a flat plate under uniform compressive stress in either the elastic or inelastic range in the following manner:

$$\sigma_c = k\frac{\pi^2 E\sqrt{\eta}}{12(1-v^2)(b/t)^2}$$

in which η = Et/E. This modification of Eq. 4.1 to adapt it to a stress higher than the proportional limit is a conservative approximation to the solution of a complex problem that involves a continuous updating of the constitutive relations depending on the axial stress carried (Stowell, 1948; Bijlaard, 1949, 1950). In combined loading the work of Stowell (1949) and Peters (1954) suggest that the inelastic buckling interaction is not the same as the elastic buckling interaction.

Under combined compressive and shear stress for loads applied in constant ratio Peters found that a circular stress-ratio interaction formula as expressed by bellow was conservative and agreed better with test results than Eq. 4.7 which was provided for elastic buckling interaction.

$$\left(\frac{\sigma_c}{\sigma_c^*}\right)^2+\left(\frac{\tau_c}{\tau_c^*}\right)^2=1$$

4.9.1 post-buckling

Post-buckling of plates may readily be understood through an analogy to a simple grillage model, as shown in Fig. 3.9. In the grillage model the continuous plate is replaced by vertical columns and horizontal ties. Under edge loading the vertical columns will buckle, if they were not connected to the ties they would buckle at the same load and no post-buckling reserve would exist. However, the ties are stretched as the columns buckle outward, thus restraining the motion and providing post-buckling reserve. In an actual plate the tension in the transverse ties is represented by membrane tension and shear. Note also that the columns nearer to the supported edge are restrained by the ties more than those in the middle. This too occurs in a real plate, as more of the longitudinal membrane compression is carried near the edges of the plate than in the centre. Thus, the grillage model provides a working analogy for both the source of the post-buckling reserve and its most important result, re-distribution of longitudinal membrane stresses.

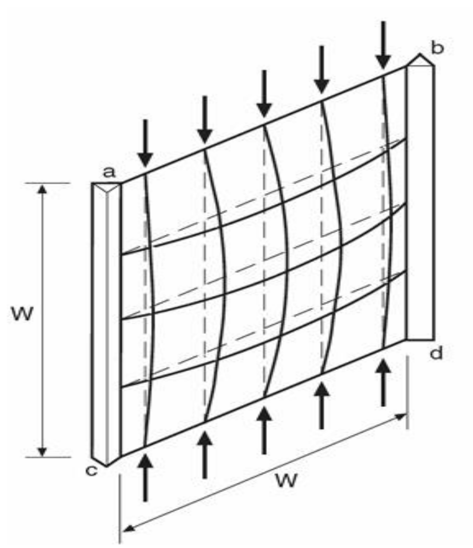


Figure 4. 11Simple model for post-buckling of a flat plate in uniform compression (after AISI 2007)

4.9.2 Post-buckling behaviour and effective width

- **Post-buckling Behaviour**

Consider a rectangular plate with all four edges simply supported and subjected to uniform compression along x-direction (Fig. 3.10). When the compressive stress equals the critical buckling stress σcr, the central part of the plate, such as the strip AB, buckles. But the edges parallel to the x-axis cannot deflect in the z-direction and so the strips closer to these edges continue to carry the load without any instability. Therefore, the stress distribution across the width of the plate in the post-buckling range becomes non-uniform with the outer strips carrying more stress than the inner strips as shown in Fig. 5(a). However, as described before, the transverse strips such as CD in Fig. 1 continue to stretch and support the longitudinal strips. This ensures the stability of the plate in the post-buckling range. Increasing the axial displacement of the plate will cause an increase in the lateral displacement. When the edge stresses approach and equal the yield stress of the material, the plate deflection would be very large and the plate, eventually, can be con9sidered to have failed when the stresses in the edge strips reach the yield stress of the material. [16]

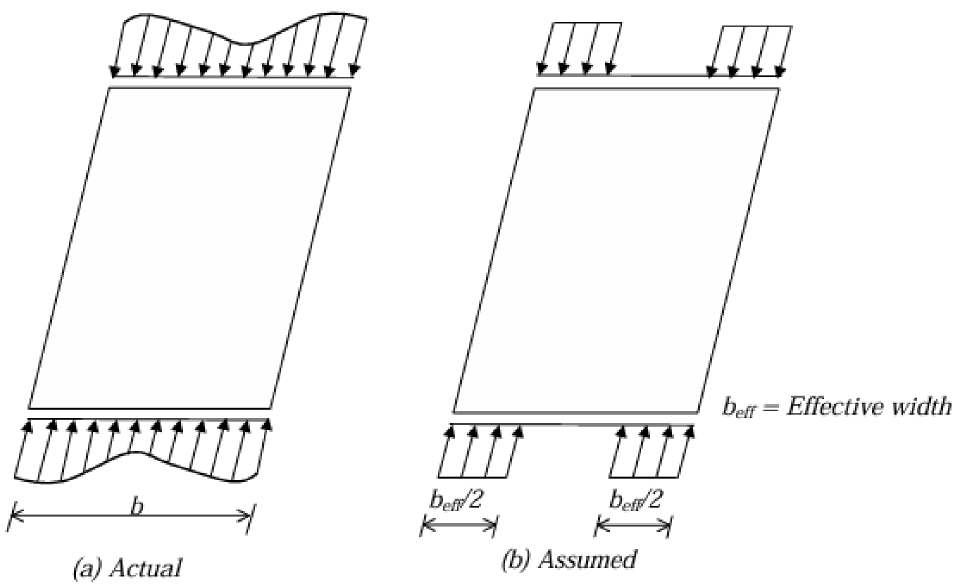


Figure 4. 12Actual and Assumed Stress Distribution in the Post-buckling

4.9.2.2 Effective Width:

To calculate the load carrying capacity of the plate in the post-buckling range, the concept of effective width is used. The concept was first proposed by von Karman. He realized that as the plate is loaded beyond its elastic buckling load, the central part such as strip AB deflects thereby shedding the load to the edge strips. Therefore, the non-uniform stress distribution across the width of the buckled plate, can be replaced by a uniform stress block of stress equal to that at the edges, over a width of $b_{eff}/2$ on either side where b_{eff} is called the effective width of the plate. This effective width can be calculated by equating the non-uniform stress blocks and the uniform stress blocks. The shape of the non-uniform stress block depends on the load and support conditions. Therefore, a number of formulae are available for calculating the effective width, each catering to a particular geometry of the plate. For the plate simply supported on all four sides, as the load is increased beyond the critical buckling load, the stress block becomes more and more non-uniform. When the stress at the outer strips reaches the yield stress, the corresponding effective width can be calculated using Winter's formula

$$\frac{b_{eff}}{b} = \sqrt{\left(\frac{\sigma_{cr}}{f_y}\right)}\left[1 - 0.22\sqrt{\left(\frac{\sigma_{cr}}{f_y}\right)}\right]$$

The yield stress fy, multiplied by the effective width gives the ultimate strength of the plate approximately. [16]

4.11. Buckling of web plates in shear

Rectangular plates loaded in shear such as web plates in a plate girder, also tend to buckle. Consider a plate loaded in shear in its own plane as shown in Fig.11. A square element in the plate, whose edges are oriented at 45° to the plate edges, experiences tensile stresses on two opposite edges and compressive stresses on the other two edges. The compressive stress can cause local buckling and as a result the plate develops waves perpendicular to them.

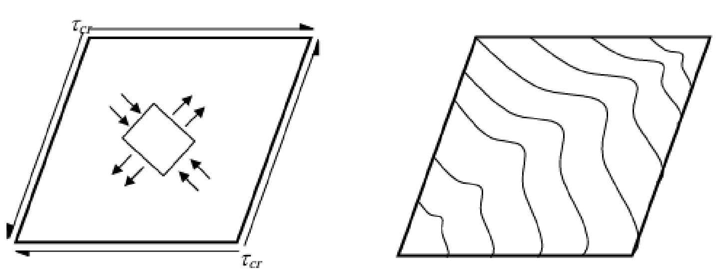


Figure 4. 13Shear buckling of a plate

The critical shear stress at which this form of buckling occurs is given by the same formula as that for plate buckling under compression, except that the value for the buckling coefficient k is different. The buckling coefficient varies with the aspect ratio a/b and increases from 5.34 for an infinitely long panel to 9.34 for a square panel. The values given in Table 1 are for the square panels.[17]

4.12 Summary Classic theory of Elastic critical shear stresses τcr

To account for the complex behaviour of steel structures, both geometric and material nonlinearities should generally be considered. Traditionally, any limit state that is not included in the structural analysis must be accounted for using appropriate design checks. For example, a linear (first-order) analysis does not capture member buckling; thus, a corresponding member buckling check is required. Advanced analysis reduced the number of required design checks by incorporating various limit states into the analysis itself.

Similar to plates subjected to uni-axial compressions, plates under edge shear forces exhibit buckling behaviour when the applied shear force reaches the shear buckling load. As shown in Figure 3.???, if a rectangular plate with length 'a', width 'h' and thickness 't' is subjected to uniform shear loads, the critical shear buckling strength of such plate can be formulated as the following (Timoshenko and Gere, 1961):

$$\tau_{cr}^{solid} = k_v^{solid} \frac{\pi^2 E}{12(1-\nu^2)(h/t)^2}$$

For plates with all edges simply supported

$$k_v^{solid} = 5.34 + \frac{4}{(a/h)^2}, \quad a/h \geq 1$$

where 'E' and 'v' are Young's modulus and Poisson's ratio, respectively. ' **τcr** , is the critical buckling stress of plates under shear loads.

τcr is the non-dimensional shear-buckling coefficient, which depends on the aspect ratio and the boundary conditions of plates. The above Equa. is the approximate relationship between the shear-buckling coefficient 'k~olid, and the aspect ratio 'a!h' for simply supported plates subjected to shear loads. 'a' is the larger dimension of plate sides and 'h' is the smaller. This will make sure that 'a!h' is always greater than 1. For example, for a simply supported square plate under uniform edge shear, the shear-buckling coefficient 'k τ , has the value of 9.34. From Equation below it can be seen that for a plate with given material properties, specific loading and boundary conditions, the critical buckling load depends on the slenderness ratio (hit) of the plate. As 'hit' decreases, "t~~lid, can be as high as the shear yield stress ('ty), or even higher. The limiting 'hit' value is when "t~~lid, is equal to "ty'. Thus, when substituting' 'ty' into Equation 4.1.1, the limiting 'hit' ratio is:

$$\frac{h}{t} = \sqrt{k_v^{solid} \frac{\pi^2 E}{12(1-\nu^2)\tau_y}}$$

Therefore, when the 'hit' ratio is below the limiting value, the plate will yield first before it buckles. Substituting the material properties for cold-formed steel, that is v=O.3 and 'ty = Fy /.J3 MPa, Equation 4.2 becomes:

$$\frac{h}{t} = \sqrt{k_v^{solid} \frac{\pi^2 E}{12(1-\nu^2)(F_y/\sqrt{3})}} \approx 1.25\sqrt{E k_v^{solid}/F_y}$$

Therefore, for a steel plate with material properties of v=0.3, and which possess on a sharp-yielding stress-strain relationship, when the slenderness ratio (hit) is greater than 1.25 ~Ek~Olid IFy , the plate will buckle before it yields.

However, for cold-formed steel with a gradual-yielding stress-strain relationship discussed in Section 2.2, during the gradually yielding process, the slope of the stress-strain curve changes continuously, which means the modulus of elasticity (E) also changes continuously. The theoretical value of critical shear buckling stress thus should be reduced according to the reduction in modulus of elasticity (E).

Worked example of the analytical evaluation of τ_{cr}:

$$\tau_{cr}^{solid} = k_v^{solid} \frac{\pi^2 E}{12(1-\nu^2)(h/t)^2}$$

$$k_v^{solid} = 5.34 + \frac{4}{(a/h)^2}, \quad a/h \geq 1$$

$$K_\tau = 5.34+(4/(a/h)^2) = 9.34$$

E=210000Mpa

V=0.3

a=1000mm; h=1000mm ; t=10mm

τ_{cr} = 177.27Mpa

CHAPTER FIVE

FINITE ELEMENT METHOD

FOR BUCKLING ANALYSIS

5.1. General

The task of a structural designer is to create steel-plated structures capable of enduring a combination of loads and deformations, spanning from regular usage to unexpected and extreme circumstances. Their objective is to design a structure that remains resilient and functional over its anticipated lifetime, meeting the demands imposed by its service requirements.

The finite element method (FEM) has become essential for predicting and simulating the physical behaviour of complex engineering systems. The commercial finite element analysis (FEA) packages have gained common acceptance among engineers in industry and researchers at universities and government laboratories. Therefore, academic engineering departments include graduate or undergraduate high-level courses that cover not only the theory of FEM but also its applications using the commercially available FEA programs. [18]

According to Torstenfelt (2007), the method has been described in several different ways but in beginning, it was described as a Rayleigh-Ritz method and later as a tool for solving partial differential equations on a weak formulation. The Finite Element Method makes it possible to simulate different scenarios during the development process of a product and therefore it helps the companies to save a lot of time and save money by reducing the experimental testing

Buckling analysis can be performed by Finite Element Method (FEM) packages. However, these analyses usually cost a lot of engineering time. This because for the analysis to make any sense, it must often be done on individual plate fields. This brings a lot of uncertainties about boundary conditions into the calculation. Therefore, buckling analysis in FEM is very labour intensive and costly. [19]

Buckling Analysis is an FEA routine that can solve all the difficult buckling problems that cannot be solved by hand calculations. Linear Buckling (LBA) is the most common Buckling Analysis. The nonlinear approach (GNLA and MNLA), on the other hand, offers more robust solutions than Linear Buckling Analysis. [20]

5.2 Some of FEM applications

Broadly speaking, the Finite Element Method (FEM) and its practical application, often known as Finite Element Analysis (FEA) is a numerical technique for finding approximate solutions of partial differential equations as well as integral equations. FEM has been applied to a number of physical problems, where the governing differential equations are available. In addition, FEA often are the only way to get an answer in the case of particular problems, such as, for example, the mechanical behaviour of a system subjected to extreme loading conditions which are impossible to duplicate in an experiment (ABAQUS, 2006).

The Majority of applications of FEM are in the realm of solid mechanics and fluid mechanics. Recently, FEM has also been used in electrical and electromagnetic problems as well as in bioengineering problems.

Categories of problems that can be solved using FEM can be divided into equilibrium, eigenvalue and transient problems. The equilibrium problems are generally steady-state (stable) problems such as determination of stress and displacement in solid mechanics-related problems, determination of temperature distribution in thermal problems, estimation of potential, velocity and pressure in fluid mechanics problems. The eigenvalue problems are also steady state in nature but include estimation of vibration and natural frequencies in solids and fluids. In the transient problems, FEM is used in propagation problems of continuum mechanics with respect to time.

Structural mechanics and aerospace engineering: FEM applications in equilibrium conditions include analysis of beams, plates, shell structures, stress and torsion analysis of various kind structures. On the other hand, eigenvalue analyses include stability of structures, viscoelastic damping, vibrations and natural frequency analysis of structures. The transient or propagation analysis using FEM includes dynamic response of structures to periodic loads, viscoelastic and thermo-elastic problems and stress wave propagation. [21]

5.3 Concept of FEA

5.3.1 General

The Finite Element Analysis (FEA) method was originally introduced by Turner et al. (1956). It, is a powerful computational technique for approximate solutions to a variety of "real-world" engineering problems having complex domains subjected to general boundary conditions. FEA has become an essential step in the design or modelling of a physical phenomenon in various engineering disciplines.

One of the key processes in finite element analysis is to build material models. When finite element models are built-up created, a variety of material models corresponding to different materials must adopted, because under the same geometry and loadings and boundary conditions, different materials may have unique behaviours. [22]

The method essentially consists of assuming the piecewise continuous function for the solution and obtaining the parameters of the functions in a manner that reduces the error in the solution. In general, a model may be defined as a simplified representation of reality. The degree of simplification depends on the problem peculiarities and on the particular aspects to investigate. In a macroscopic phenomenological perspective, the majority of the surrounding objects and systems as continuous. [23]

A physical phenomenon usually occurs in a continuum of matter (solid, liquid, or gas) involving several field variables. The field variables vary from point to point, thus possessing an infinite number of solutions in the domain. Within the scope of this book, a continuum with a known boundary is called a domain.

5.3.2 Basis of FEM

The basis of FEA relies on the decomposition of the domain into a finite number of subdomains (elements) for which the systematic approximate solution is constructed by applying the variational or weighted residual methods. In effect, FEA reduces the problem to that of a finite number of unknowns by dividing the domain into elements and by expressing the unknown field variable in terms of the assumed approximating functions within each element. These functions (also called interpolation functions) are defined in terms of the values of the field variables at specific points, referred to as nodes. Nodes are usually located along the element boundaries, and they connect adjacent elements.

The ability to discretize the irregular domains with finite elements makes the method a valuable and practical analysis tool for the solution of boundary, initial, and eigenvalue problems arising in various engineering disciplines. Since its inceptio4n, many technical papers and books have appeared on the development and application of FEA. The books by Desai and Abel (1971), Oden (1972), Gallagher (1975), Huebner (1975), Bathe and Wilson (1976), Ziekiewicz (1977), Cook (1981), and Bathe (1996) have influenced the current state of FEA. Representative common engineering problems and their corresponding FEA discretization are illustrated in Fig. 5.1

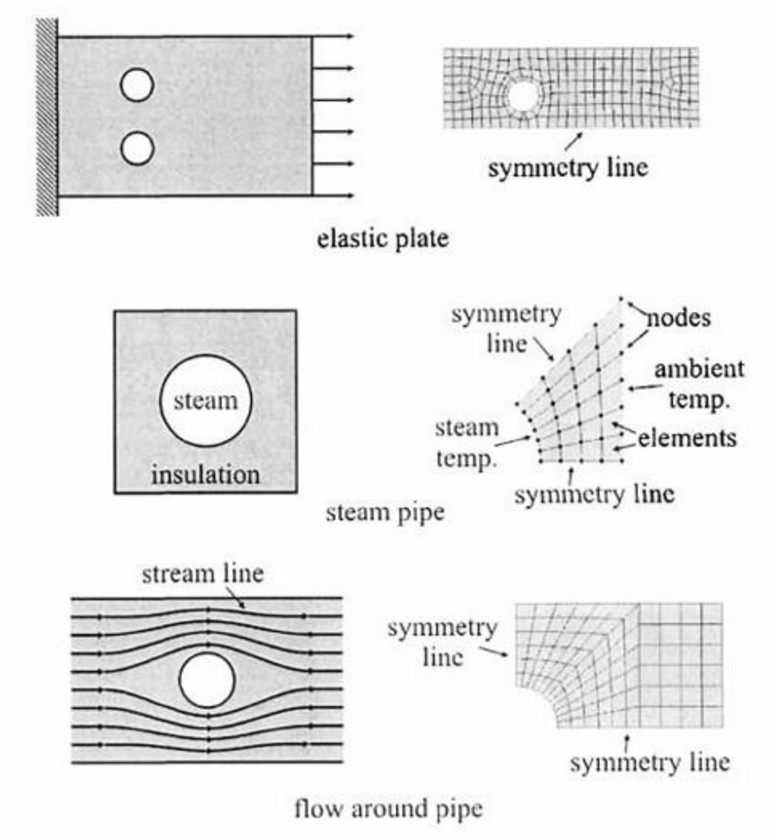


Figure 5. 1FEA representation of practical engineering problems

5.3.3 Steps of FEA

The finite element analysis method involves major steps to be followed:

• Discretization of the domain into a finite number of sub-domains (elements).

• Selection of interpolation functions.

• Development of the element matrix for the sub-domain (element).

• Assembly of the element matrices for each sub-domain to obtain the global matrix for the entire domain,

• Imposition of the boundary conditions.

• Solution of equations.

• Additional computations (if desired).

5.4. Nodes

As shown in Fig. 5.2, the transformation of the practical engineering problem to a mathematical representation is achieved by discretizing the domain of interest into elements (subdomains). These elements are connected to each other by their "common" nodes. A node specifies the coordinate location in space where degrees of freedom and actions of the physical problem exist. The nodal unknown(s) in the matrix system of equations represents one (or more) of the primary field variables. Nodal variables assigned to an element are called the degrees of freedom of the element.

The common nodes shown in Fig. 5.2 provide continuity for the nodal variables (degrees of freedom). Degrees of freedom (DOF) of a node are dictated by the physical nature of the problem and the element type. Table 1.1 presents the DOF and corresponding "forces" used in FEA for different physical problems. [18]

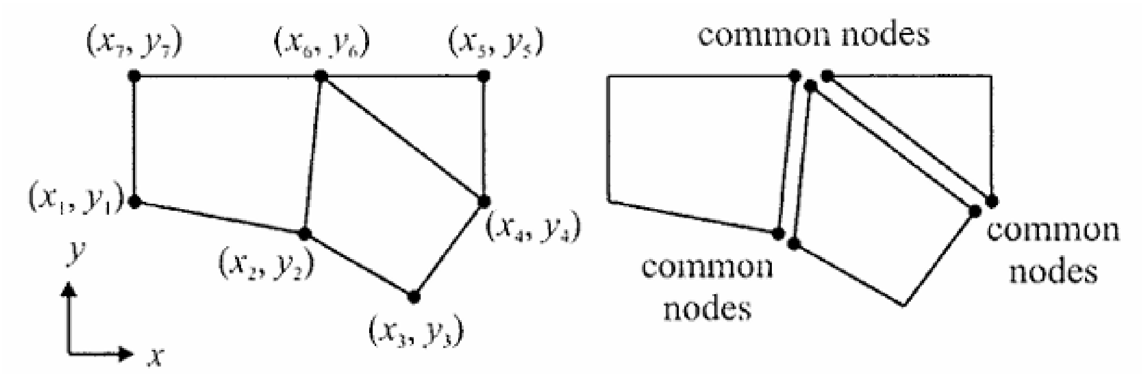


Figure 5. 2Common nodes with the corresponding degree of freedom

Table 5. 1Degrees of freedom and force vectors in FEA for different engineering disciplines.

Discipline	DOF	Force Vector
Structural/solids	Displacement	Mechanical forces
Heat conduction	Temperature	Heat flux
Acoustic fluid	Displacement potential	Particle velocity
Potential flow	Pressure	Particle velocity
General flows	Velocity	Fluxes
Electrostatics	Electric potential	Charge density
Magnetostatics	Magnetic potential	Magnetic intensity

5.5. Elements

Every type of element itself can have various properties, which are strongly dependent on the element formulation. As shown in Fig.5.4, there are four main groups of elements:

- Zero-dimensional elements
- One-dimensional elements

- Two-dimensional elements
- Three-dimensional elements

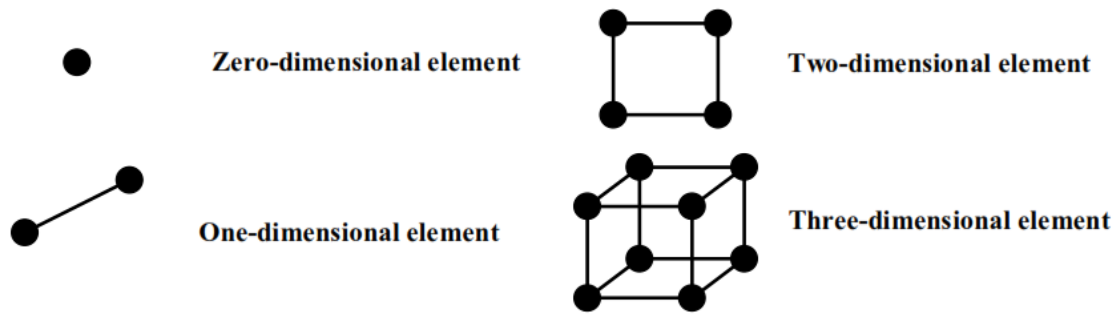


Figure 5. 3Various types of elements that is available in commercial FE software.

Depending on the geometry and the physical nature of the problem, the domain of interest can be discretized by employing line, area, or volume elements. Some of the common elements in FEA are shown in Fig. 5.5. Each element, identified by an element number, is defined by a specific sequence of global node numbers. The specific sequence (usually counter clockwise) is based on the node numbering at the element level. The node numbering sequence for the elements shown in Fig. 5.5.

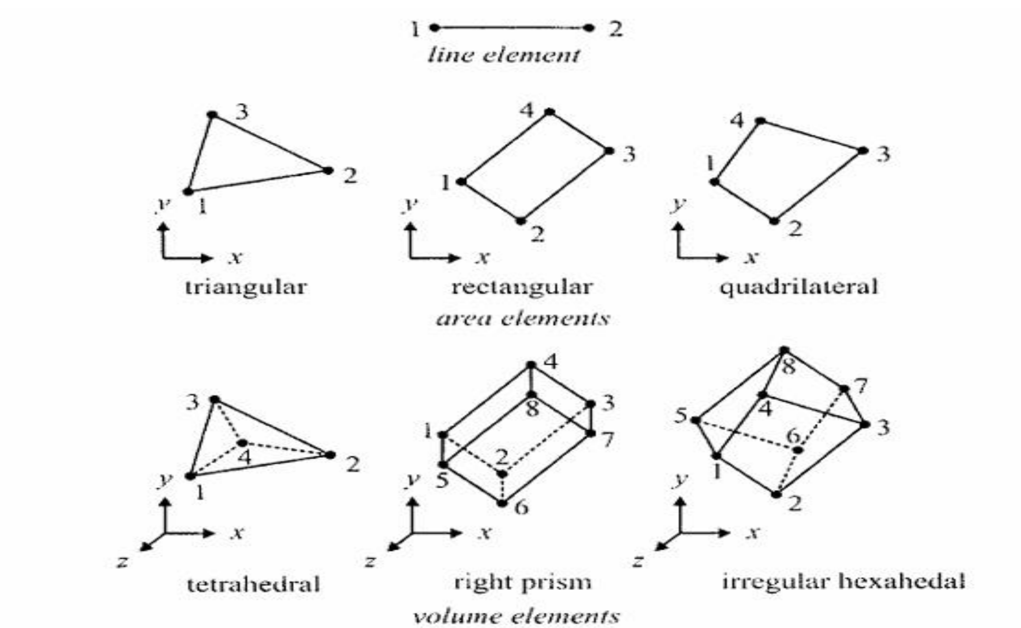


Figure 5. 4Description of line, area, and volume elements with node numbers at the element level.

5.6 Approaching methods used in structural engineering

5.6.1 General

It is possible to obtain exact analytical solutions for few simplified engineering problems. The exact solution can be obtained by direct integration of the concerned differential equations. Generally, the direct integration is made possible by:

(i) separation of variables;

(ii) similarity solutions; or

(iii) Fourier and Laplace transformations. Since most engineering problems are complex in nature, number of problems having exact solutions is severely limited. Hence, approximate solutions are generally sought.

Generally used approaching methods are as follows.

1. Perturbation Techniques
2. Power Series Solutions
5. Probability Methods
5. Finite Difference Method (FDM)
5. Method of Weighted Residuals (MWR)
6. Rayleigh–Ritz Method
7. Finite Element Method (FEM)

Usefulness of the perturbation method is limited because they are applicable only when the nonlinear terms in the equation are small in relation to the linear terms. The power series method, on the other hand, has been employed with some success. However, as the method requires generation of a coefficient for each term in the series, it is relatively tedious to apply. Also, it is difficult to prove that the series converges. The probability methods are used if the desired variables in the problem concerned are statistical parameters. The probability method is used to obtain statistical estimate of variables by the process of random sampling. The first three methods briefly mentioned above have limited use in engineering problems.

Due to wide applicability and simplicity, the most commonly used approximating methods include FDM, MWR, Ritz Method and FEM. Out of various approximating methods, FDM is the most popular method due to its simplicity and wide applications. As discussed earlier, FDM has some merits and demerits compared to FEM. Incorporation of variation of physical properties of the medium in the solution domain is a difficult task in application of FDM to engineering problems. A large number of text books are available describing various applications of FDM such as Forsythe and Wasow (1960), Mitchell and Griffiths (1980), Davis (1986) and Thomas (1995). The MWR, Ritz method and FEM are related methodologies. Hence, discussion here is limited only to methods of weighted residual and Raleigh–Ritz, which are important in obtaining element equations in finite element analysis [21]

5.6.2 Rayleigh–Ritz method

Form of the unknown solution is assumed in terms of known functions (trial functions) with unknown adjustable parameters in the Rayleigh–Ritz Method. From the family of trial functions, the function that renders the functional stationary are selected and substituted into the functional, which is a function of the functions. Thus, the functional is expressed in terms of the adjustable parameters.

The resulting functional is differentiated with respect to each parameter and the resulting equation is set equal to zero. If there are n unknown parameters in the functional, there will be n simultaneous equations to be solved for the parameters. Thus, the best solution is obtained from the family of assumed solutions. Accuracy depends on the trial functions chosen. Generally, the trial function is constructed from the polynomials of successively increasing degree. It may be noted that the method requires the knowledge of the functional.

Thus, the main aim of Rayleigh–Ritz method is to replace the problem of finding the minima and maxima of integrals by finding the minima and maxima of functions of several variables. For example, consider search of a function L(x) that will extremize certain given functional (L). As mentioned, L(x) can be approximated by a linear combination of suitably chosen coordinate functions c1(x), c2(x),………cn(x). Then L(x) can be written as

$$L(x) = g_1 c_1(x) + g_2 c_2(x) + \dots\dots + g_n c_n(x)$$

where gi are unknown constants to be found. Since each of ci(x) is an admissible function, the functional I(L) become a function of g. By taking the differential of the function, unknown g can be determined as follows.

$$\frac{\partial I}{\partial g_j} = 0 \quad (j = 1, 2, 3, \dots . n)$$

Using Eq. (2.36), n algebraic equations are obtained from which the unknown constants gj are determined. Method has been explained in the following example. [21]

5.7 Finite Element Method for Structural Analysis

5.7.1 Introduction

Most conventional methods are cumbersome, uneconomical and time consuming to apply to many engineering applications, due to introduction of new materials such as composites, fibre reinforced materials etc. and complicated geometries to be modelled. FEM has become one of the most popular numerical methods to deal with complex geometries and material properties with the availability of high-speed digital computers. Generally, FEM can be considered as a variant of Rayleigh–Ritz method in combination with the variational principle applied to continuum mechanics. In the Rayleigh–Ritz method, trial functions are used to represent the entire domain. On the other hand, the trial functions used in FEM are approximated over a sub-domain, referred to as an element. Hence, FEM can better handle any material inhomogeneity within the domain and can model any irregular geometry by using varieties of shapes of elements. General steps used in FEM are explained in the following section.[21]

5.7.2. Steps for performing FEA

Even though FEM procedure may vary depending upon problem and approach used for solution, general steps in the FEM analysis remain more or less the same. The first step to the solution of the considered problem is to define the problem in detail with the available data. Based on the problem statement and available data, a mathematical model is developed defining the geometry of the problem, material properties, assumptions and simplifications used, governing equations, boundary and initial conditions.

In this section, the general steps used in the solution using the finite element method are described with explanations. [21]

Step 1: Discretize and select element types

Step 2: Select approximation functions

Step 3: Define the gradients of the unknown quantity and constitutive relationships

Step 4: Derive element equations

Step 5: Assemble element equations to obtain the global or total equation and introduce boundary conditions

Step 6: Solve for the unknown DOF (primary unknowns)

Step 7: Solve for secondary quantities

Step 8: Interpret the results

5.7.3. Different approaches used in FEM

There are three main approaches to constructing an approximate solution based on the concept of FEA:

Direct Approach: This approach is used for relatively simple problems, and it usually serves as a means to explain the concept of FEA and its important steps

Weighted Residuals: This is a versatile method, allowing the application of FEA to problems who's functional cannot be constructed. This approach directly utilizes the governing differential equations, such as those of heat transfer and fluid mechanics

Variational Approach: This approach relies on the calculus of variations, which involves extremizing a functional. This functional corresponds to the potential energy in structural mechanics

In matrix notation, the global system of equations can be cast into:

$$Ku = F$$

where K is the system stiffness matrix, u is the vector of unknowns, and F is the force vector. Depending on the nature of the problem, K may be dependent on u, i.e., K = K(u) and F may be time dependent, i.e., F = F(t). [21]

5.8 Sources of nonlinearity (Abaqus 2006)

There are three sources of nonlinearity in structural mechanics simulations:

- Material non-linearity.
- Boundary nonlinearity.
- Geometric nonlinearity.

Resources of nonlinearities

- **Geometrically non-linear elastic analysis (GNA)**

- **Materially non-linear analysis (MNA)**

Displacements are small but strains are large. Analysis may be performed on the basis of the initial, un-deformed geometry of the structure but the effects of non-elastic irreversible strains must be taken into account. This analysis is also called 1st order plastic analysis.

5.8.1 Material nonlinearity

This type of nonlinearity is probably the one that you are most familiar with. Most metals have a fairly linear stress/strain relationship at low strain values, but at higher strains the material yields, at which point the response becomes nonlinear and irreversible (5.5). Displacements are small but strains are large. Analysis may be performed on the basis of the initial, un-deformed geometry of the structure but the effects of non-elastic irreversible strains must be taken into account. This analysis is also called 1st order plastic analysis.

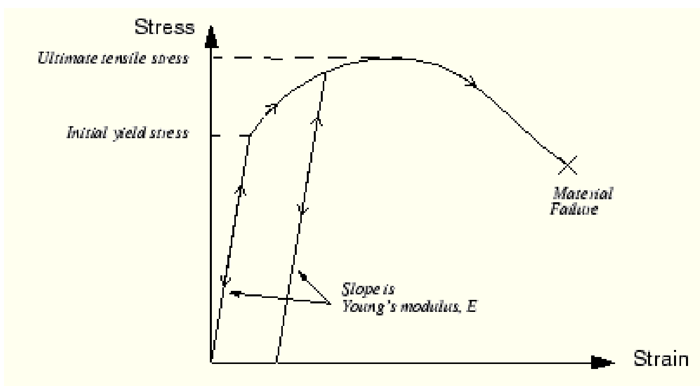


Figure 5. 5Stress-strain curve for an elastic-plastic material under uniaxial tension [Abaqus Manual, 2006].

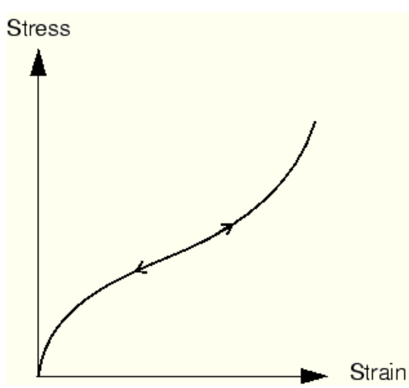


Figure 5. 6Stress-strain curve for a rubber-type material [Abaqus Manual, 2006].

Material nonlinearity may be related to factors other than strain. Strain-rate-dependent material data and material failure are both forms of material nonlinearity. Material properties can also be a function of temperature and other predefined fields.

5.8.2 Boundary nonlinearity

Boundary nonlinearity occurs if the boundary conditions change during the analysis. Consider the cantilever beam, shown in Figure 5.7, that deflects under an applied load until it hits a "stop."

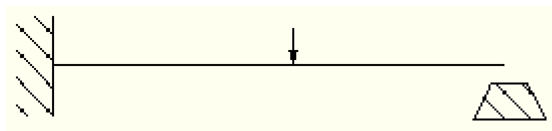


Figure 5. 7Cantilever beam hitting a stop [Abaqus Manual, 2006].

The vertical deflection of the tip is linearly related to the load (if the deflection is small) until it contacts the stop. There is then a sudden change in the boundary condition at the tip of the beam, preventing any further vertical deflection, and so the response of the beam is no longer linear. Boundary nonlinearities are extremely discontinuous: when contact occurs during a simulation, there is a large and instantaneous change in the response of the structure. Another example of boundary nonlinearity is blowing a sheet of material. The sheet expands relatively easily under the applied pressure until it begins to contact. From then on, the pressure must be increased to continue forming the sheet because of the change in boundary conditions. Boundary nonlinearity is covered in Chapter 11, "Contact."

5.8.3 Geometric nonlinearity

The third source of nonlinearity is related to changes in the geometry of the model during the analysis. Due to large displacements, equilibrium is defined in the deformed state of the structure under loading. Material behaviour is elastic. This analysis is also called elastic analysis according to 2nd order theory. It is appropriate for stability investigations up to the buckling load under moderate displacements.

Geometric nonlinearity occurs whenever the magnitude of the displacements affects the response of the structure. This may be caused by:

- Large deflections or rotations.
- "Snap through."

- Initial stresses or load stiffening.

For example, consider a cantilever beam loaded vertically at the tip Figure 5.7

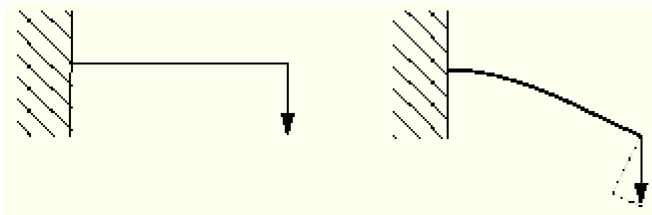


Figure 5. 8Large deflection of a cantilever beam [Abaqus Manual, 2006].

If the tip deflection is small, the analysis can be considered as being approximately linear. However, if the tip deflections are large, the shape of the structure and, hence, its stiffness changes. In addition, if the load does not remain perpendicular to the beam, the action of the load on the structure changes significantly. As the cantilever beam deflects, the load can be resolved into a component perpendicular to the beam and a component acting along the length of the beam. Both of these effects contribute to the nonlinear response of the cantilever beam (i.e., the changing of the beam's stiffness as the load it carries increases).

large deflections and rotations can be expected to have a significant effect on the way that structures carry loads. However, displacements do not necessarily have to be large relative to the dimensions of the structure for geometric nonlinearity to be important. Consider the "snap through" under applied pressure of a large panel with a shallow curve, as shown in Figure 5.9.

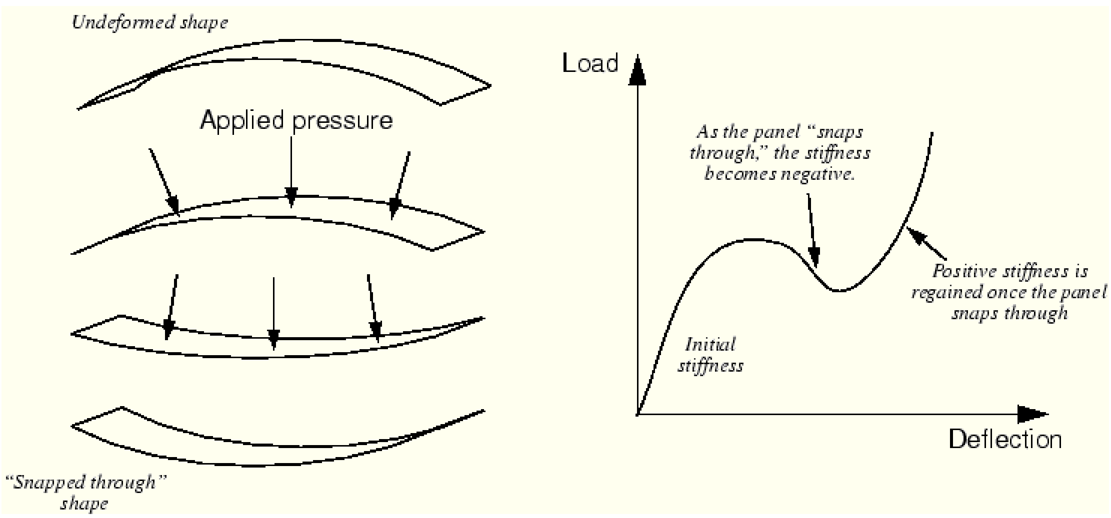


Figure 5. 9Snap-through of a large panel [Abaqus Manual, 2006].

In this example there is a dramatic change in the stiffness of the panel as it deforms. As the panel "snaps through," the stiffness becomes negative. Thus, although the magnitude of the displacements, relative to the panel's dimensions, is quite small, there is significant geometric nonlinearity in the simulation, which must be taken into consideration.

5.9 Buckling and Finite Element Approach

5.9.1 Introduction

Most steel constructions such as cranes, ships, bridges and building frames experience compressive loads and are thus susceptible to buckling. Buckling of construction members is usually a very sudden process without much warning in advance. Therefore, it is important to account for the effect of buckling in the design of a construction.

Buckling analysis can be performed by Finite Element Method (FEM) packages. However, these analyses usually cost a lot of engineering time. This because for the analysis to make any sense, it must often be done on individual plate fields. This brings a lot of uncertainties about boundary conditions into the calculation. Therefore, buckling analysis in FEM is very labour intensive and costly.

A much simpler way to check for buckling is by means of a standard, such as Eurocode 3(EC3). Engineers use standards to validate whether the dimensions of a design will be able to resist all loads acting on a structure. These standards are made up from mathematical models that describe buckling combined with empirical data from real life experiments. The recommendations ensure, when followed correctly, that a structure will be strong enough to resist certain applied loads[14]

5.9.2 Fundamentals of Buckling Analysis using FEM

According to Campbell et al. (1993) and Lee (2001) the procedure from an FE point of view when performing buckling analysis of a structure consists of three steps. First, a linear buckling analysis is performed. Cook et al. (2002) describes the linear buckling analysis as an eigenvalue problem whose smallest root defines the smallest level of external load for which there is bifurcation i.e. where two equilibrium paths intersect.

The eigenvalue problem looks like the following:

$$\left([K] + \lambda_{cr}[K_{\sigma}]_{ref}\right)\{\delta u\} = 0$$

where [K] is the stiffness matrix, λ_{cr} is the eigenvalue, $[K_\sigma]_{ref}$ is the stress stiffness matrix and **{u}.** is the eigenvector.

The magnitude of the eigenvector is normalized in a linear buckling analysis problem meaning that it defines the shape of the eigenmode but not the amplitude. The linear buckling analysis consists of two subcases. In the first subcase the load is applied and in the second subcase, the eigenpairs are calculated using an extraction method e.g. the Lanczos method or the Subspace method.

According to Bathe (2012), both the Subspace method and the Lanczos method are the two widely used schemes which both have different capabilities and attractive properties. The Subspace iteration method is very robust but the speed is dependent on the starting subspace while the Lanczos method is very effective in the case when solving for many eigenpairs.

Generally, the buckling load obtained from a linear buckling analysis is higher than the true buckling load of a structure. The reason is that there are no imperfections included in a linear buckling analysis, which are present when doing experimental testing.

5.9.3 Nonlinear situation

The nonlinear buckling analysis is a simulation procedure that allows for large deformations and geometrical and/or material nonlinearities. This type of analysis is the second step after a linear buckling analysis approach.

There are three main types of nonlinearities:

- *Material nonlinearity*, in which material properties are functions of stress or strain.
- *Contact nonlinearity*, in which gaps between parts can open/close or the contact area between adjacent parts change.
- *Geometric nonlinearity*, in which deformations are large enough to make the small displacement assumption invalid.

In nonlinear analysis, the stiffness matrix **[K]** and/or the load vector {f} in the structural equations, **[K] {u} = {f},** become functions of the displacements, **{u}.**

This makes it no longer possible to solve the systems of equations directly for the displacements by inverting the stiffness matrix. Consequently, a tangent stiffness matrix, $[K]_t$, is created, which includes both the effect of changing geometry as well as stiffening due to stress. The procedure then becomes to solve **[K] {u} = {f}** by use of an incremental scheme, e.g. the Newton-Raphson method,

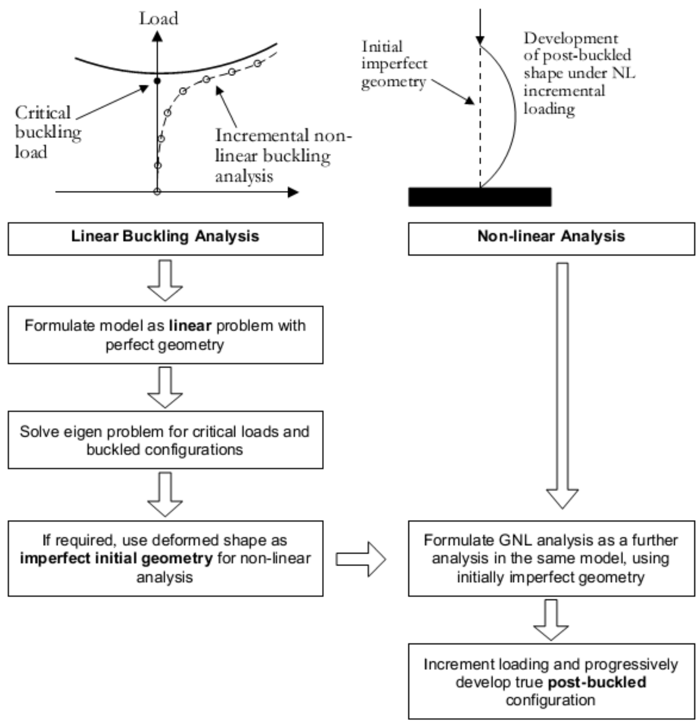


Figure 5. 10Flowchart describing steps to perform for linear and nonlinear buckling analysis [Lusas ver.14]

Once again, according to Cook et al. (2002) the Newton-Raphson method is the most rapidly convergent process and has a quadratic convergence rate and is also the most commonly implemented solution scheme in commercial FE software.

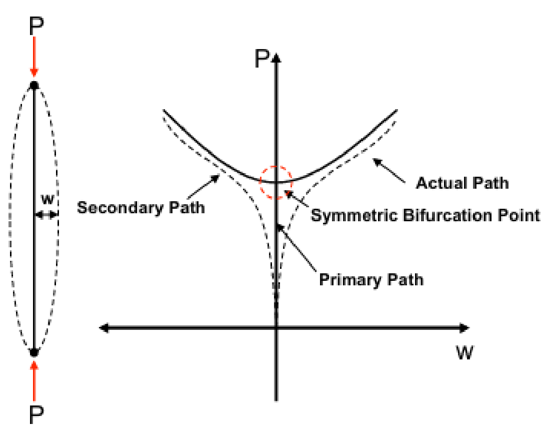


Figure 5. 11The primary and secondary paths for a perfect plate and the actual part obtained due to imperfections.

5.10 Structural modelling and FEM analysis:

5.10.1. Classification of the problem

The first step in the solution of a problem is the identification of the problem itself. Hence, before we can analyse a structure, we must ask ourselves the following questions: Which are the more relevant physical phenomena influencing the structure? Is the problem of static or dynamic nature? Are the kinematics or the material properties linear or non-linear? Which are the key results requested? What is the level of accuracy sought? The answers to these questions are essential for selecting a structural model and the adequate computational method. [21]

5.10.2Conceptual, structural and computational models

Computational methods, such as the FEM, are applied to conceptual models of a real problem, and not to the actual problem itself. Even experimental methods in structural laboratories make use of scale reproductions of the conceptual model chosen (also called physical models) unless the actual structure is tested in real size, which rarely occurs.

A conceptual model can be developed once the physical nature of a problem is clearly understood. In the derivation of a conceptual model, we should aim to exclude superfluous details and include all the relevant features of the problem under consideration so that the model can describe reality with enough accuracy.

A conceptual model for the study of a structure should include all the data necessary for its representation and analysis. Clearly different persons will have different perceptions of reality and, consequently, the conceptual model for the same structure can take a variety of forms.

After selecting a conceptual model of a structure, the next step for the numerical (and analytical) study is the definition of a structural model (sometimes called mathematical model).

A structural model must include three fundamental aspect that is:

The geometric description of the structure by means of its geometrical components (points, lines, surfaces, volumes) ;

The mathematical expression of the basic physical laws governing the behaviour of the structure (i.e. the force-equilibrium equations and the boundary conditions) usually written in terms of differential and/or integral equations;

The specification of the properties of the materials and of the loads acting on the structure.

Clearly the same conceptual model of a structure can be analysed using different structural models depending on the accuracy and/or simplicity sought in the analysis. As an example, a beam can be modelled using the general 3D elasticity theory, the 2D plane stress theory or the simpler beam theory.

Each structural model provides a different set out for the analysis of the actual structure. We should bear in mind that a solution found by starting from an incorrect conceptual or structural model will be a wrong solution, far from correct physical values, even if obtained with the most accurate numerical method.

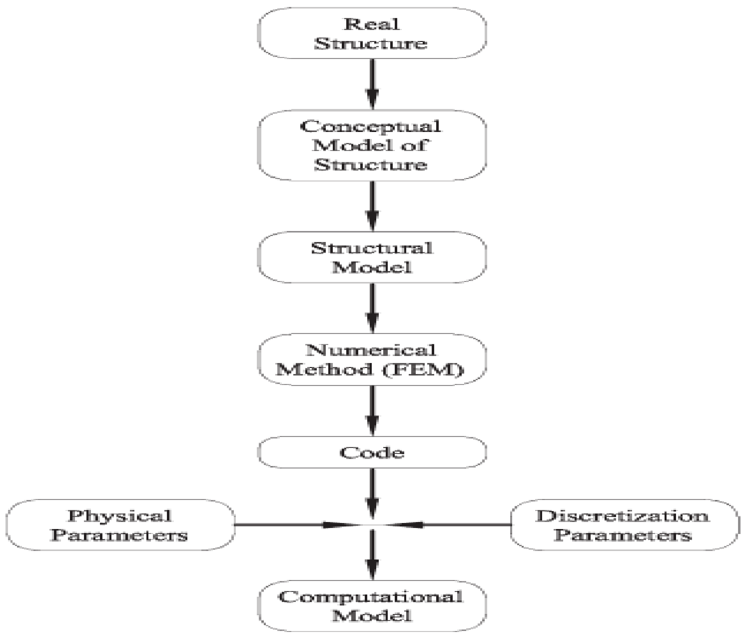


Figure 5. 12The computational model for the analysis of a structure

The next step in the structural analysis sequence is the definition of a numerical method, such as the FEM. The application of the FEM in variably requires its implementation in a computer code. The outcome of this process is what we call a computational model for the analysis of a structure as described in Figure 5.11. [21]

5.11 Buckling and Finite Element Approach

In order to answer this question, usually an imperfection sensitivity analysis has to be carried out. The information obtained in the sensitivity analysis makes it possible to quantify how big influence the imperfection has on the buckling load of the structure. If the load versus out of plane deflection paths in Figure 5.12 is considered, it is possible to see the effect of imperfections in contrast to an ideal buckling path and realize that including imperfections reduces the buckling load of a structure. The path later on converges and coincides with the ideal bifurcation path, in other words the post buckling path is not affected by the imperfections.

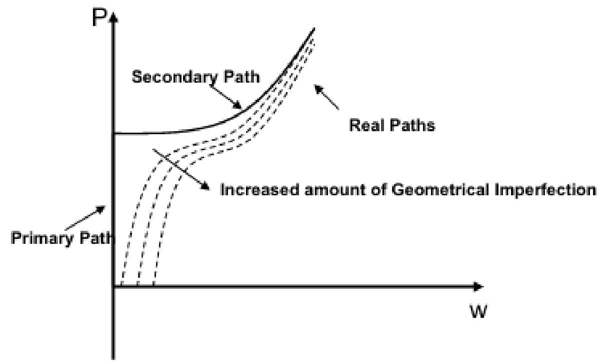


Figure 5. 13The ideal path (primary and secondary) in contrast to the real path obtained due to presence of imperfections.

A third analysis may be carried out for investigating the post buckling of the structure. It can be of interest to see if the structure continues to carry the load after it has reached its critical limit or if it loses all its stiffness and collapses.

When investigating the post buckling, the used iteration scheme is called the RIKS method and is a variant of the Arc Length method1. Unlike the Newton-Raphson method, this method puts in an extra constraint when iterating for the solution and this constraint allows the solver to lower the applied load and find equilibrium. This property of the Riks method makes it possible to trace the behaviour after a limit point is reached, even though that the stiffness matrix is not positive definite.

According to Abaqus manual (Abaqus Version 6.11, 2011), the loading during a step in the Riks solution scheme is always proportional to the current load magnitude:

$$P_{tot} = P_0 + \zeta(P_{ref} - P_0)$$

where P is the dead load, Pref is a reference load and ζ is the Load Proportionality Factor (LPF).

Post buckling can be divided into two different types:

The first type is called stable post buckling and the second is called unstable post buckling. The characteristic of stable post buckling behaviour is when the structure continues to carry the load that it is subjected to and keep its stiffness or in FE terms explained as having a positive definite stiffness matrix, see Figure 5.15.

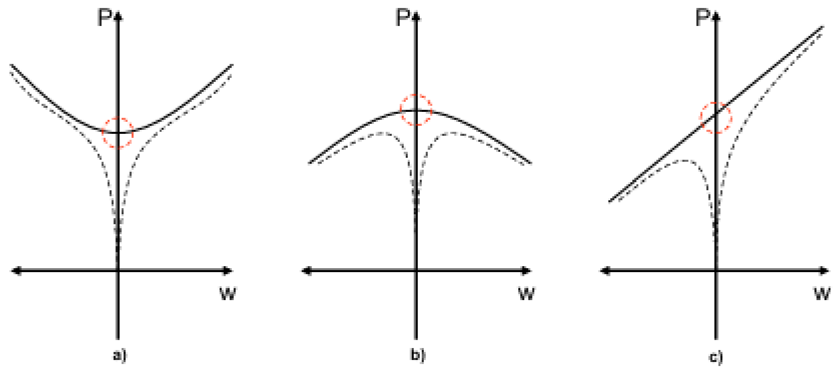


Figure 5. 14Different types of buckling and post buckling paths; a) Stable post buckling, b) Unstable post buckling and c) Stable/Unstable Post buckling

The definition of unstable post buckling is when the structure loses its stiffness and is no more able to carry the same amount of load. This often leads to that the structure starts to undergo very large geometrical changes for decreased or unchanged loading, see Figure 5.13b.

5.11.2 Estimation of Buckling load in Nonlinear Analysis

When performing a nonlinear buckling analysis, the load is applied to the nodes in increments using the LPF and then a force versus out-of-plane displacement curve is obtained. From this curve, it is possible to find out for which load, P, the out-of-plane displacement, w, increases rapidly, i.e. the structure buckles. At this point the load goes from P to P, which is the force utilized when calculating the buckling load of the structure.

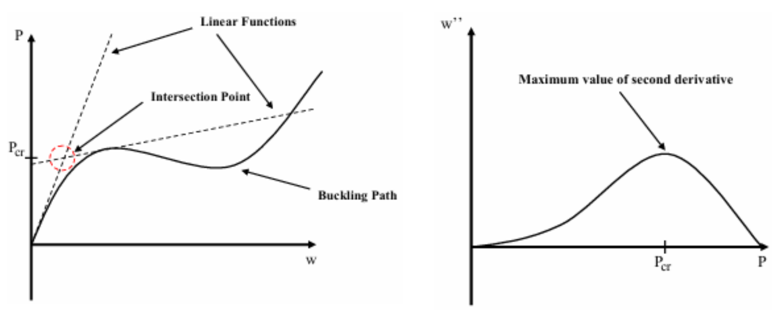


Figure 5. 15The Linearization method (left) and the Second derivative method (right), both utilized for estimating the buckling load in nonlinear buckling analyses

For some load cases or geometries, the buckling process can be somewhat diffuse and not occur so rapidly, which makes it hard to concretize the buckling load. Consequently, there are some methods that can be utilized for estimating the critical buckling load in nonlinear analysis and in this thesis, two different approaches are tested and evaluated against each other.

The following two methods are evaluated:

• The Linearization method, in which the idea is to create two linear functions, one that starts from the origin and follows the slope of the pre-buckling force path versus out of-plane displacement curve and another one that follows the post buckling slope path, i.e. where the path shows a great increase in displacement for a small increase of the load. The intersection point between these two linear functions then gives the estimated critical buckling load, see Figure 5.15.

• The Second derivative method, where a curve fitting or linearization between the points has to be made with respect to the obtained force versus out-of-plane displacement curve from the nonlinear analysis. In this thesis, for this particular method, the force is plotted on the x-axis and the out-of-plane displacement on the y axis. After an equation describing the out-of-plane displacement versus force curve is obtained from the curve fitting, it is differentiated two times with respect to the force and then values of this function is calculated for all load increments. Mathematically, at the load increment where the largest value of the second derivative is obtained is the corresponding part where the largest curvature is obtained on the original out-of-plane displacement versus force curve, which is also where buckling occurs. This can be seen in Figure 5.15.

Both of the methods contain a lot of approximations and are strongly dependent on the quality of the nonlinear solution, especially the second derivative approach because of the curve fit step where it can be quite complicated to make a good approximation if the structure shows an unstable post buckling path. Using the linearization method can result in a bit arbitrary approximation depending on the defined slopes of the linear functions and this method is also a bit sensitive to where the second function is chosen to cross the y-axis.

5.11.3 Estimation of Error:

The method used for calculating the error when two cases are evaluated against each other is an absolute relative error method, which is calculated by subtracting the difference between the two results and dividing by the value that is constant and compared against. The following equation is used for estimating the absolute relative error with respect to the buckling load obtained numerically versus the handbook result:

$$error = \left| \frac{\sigma_{cr}^{NUMERICAL} - \sigma_{cr}^{THEORETICAL}}{\sigma_{cr}^{THEORETICAL}} \right|$$

5.12 FE Modelling

5.12.1 General

The procedure of setting up an FE model using a pre-processor is very straightforward. First, the pre-processor program is selected and then the desired geometry is modelled. After the desired geometry is obtained, the surface is meshed with the proper type of element. When satisfied with the mesh, appropriate boundary conditions and loads are applied to the model. Further on, depending on what kind of an analysis that is going to be performed and the desired output variables, corresponding control cards have to be selected.

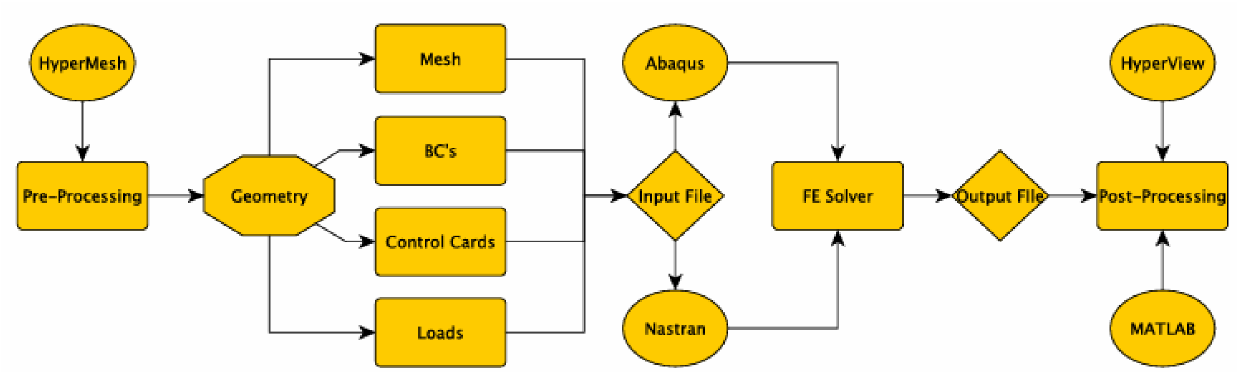


Figure 5. 16Flowchart describing how to set up an FE structural analysis

After this is done, the pre-processor compiles and exports the FE data as a bulk data file, which is then submitted into the selected FE solver. After solving the problem, the FE solver generates an output file that contains the results from the analysis. This output file is then used as input to the post-processing program, which is used for formatting and presenting the results in a desired way. See Figure 19 for a schematic overview of the process describing how to setup an FE analysis.

5.12.2. Boundary conditions and Load types

The type of boundary condition used for the largest part of the models is simply supported conditions, but for the convergence study with respect to the mesh, also clamped conditions were studied. In reality, neither simply supported nor clamped conditions are sufficient to describe the behaviour of a structure because the mobility is something in between these two.

If the plate is modelled according to Figure 20 and the coordinate system is oriented in the way, then following the constraints were applied to the structure when simulating simply supported conditions

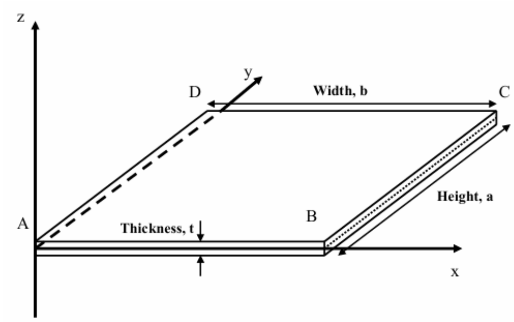


Figure 5. 17Definition of the different geometrical parameters and the coordinate system.

Table 5. 2Definition of the displacement constraints for a simply supported plate where U1, U2, U3 are the translational degrees of freedom and R1, R2, R3 are the rotational degrees of freedom at each node.

FE Modeling	U1	U2	U3	R1	R2	R3
Edge A→B			x			
Edge B→C			x			
Edge C→D			x			
Edge D→A			x			
Point A	x	x	x			
Point B						
Point C		x				
Point D						

The corresponding constraints for simulating a clamped plate were the following:

Table 5. 3Definition of the displacement constraints for a clamped plate where U1, U2, U3 are the translational degrees of freedom and R1, R2, R3 are the rotational degrees of freedom at each node.

FE Modeling	U1	U2	U3	R1	R2	R3
Edge A→B			x	x		
Edge B→C			x		x	
Edge C→D			x	x		
Edge D→A			x		x	
Point A	x	x	x			
Point B						
Point C		x				
Point D						

Two different types of loads are considered in the FE models, in-plane shear load and uniaxial compression load, see Figure 21. As magnitude, in all of the linear buckling analyses, a unit load is applied to the involved edges. In the nonlinear buckling analyses, the load is applied incrementally and is therefore dependent on the load proportional factor (LPF).

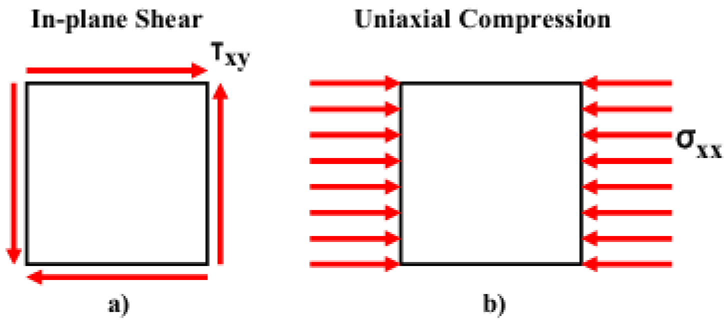


Figure 5. 18Two types of loads are used in the parametric studies.

a) In-plane shear load b) Uniaxial compression load

5.13 Software introduction used in this thesis:

In this section, the following two commercial structural engineering software tools will be introduced:

- Abaqus 3D
- EBPLATE

5.13.1. ABAQUS Software for elastic and inelastic buckling:

Many different types of analyses are available with ABAQUS. However, only one single type of analysis is covered by the guide, namely large displacement analysis with used to examine instability. The instability is triggered by introducing a small geometric imperfection in the structure.

ABAQUS has two different ways of providing input: Via a graphical user interface (the so-called CAE) or directly via an input text file. Only the first method will be described in the following. Advanced users may prefer the second option as it provides some freedom compared with the CAE.

Getting started: Run ABAQUS CAE and choose **Create Model Database with Standard/Explicit Model**. You should now see the screen visualised in Figure 5.10. On the left you have the project tree and, on the right, you have the working area. The field right above the working area is denoted the context bar.

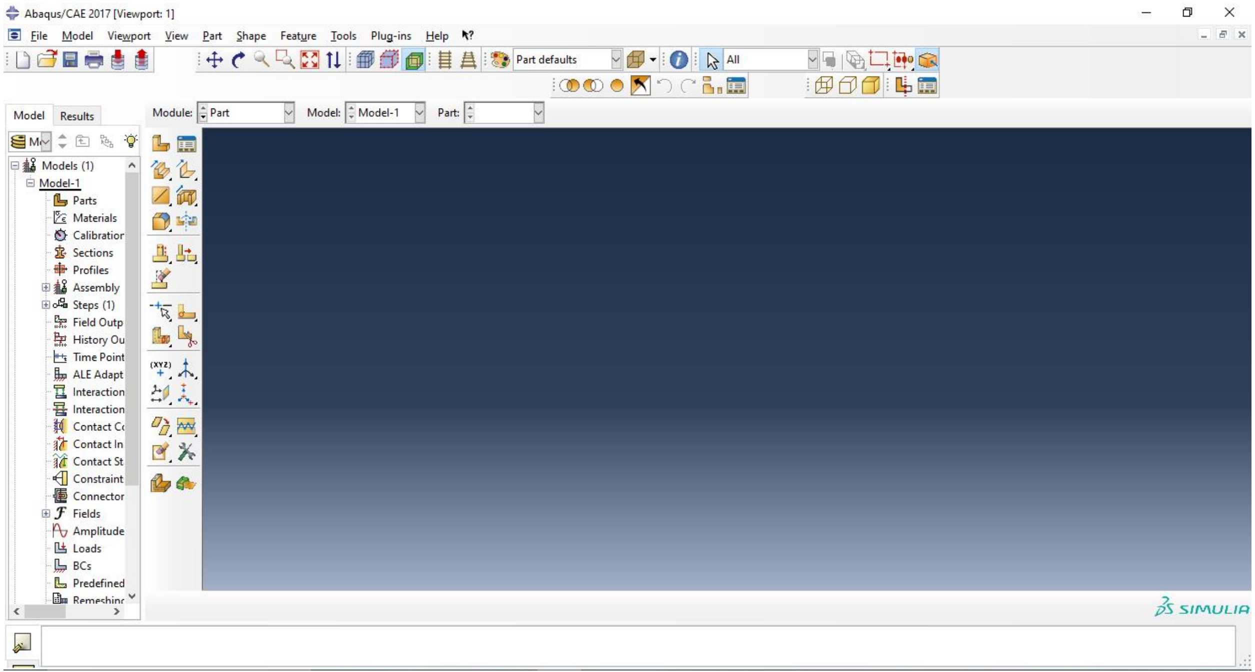


Figure 5. 19The main window of Abaqus

ABAQUS CAE has a number of so-called modules. Each module is utilized to define some part of the model, e.g. the geometry or the boundary conditions. You work in one module at a time and can change module in the drop-down menu **Module** which can be found in the context bar as indicated by the red rectangle in Figure 5.10.

The modules are:

1. **Part** –defines the geometry of a structural element or model to be used in the analysis.
2. **Property** –defines materials and cross sections.
3. **Assembly** –assembles a number of parts to form the global geometry of a model.
4. **Step** –defines the different analyses to be carried out.
5. **Interaction** –defines connections and interface conditions between different parts.
6. **Load** –defines the boundary conditions of the model.
7. **Mesh** –provides the discretization of the model into finite elements.
8. **Job** –defines the jobs to be carried out by the analysis program.
9. **Visualization** –is utilized for viewing and post processing the results.

10. **Sketch** –can be used as a simple CAD programme for making additional drawings.

As indicated in Figure 5.10, you will start out in the module Part.

Examples

- **Elastic eignen values analysis**

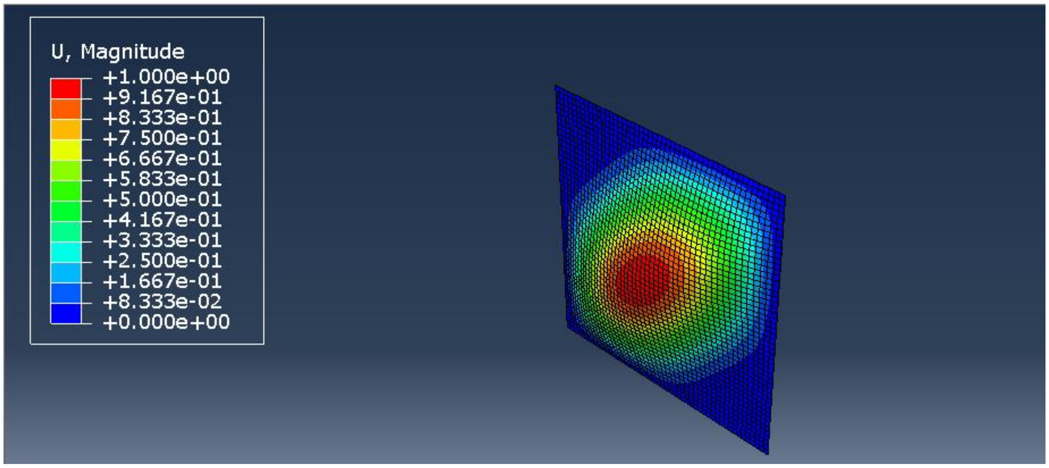


Figure 5. 20Square plate with Simply supported

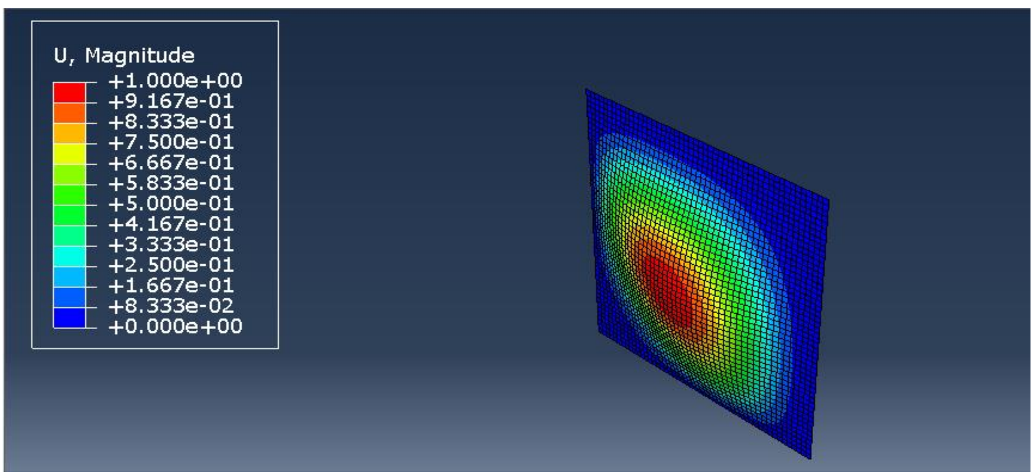


Figure 5. 21square plate with Fixed support

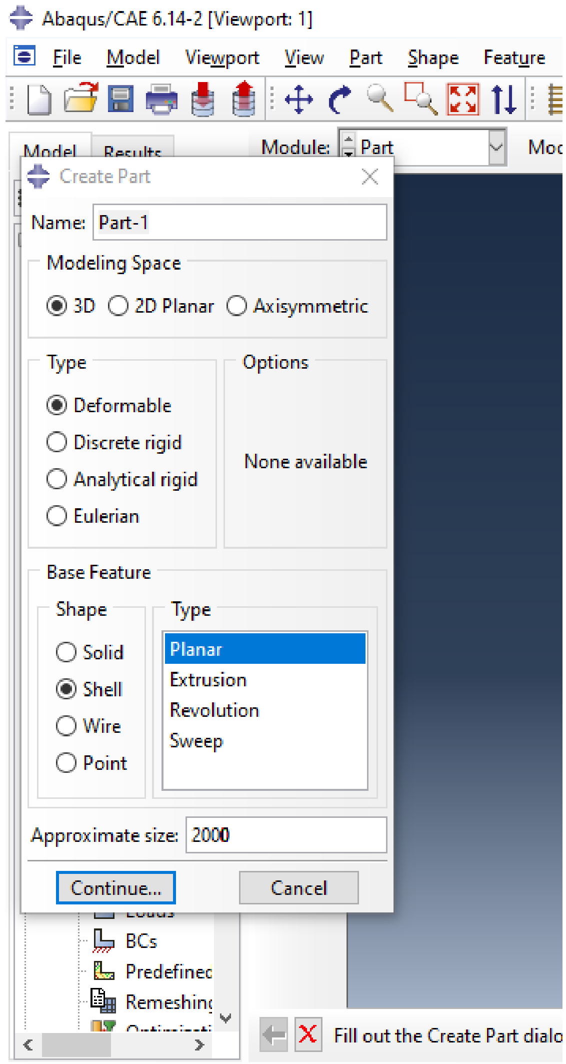
5.14.Modelling using ABAQUS and EBPLATE software:

5.14.1.Introduction to Abaqus Modules

ABAQUS modelling process is executed through a group of commands gathered each in a number of modules, each module contains commands and tools that serves specific tasks in modelling process. The following shown list eliminate the available modules in Abaqus.[4]

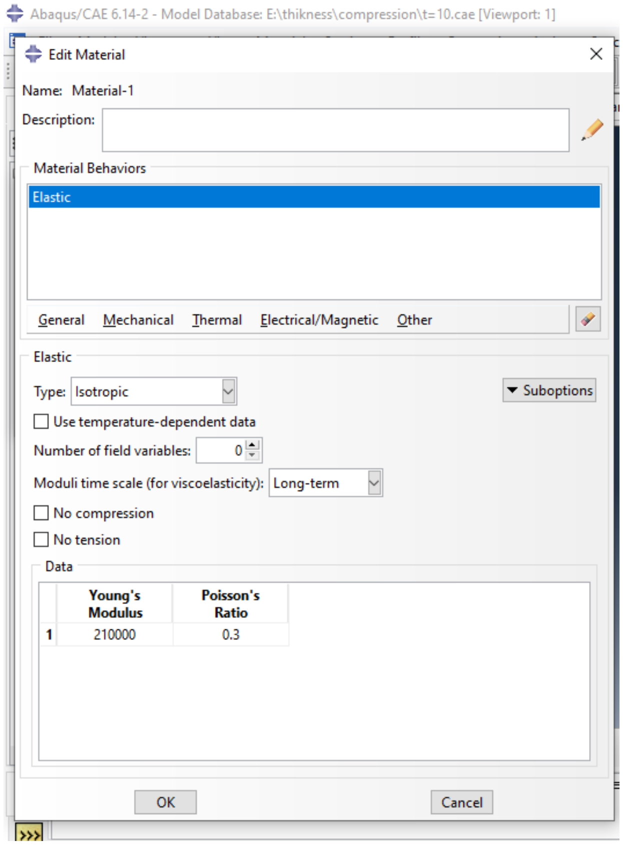
1-Part Module:

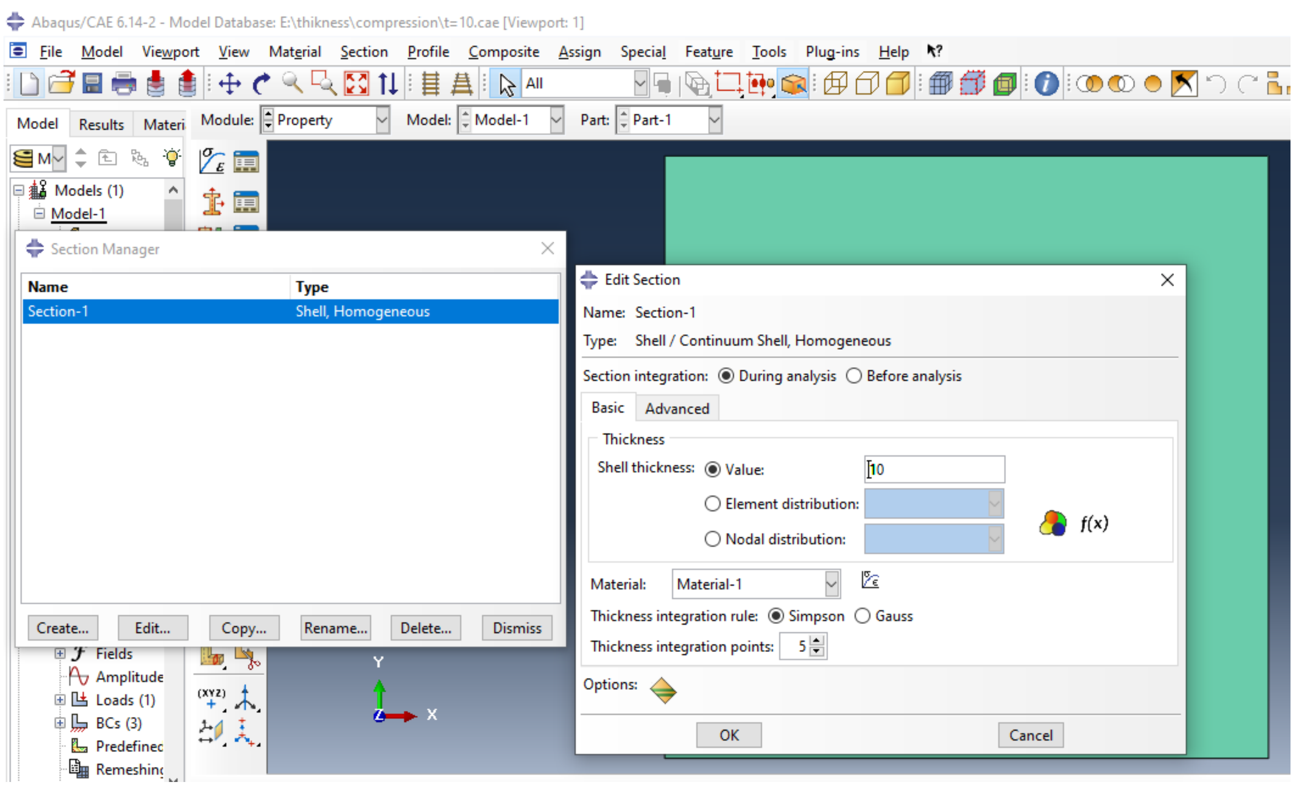
The Part module main objective is creating individual parts to simulate their geometry, and also to assign pre-defined section to each part. You can draw parts in ABAQUS or importing them from other software such as Solid Work.



2-Property Module

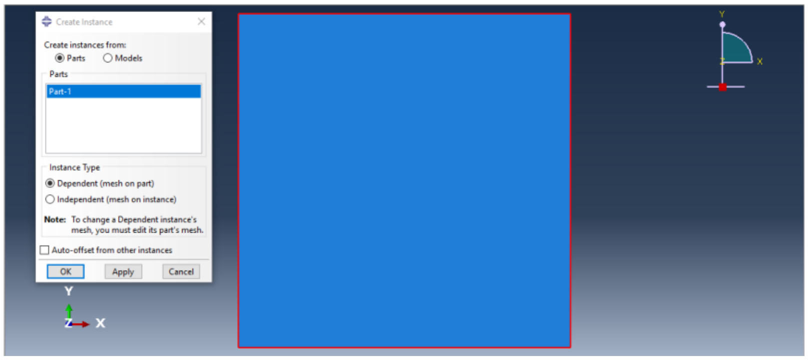
The Property module main objective is defining material properties and cross sections properties to use them later, and assign them to each pre-sketched parts.


Abaqus/CAE 6.14-2 - Model Database: E:\thikness\compression\t=10.cae [Viewport: 1]
Edit Material
Name: Material-1
Description:
Material Behaviors
Elastic
General
Mechanical
Thermal
Electrical/Magnetic
Other
Elastic
Type: Isotropic
Suboptions
Use temperature-dependent data
Number of field variables: 0
Moduli time scale (for viscoelasticity): Long-term
No compression
No tension
Data
Young's Modulus
Poisson's Ratio
1
210000
0.3
OK
Cancel


Abaqus/CAE 6.14-2 - Model Database: E:\thikness\compression\t=10.cae [Viewport: 1]
File Model Viewport View Material Section Profile Composite Assign Special Feature Tools Plug-ins Help
All
Model Results Materi
Module: Property
Model: Model-1
Part: Part-1
Models (1)
Model-1
Section Manager
Name
Type
Section-1
Shell, Homogeneous
Create...
Edit...
Copy...
Rename...
Delete...
Dismiss
Fields
Amplitude
Loads (1)
BCs (3)
Predefinec
Remeshing
Edit Section
Name: Section-1
Type: Shell / Continuum Shell, Homogeneous
Section integration: During analysis Before analysis
Basic
Advanced
Thickness
Shell thickness: Value: 10
Element distribution:
Nodal distribution:
f(x)
Material: Material-1
Thickness integration rule: Simpson Gauss
Thickness integration points: 5
Options:
OK
Cancel
Y
X

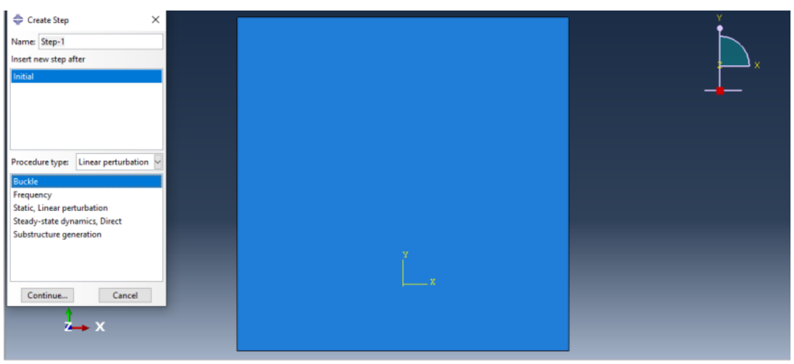
3-AssemblyModule:

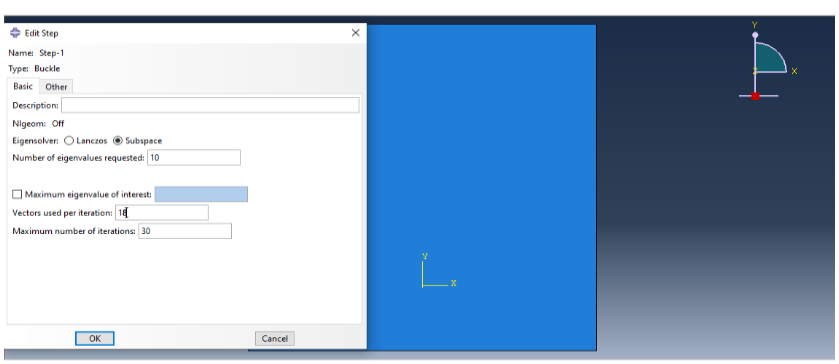
The Assembly module main objective is creating instances of your parts, and place them in the right position relative to other instances, to be ready later on to define the interactions and constraints between them



4-StepModule:

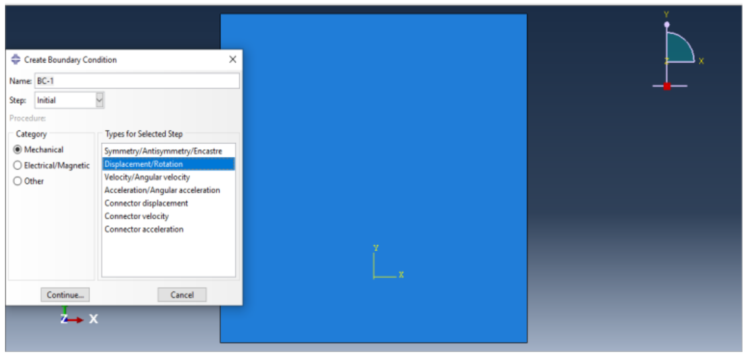
The Step module main objective is configuring the analysis step, and the accompanied field output request and history output request.

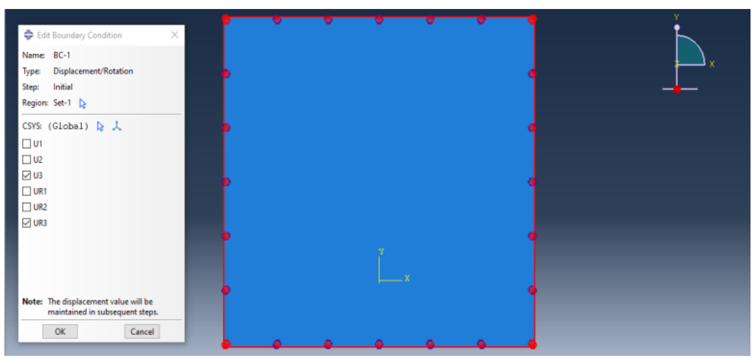


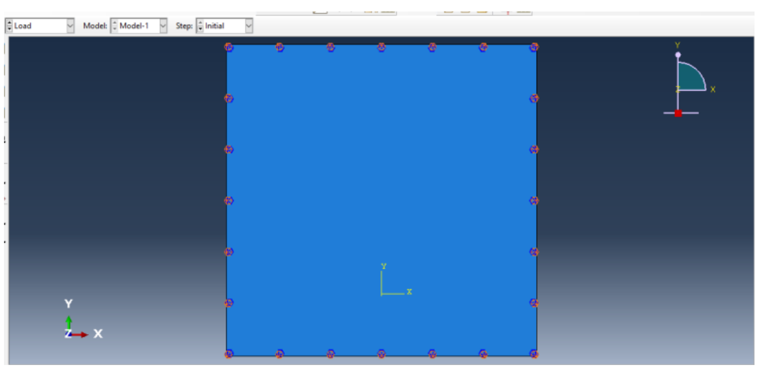


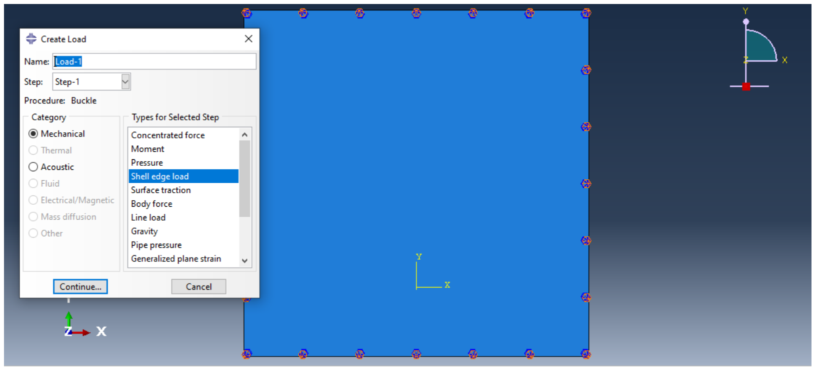
5-LoadModule:

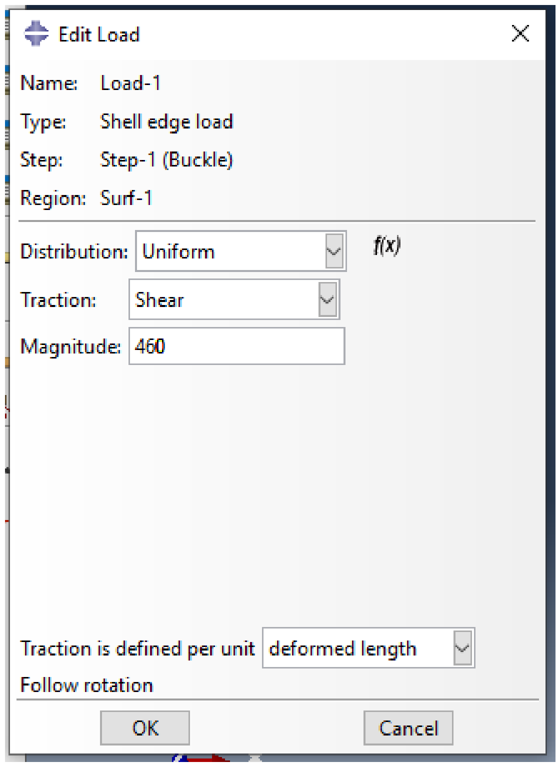
The Load module main objective defining boundary conditions and loads .To create the load, Choose Load module from the Load Manager Create a load case and fill the box with shear as General and Magnitude=460 Select the edges of the geometry for assigning the load to the model. From the same module define the boundary conditions of the model.

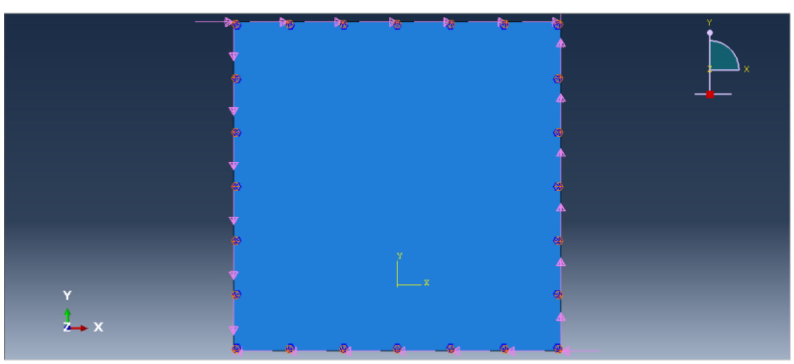







Create Load
Name: Load-1
Step: Step-1
Procedure: Buckle
Category
Mechanical
Thermal
Acoustic
Fluid
Electrical/Magnetic
Mass diffusion
Other
Types for Selected Step
Concentrated force
Moment
Pressure
Shell edge load
Surface traction
Body force
Line load
Gravity
Pipe pressure
Generalized plane strain
Continue...
Cancel

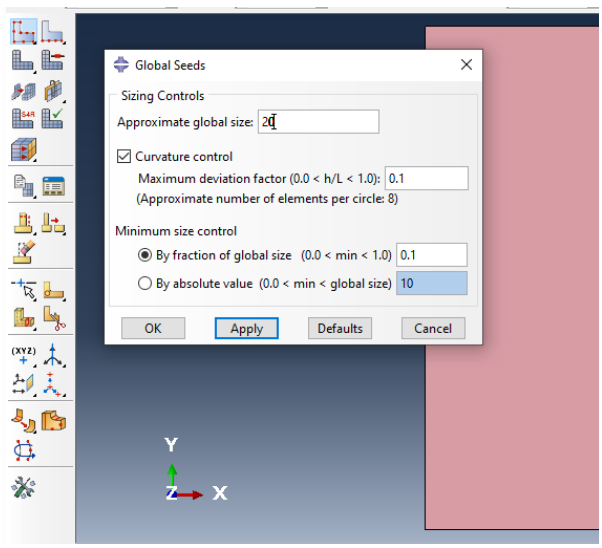

Edit Load
Name: Load-1
Type: Shell edge load
Step: Step-1 (Buckle)
Region: Surf-1
Distribution: Uniform
f(x)
Traction: Shear
Magnitude: 460
Traction is defined per unit deformed length
Follow rotation
OK
Cancel

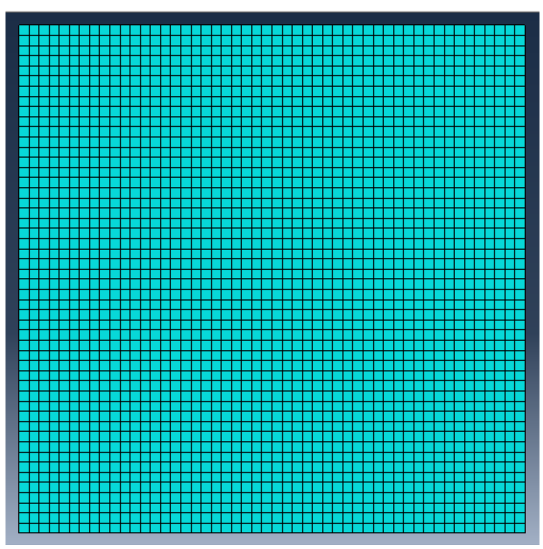


6-MeshModule:

The Mesh module main objective is creating the desired finite element mesh

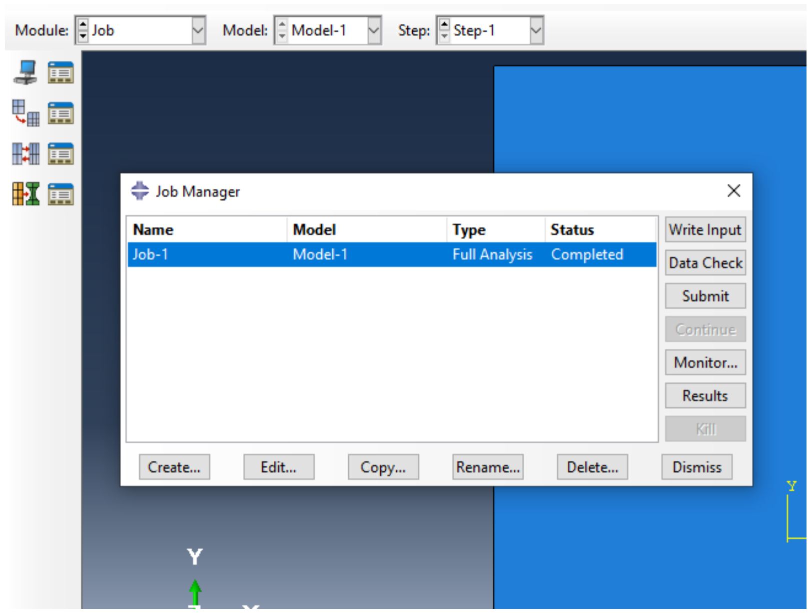
Select Mesh Module. From Seed Part make an approximation for global size of part and the number of nodes on each edge. Smaller values mean smaller finite element size and more execution time. Choose Mesh Part and click Ok. The mesh will be creating as Figure 4.18. This automatic mesh is good enough for our analysis and will produce reliable results, therefore keep it as it is and keep going to the next step.





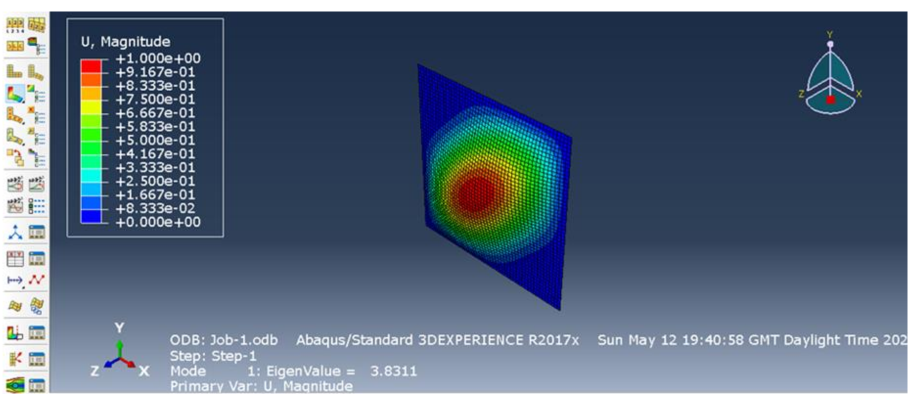
7-JobModule:

The Job module main objective is to run the model or analyze the model after finishing model definition process.



8-VisualizationModule:

The Visualization module main objective to display results graphically or import results in the form of printed reports. You should specify the desired output before running the model from output request as explained previously.



5.14.2 EBPLATE for elastic buckling analysis

- **General:**

EBPLATE software provides accurate values of elastic critical stresses for a wide field of practical cases of rectangular plates subject to in-plane loading. Plates can be either stiffened or unstiffened, the stiffening being provided by longitudinal and/or transverse stiffeners; orthotropic plates can also be treated. Complex stresses patterns can easily be defined, including patch loads. Numerous display possibilities are available for data and results. EBPLATE tool was designed by CTICM in the frame of the European research project COMBRI (RFS-CR-03018), with partial funding of the Research Fund for Coal and steel.[5]

Scope of EBPLATE:

The scope of EBPLATE may be summarized as follows see Figure

Plate: -rectangular (a×b), uniform thickness t

-possibly orthotropic (Dx, Dy) due to smeared stiffeners

-plate stretching neglected

Supports: -laterally supported along its 4 edges (out-of-plane displacement w = 0)

-support conditions in rotation at edges: hinged or restrained (full or elastic – rotational (Kr) and/or torsional (J) stiffnesses accounted for)

Stiffeners: -longitudinal and/or transverse, single or multiple, with different properties .

-axial (due to area As), flexural (due to flexural inertia Is) and torsional (due to torsional constant Js) stiffnesses accounted for

-smearing of identical and regularly spaced stiffeners (Dx, Dy)

-automatic calculations of stiffener properties for commonly used cross-sections

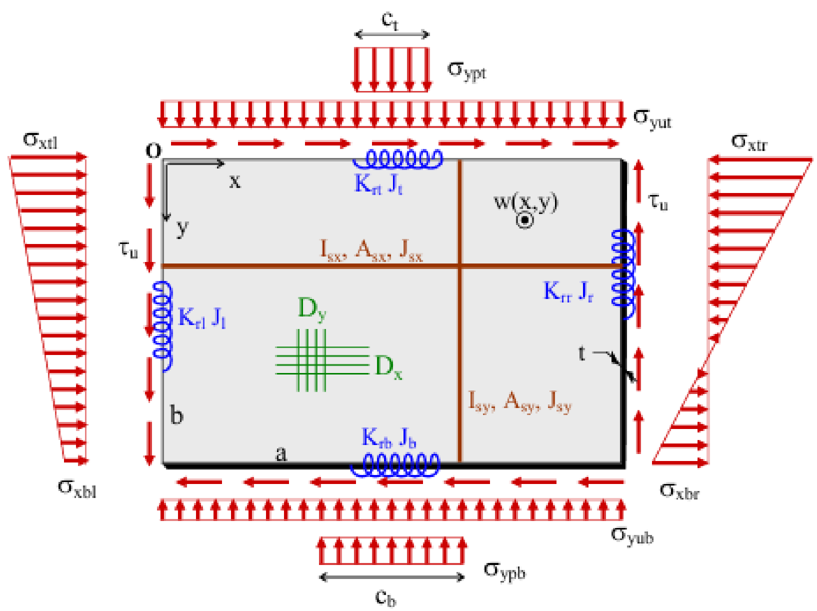
-special treatment for closed (trapezoidal) stiffeners, accounting for the distance between their junctions to the plate, their torsional stiffness, the flexibility of their cross-section.

Stresses: -normally, generated by stress patterns acting along edges in the mid-plane

-possibility of linear variation of longitudinal stress σx along the plate length

-possibility of generating any stress distribution by defining directly stresses (σx,σy,τ) within the elements of a regular meshing of the plate

-possibility to impose one or more stresses at buckling state and to perform the buckling analysis on the remaining free stresses [6]



EBPLATE calculates the minimum factor φcr – also called "critical factor" – to be applied to the reference stresses (σx, σy, τ) defined by the user and acting within the plate for reaching the elastic plate buckling. Critical stresses are then obtained from:

$$\sigma_{x,cr} = \phi_{cr} \cdot \sigma_x \qquad \sigma_{y,cr} = \phi_{cr} \cdot \sigma_y \qquad \tau_{cr} = \phi_{cr} \cdot \tau$$

Presentation:

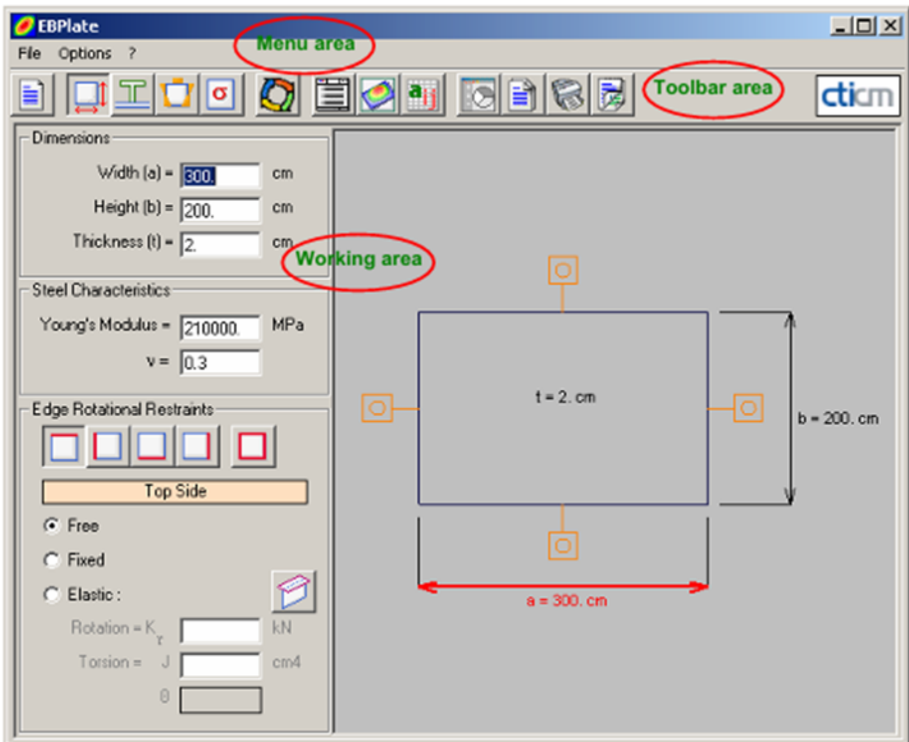
EBPLATE is a piece of software designed in VB language for Windows systems, from versions Windows 98 to Windows XP. It is a freeware that can be freely downloaded from the website of CTICM (http//:www.steelbiz.com/file/descfile.asp?idfile=200 at the time being). When EBPLATE is launched and if the connection to internet is available, the availability of a more recent version of the software is automatically controlled.

EBPLATE is built up with a main sizeable window, which has the following parts:

- a menu area, which gives access to files functions and to options

- a toolbar area, which gives access to the most common function and allows to select the working area

-a working area, where the parameters are defined, the calculations carried out and the results displayed[6]



Menus:

The following menus are available:

-“File” menu:

The parameters of the study, but not the results, can be saved in an ASCII file, which extension is .EBP. The following operations are allowed: opening a file, saving or saving as a file and opening directly one of the last four files.

- “Options” menu:

The options menus enable the user to modify several parameters of EBPLATE: o the language of the interface – French, English, German and Spanish are available; o the default working directory, where files are opened or saved; o the unit in which length are defined by the user (m, cm or mm);

o the table of colours with whom the results are displayed; o the coefficient which defines the length of plate to be attached to a stiffener to calculate its second moment of inertia.

-“?” menu:

This menu give access to the “About” window and to the Help Files. The “About” windows propose email addresses to be contacted in case of troubles. EBPLATE is delivered with a French Help File, associated to the French interface and with an English Help File, associated to all other language of the interface.

Toolbars:

The most useful functions and the working areas of EBPLATE can be activated directly by the buttons of the toolbars. The functions associated with the buttons of the main toolbar are described hereafter:

Definition of the references of the study

Activation of the “plate’s parameter” working area – see 2.4.1

Activation of the “Stiffening” working area

Activation of the “Define the stresses” working area

Activation of the “Check the stresses” working area

Activation of the “Calculation” working area

Edition of the calculation sheet. Available only after the calculation is done.

Activation of the “Post processing” working area. Available only after the calculation is done.

Edition of the deformed shape coefficients. Available only after the calculation is done.

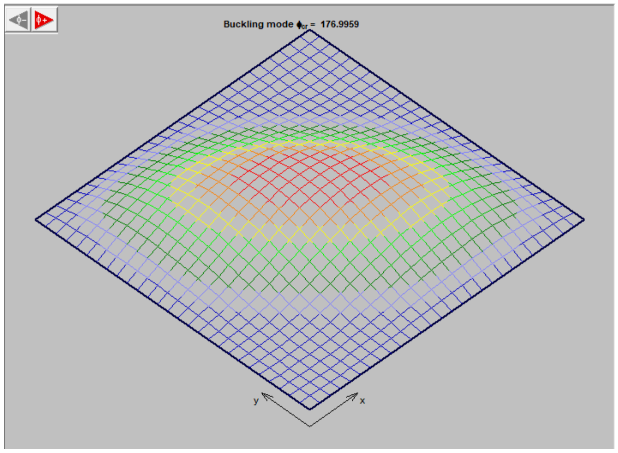
Modification the background colour.

Copy of the present drawing in the Clipboard

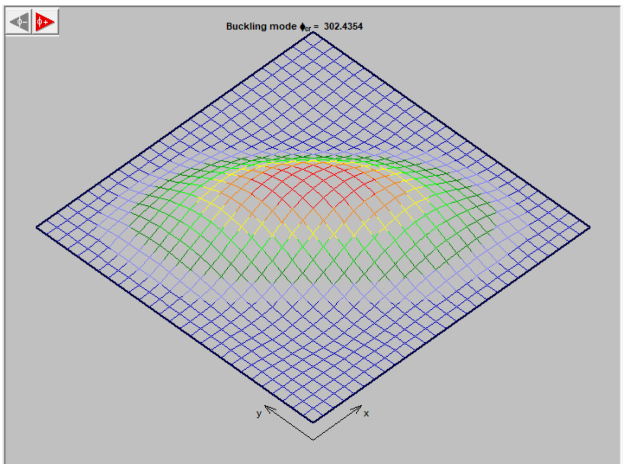
Printing of the picture which is displayed in the working area

- **Examples:**

- Elastic case:



Square plate with hinged support: elastic buckled shape under shear



Square plate with fixed support: elastic buckled shape under shear

CHAPTER SIX

RESULTS AND DISCUSSION OF ELASTIC AND INELASTIC SHEAR BUCKLING OF HSS STEEL PLATE

6.1 General

It is well known that the quality of the results obtained from FEA depends up on the density of the mesh, element type and the input properties of the element. Some preliminary analyses were conducted to study the effect of mesh refinement, and to determine whether reduced integration elements could be used to improve computational time without loss of significant accuracy.

In this chapter, firstly the results of linear elastic buckling analysis of a parametric study in terms of some key parameters believed to have an influence on the buckling of a thin plate, namely the steel grades, the ratio aspect and the support conditions will be presented and fully discussed. It is worth to recall that three means have been used: classical analytical procedure, EBPLATE software and ABAQUS package. A brief presentation of the three means of investigation will be also recalled.

A compressive series of finite element analysis (FEA) modelling has been carried out performed on thin steel plates having different characteristics. For validation purposes, the FEA elastic models were executed and compared with results of the classical approach detailed in chapter 3.

Following this, linear buckling analysis was restarted to conduct nonlinear buckling analysis on the same models. The objective of this process is to enhance confidence in the numerical analysis of these models, as they will be subsequently used for more complex analyses, including nonlinear and inelastic buckling analysis.

As detailed in Chapter 4, the numerical analyses are conducted using two software programs: Abaqus and EBPLATE. The validation focuses on models subjected to shear stresses with varying support conditions and geometries. This validation is achieved by comparing the values obtained from the elastic eigenvalue buckling analysis.

6.2 Elastic linear buckling analysis

6.2.1 Introduction

An accurate solution to a buckling problem requires more efforts than just following a numerical procedure, there are a number of factors to consider before a buckling solution can be accepted with confidence. In fact, a starting step should be a linear buckling analysis. Nonlinear buckling analysis capability can be performed restart the linear buckling analysis.

In this study, a series of simulations using commercial packages to investigate the elastic shear buckling behaviour of square and rectangular steel plates made from different steel grade and under various support conditions.

This parametric study aims to investigate and compare the effects of different support conditions—both simply supported and clamped—on plates with dimensions of 1000x1000 mm, 1000x1200 mm, and 1000x1500 mm. Additionally, these plates were modelled using different grades of steel to understand the influence of material properties on their buckling performance.

6.2.2 Objective:

The primary focus of the research presented herein was to determine the critical buckling load for every configuration. For each case, the plates were subjected to a uniform shear load, and the critical buckling load was identified as the point at which the plates exhibited instability.

As far as the support conditions are concerned, the simply supported boundary conditions allowed the plates to freely rotate along their edges while preventing any translational movement, whereas the clamped boundary condition restricted both rotational and translational movements along the edges.

By comparing the elastic buckling behaviour across the different plate sizes and support conditions, the study aimed to provide insights into how plate dimensions and boundary constraints affect the structural stability of steel plates. The results from these simulations are expected to contribute valuable data for the design and optimization of steel structures, ensuring better performance under critical loading conditions.

The classical theory of plates or shells, which formulates and solves problems from the point of view of rigorous mathematical analysis, is an important application of the theory of elasticity. Classical buckling problems, by the so-called equilibrium method and the conditions that result in the lowest eigenvalue, or the actual buckling load, are not at all obvious in many situations.

Buckling Analysis is an FEA routine that can solve all the difficult buckling problems that cannot be solved by hand calculations. Linear Buckling (LBA) is the most common Buckling Analysis is essentially an eigenvalues analysis. The nonlinear approach (GNLA and MNLA), on the other hand, offers more robust solutions than Linear Buckling Analysis.

6.2.3 Studied cases considered:

- **Case 1: Simply Supported Square Plate (1000x1000 mm)**

This case examines a simply supported square plate with dimensions 1000x1000 mm. All characteristics used in the study are summurazied in Table 6.1 bellow. The general geometry shape is given in Figure 6.1.Obvisiouly, the objective is to determine the critical shearing stresses and analyze the buckling patterny mean of the classical elastic buckling (Chapter 3) and the FEA approch as described in Chaptre 4. The analysis will be performed for different grades steed: S235; S355; and high-strength steel S420 and S460.

Table 6. 1Geometrical and material characteristics of the models

Characteristic	**Value**
Young's modulus (*E*).	**210000Mpa**
Poisson's ratio (*v*)	**0.3**
width of plate (*H*)	**1000 mm**
length of plate (*a*)	**1000 mm**
thickness (t)	**10 mm**

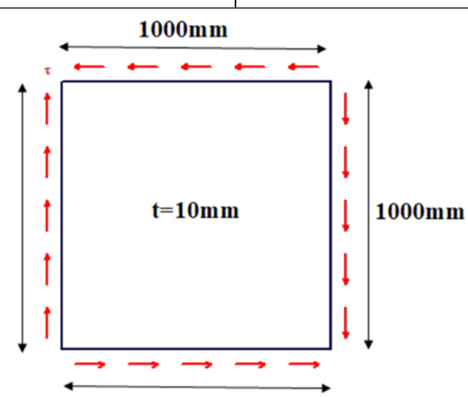


Figure 6. 1Geometry of the square thin plates(1000x1000x10mm)

- **Using the analytical evaluation of τ_{cr}:**

$$\tau_{cr}^{solid} = k_v^{solid} \frac{\pi^2 E}{12(1-\nu^2)(h/t)^2}$$

$$k_v^{solid} = 5.34 + \frac{4}{(a/h)^2}, \quad a/h \geq 1$$

$k_\tau = 6.34+(4/(a/h)^2) = 9.34$

E=210000MPa

υ=0.3

a=1000mm ; h=1000mm ; t=10mm

τcr = 176.2Mpa

The obtained results are as follows

S235: τ_{cr} = 0.754MPa

S355: τ_{cr} = 0.5MPa

S420: τ_{cr} = 0.422MPa

S460: τ_{cr} = 0.385MPa

The first impression of the above results leads to the conclusion that as the steel grade becomes higher, the value of the critical shear stress decreases.

- **Using EBPLATE**

In the following, the section concerns only the final results obtained. The technics of modelling are as fully detailed in Chapter 4.

The results will be presented by a is the deformed shape of the plate along with its critical shear stresses for each steel grade considered. Figures 6.2 up to 6.

- **Outcomes**

The numerical results for the critical shear buckling load are presented. Illustrates the buckled shape of the plate under shear stresses with simply supported using EBPLATE

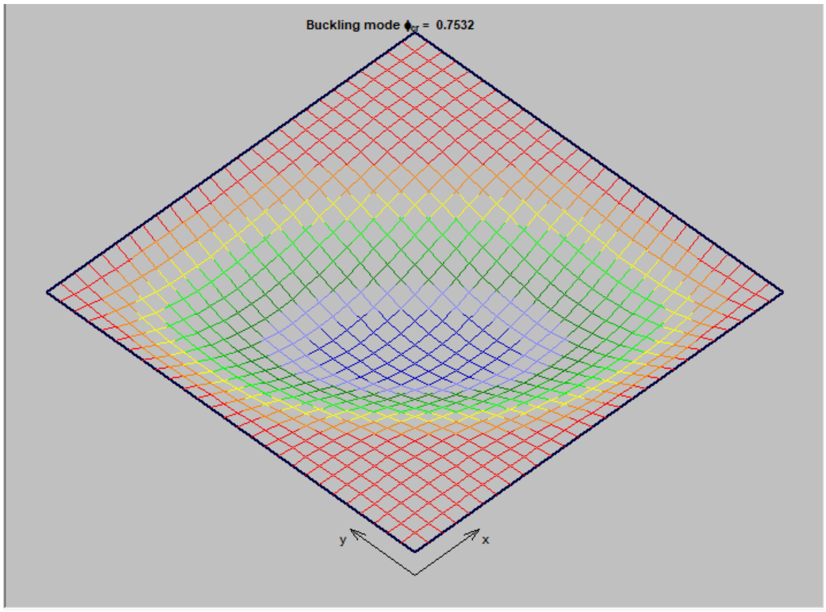


Figure 6. 2Case 1 Steel grade S235 Buckled shape of square plate under shear stresses with simply support by EBPLATE

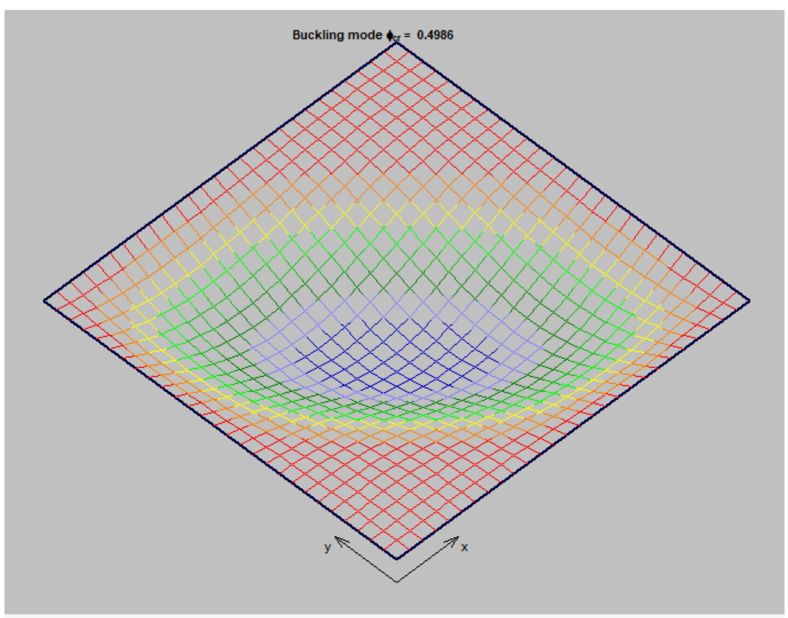


Figure 6. 3Case 1 Steel grade S335 Buckled shape of square plate under shear stresses with simply support by EBPLATE

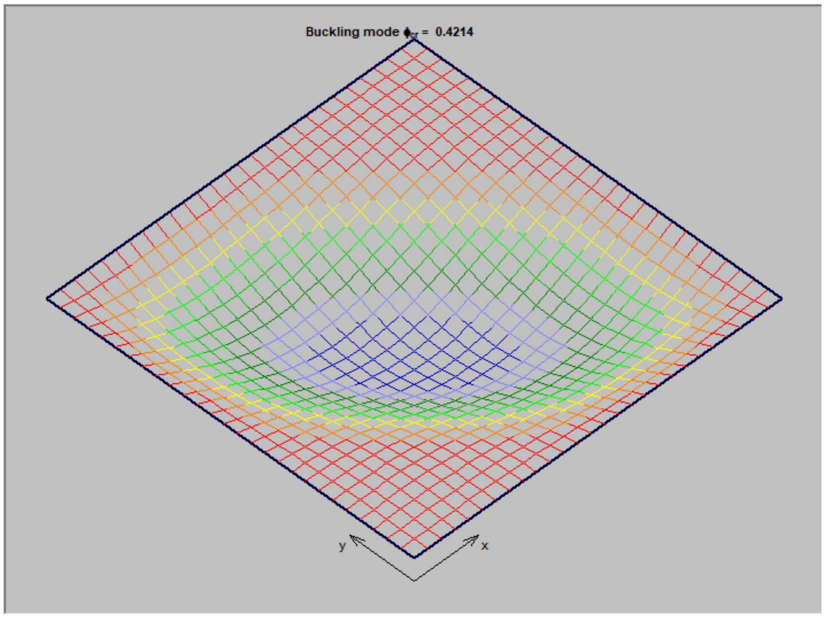


Figure 6. 4Case 1 Steel grade S420 Buckled shape of square plate under shear stresses with simply support by EBPLATE

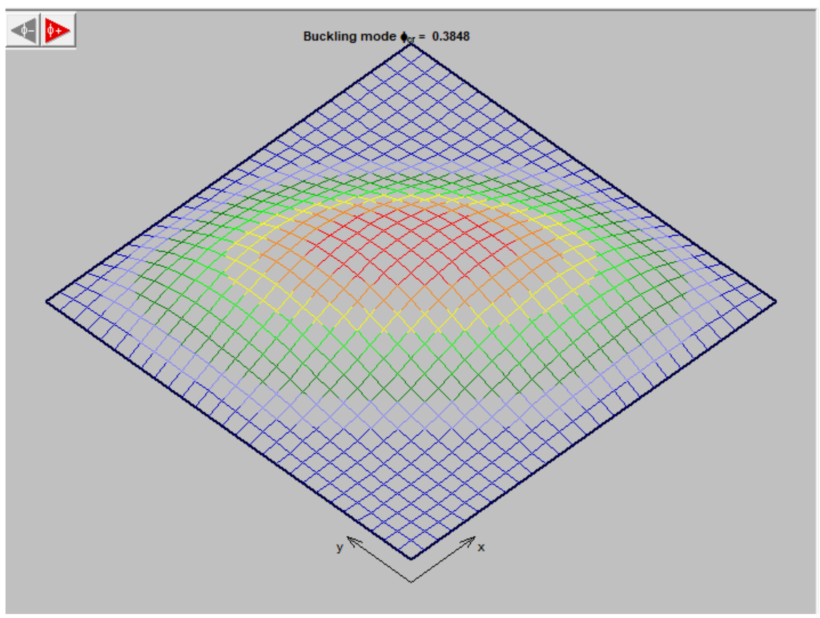


Figure 6. 5Case 1 Steel grade S460 Buckled shape of square plate under shear stresses with simply support by EBPLATE

- **Comparison of results**

From Table 6.2, it can be easily seen that the results are very close to each other's and then the EBPLATE model can be trusted.

Table 6. 2Comparison of τcr case 1 analytical and EBPLATE results

Steel grades (MPa)	τ_{cr}		Difference %
	EBPLATE	Analytic	
235	0.753	0.754	0.13
355	0.499	0.5	0.2
420	0.42	0.422	0.47
460	0.385	0.385	0

- **Using Abaqus**

A summary of the procedures to follow is outlined in the following.

Type of Element: In all numerical simulations, the ABAQUS software utilised a reduced integration eight-node thin shell element. This element features six degrees of freedom at each node: three translations (u, v, w) and three rotations (θx, θy, θz).

Meshing: The plate was discretized using rectangular elements with a side length of 20.00 mm. (Figure 6.7)

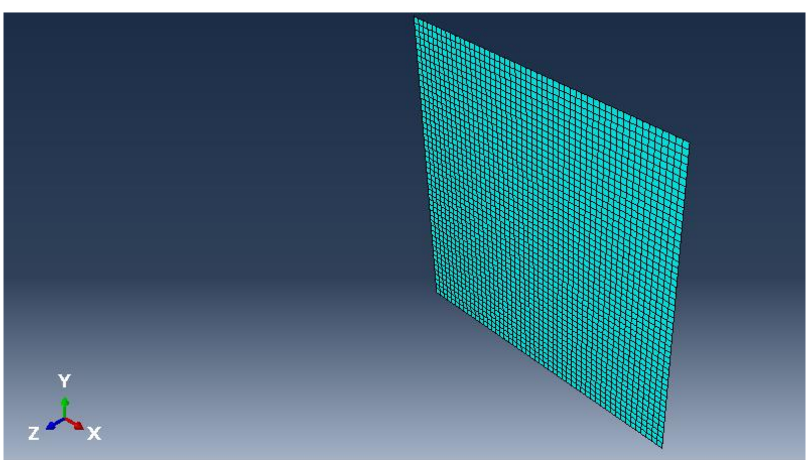


Figure 6. 6Finite element mesh of the plate by Abaqus.

Outcomes (results):

In the same manner, the numerical results of ABAQUS will be presented with a buckled (deformed shape) then the critical buckling load under shear stresses with simply supported. See Figure 6.8 to 6.11.

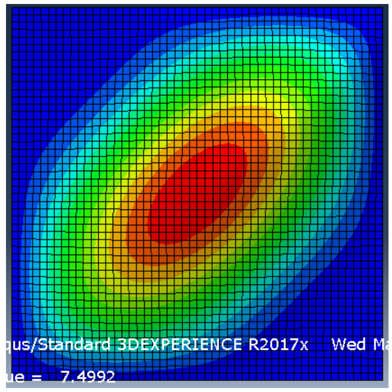


Figure 6. 7Case 1 Steel grade S235 Buckled shape of square plate under shear stresses with simply support by ABAQUS

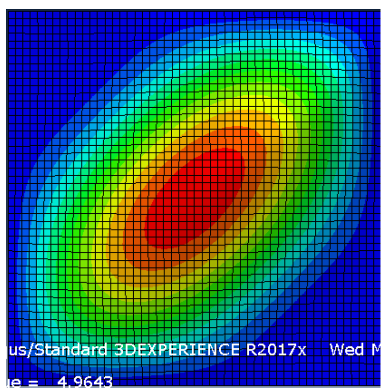


Figure 6. 8Case 1 Steel grade S335 Buckled shape of square plate under shear stresses with simply support by ABAQUS

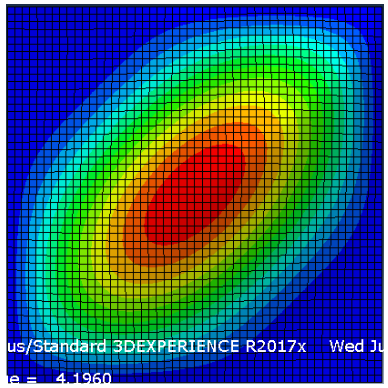


Figure 6. 9Case 1 Steel grade S420 Buckled shape of square plate under shear stresses with simply support by ABAQUS

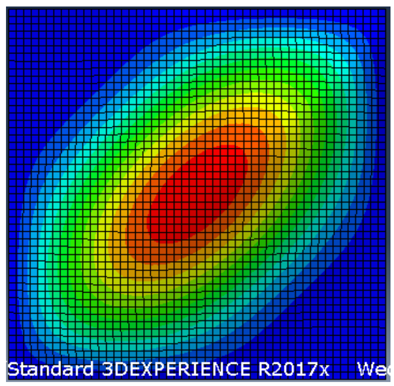


Figure 6. 10Case 1 Steel grade S460 Buckled shape of square plate under shear stresses with simply support by ABAQUS

Table 6. 3Comparison of τcr case 1 analytical and ABAQUS 3D results

Steel grade (MPa)	τ_{cr}		Difference %
	Abaqus	Analytic	
235	0.75	0.754	0.53
355	0.496	0.5	0.8
420	0.42	0.422	0.47
460	0.383	0.385	0.52

- **Comparison**

From Table 6.3, it can be easily seen, once again, that the results are very close to each other's and then the ABAQUS model can be trusted it will be used later in much more complicated study: inelastic buckling of thin plates.

- **Case 2: Simply Supported Rectangular Plate (1000x1200 mm)**

With the aspect ratio: 1.2

Table 6. 4Geometrical and material characteristics of the models

Characteristic	**Value**
Young's modulus *(E)*.	**210000Mpa**
Poisson's ratio (*ν*)	**0.3**
width of plate (*H*)	**1000 mm**
length of plate (a)	**1200mm**
thickness (t)	**10mm**

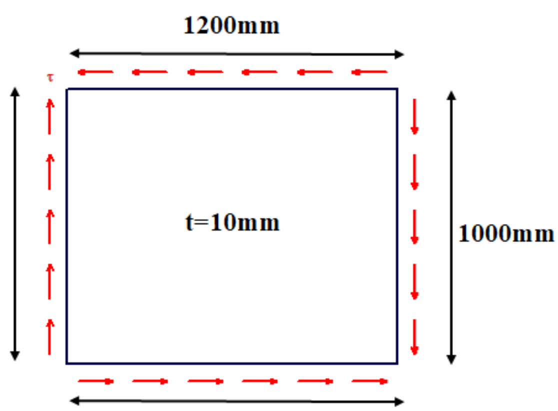


Figure 6.12 Geometry of rectangular thin plates(1000x1200x10mm)

- **Analytical evaluation of τ_{cr}**

$$\tau_{cr}^{solid} = k_v^{solid} \frac{\pi^2 E}{12(1-\nu^2)(h/t)^2}$$

$$k_v^{solid} = 5.34 + \frac{4}{(a/h)^2}, \quad a/h \geq 1$$

$K\tau$ = 6.34+(4/(a/h) ²) =8.11

E=210000MPa

υ=0.3

a=1200mm ; h=1000mm ; t=10mm

τ_{cr} = 153.92MPa

S235: τ_{cr} = 0.655MPa

S355: τ_{cr} = 0.434MPa

S420: τ_{cr} = 0.367MPa

S460: τ_{cr} = 0.335MPa

- **EBPLATE**

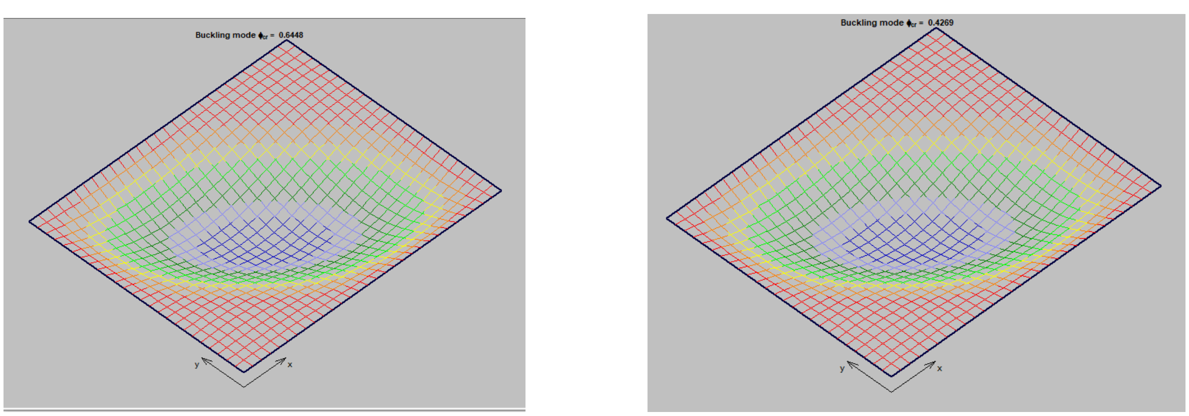
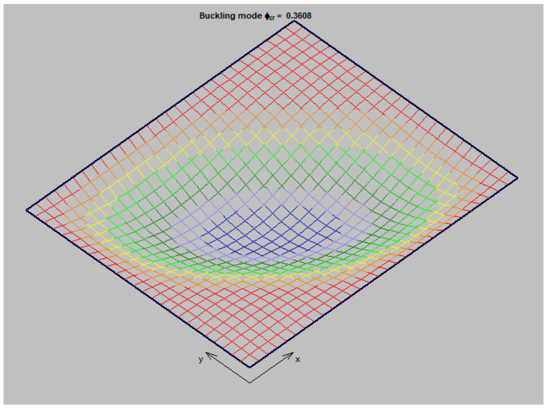


Figure 6. 11Case 2 Steel grades S235 and S355 Buckled shape of rectangular plate under shear stresses with simply support by EBPLATE



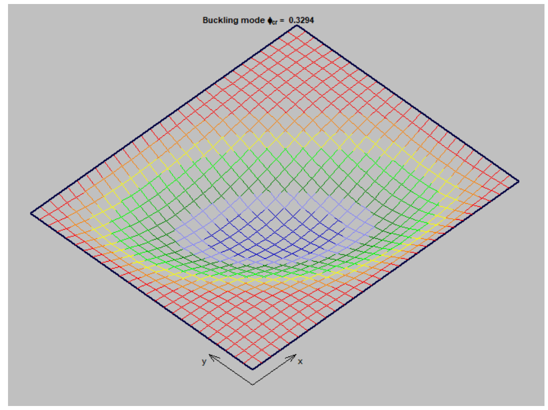


Figure 6. 12Case 2 Steel grades S420 and S460 Buckled shape of rectangular plate under shear stresses with simply support by EBPLATE

Table 6. 5Comparison of τcr case 2 analytical and EBPLATE results

Steel grade (MPa)	**τ_{cr} (MPa)**		**Difference %**
	EBPLATE	**Analytic**	
235	**0.645**	**0.655**	**1.52**
355	**0.425**	**0.434**	**2.07**
420	**0.359**	**0.367**	**2.17**
460	**0.328**	**0.335**	**2.08**

- **Comparison**

Once again, EBPLATE gives results very close to those found by the classical formulas, however with higher differences compared the case 1 for all steel grades.

- **ABAQUS**

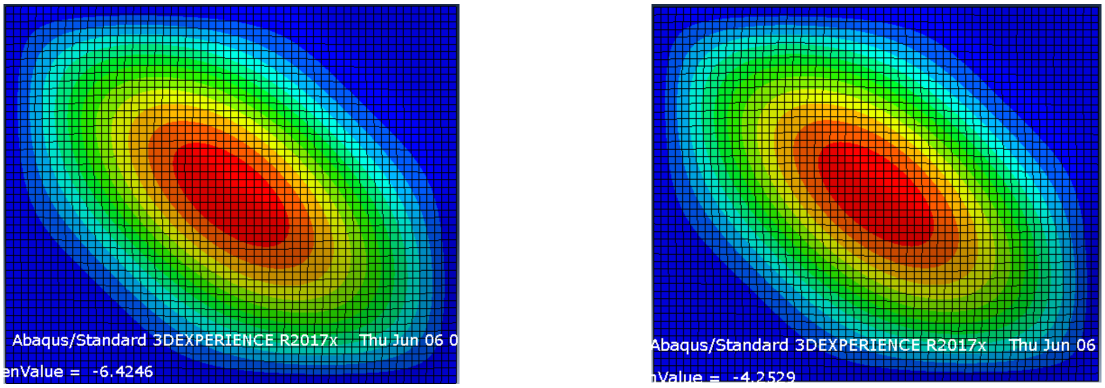


Figure 6. 13Case 2 Steel grades S235 and S355 Buckled shape of rectangular plate under shear stresses with simply support by ABAQUS

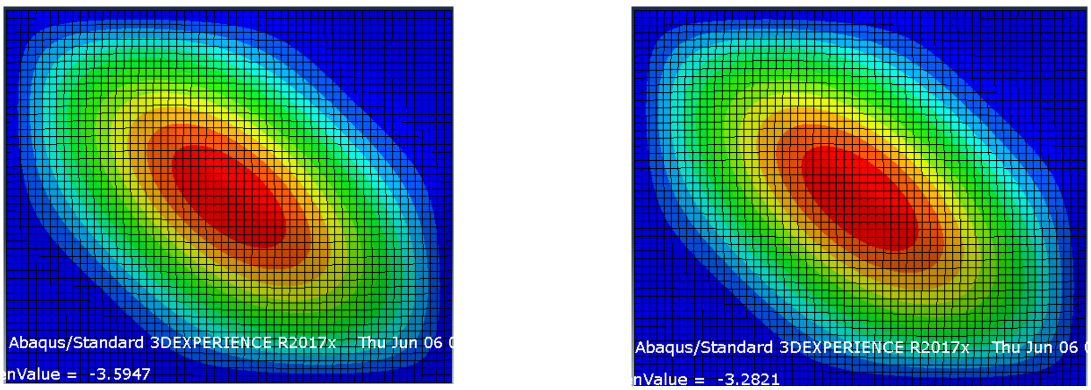


Figure 6. 14Case 2 Steel grades S420 and S460 Buckled shape of rectangular plate under shear stresses with simply support by ABAQUS

Table 6. 6Comparison of τcr case 2 analytical and ABAQUS results

Steel grade (MPa)	**Tcr**		**Difference %**
	Abaqus	**Analytic**	
235	**0.642**	**0.655**	**1.98**
355	**0.425**	**0.434**	**2.07**
420	**0.359**	**0.367**	**2.17**
460	**0.328**	**0.335**	**0.7**

Same remarks can be made as above.

- **Case 3: Simply Supported Rectangular Plate (1000x1500 mm)**

with the aspect ratio: 1.5

Table 6. 7Geometrical and material characteristics of the models

Characteristic	**Value**
Young's modulus *(E)*.	**210000MPa**
Poisson's ratio (*v*)	**0.3**
width of plate (*H*)	**1000 mm**
length of plate (a)	**1500mm**
thickness (t)	**10mm**

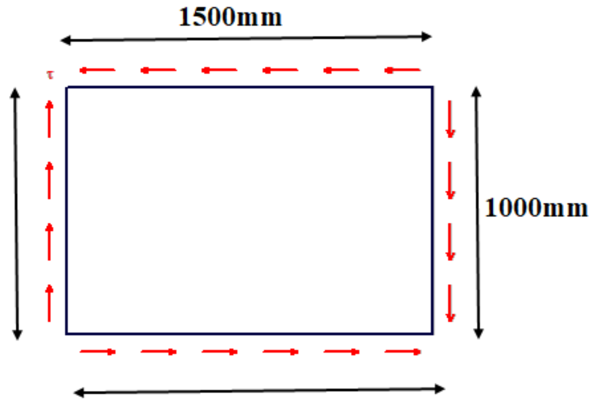


Figure 6. 15Geometry of rectangular thin plates(1000x1500x10mm)

- **Analytical evaluation**
- **Worked example of the analytical evaluation of τcr:**

$k\tau = 6.34+(4/(a/h)^2) = 6.11$

E=210000MPa

υ=0.3

a=1500mm ; h=1000mm ; t=10mm

τ_{cr} = 134.96MPa

S235: τ_{cr} = 0.574MPa

S355: τ_{cr} = 0.38MPa

S420: τ_{cr} = 0.321MPa

S460: τ_{cr} = 0.293MPa

- **EBPLATE:**

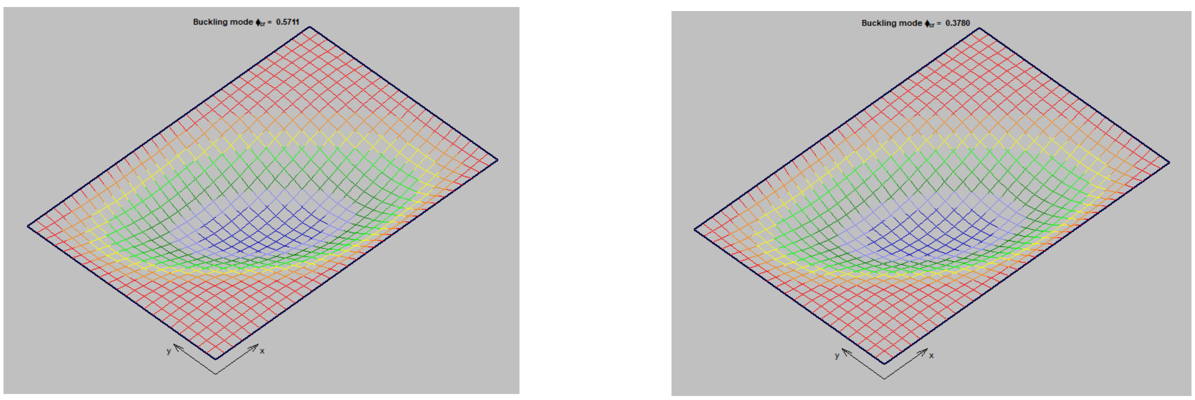


Figure 6. 16Case 3 Steel grades S235 and S355 Buckled shape of rectangular plate under shear stresses with simply support by EBPLATE

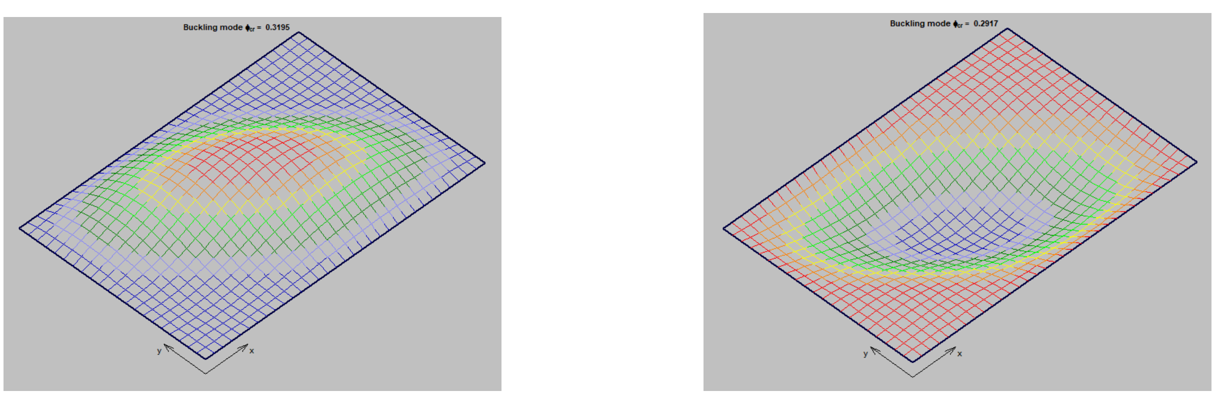


Figure 6. 17Case 3 Steel grades S420 and S460 Buckled shape of rectangular plate under shear stresses with simply support by EBPLATE

Table 6. 8Comparison of τcr case 2 analytical and EBPLATE results

Steel grade (MPa)	τ_{cr}		Difference %
	EBPLATE	Analytic	
235	0.57	0.577	1.21
355	0.38	0.38	0
420	0.32	0.32	0
460	0.29	0.293	0.3

As can be deducted from table 6.9 and surprisingly enough, the EBPLATE gives exactly the same results as the classical approach for higher steel grades, that is S355 and S420.

- **Abaqus:**

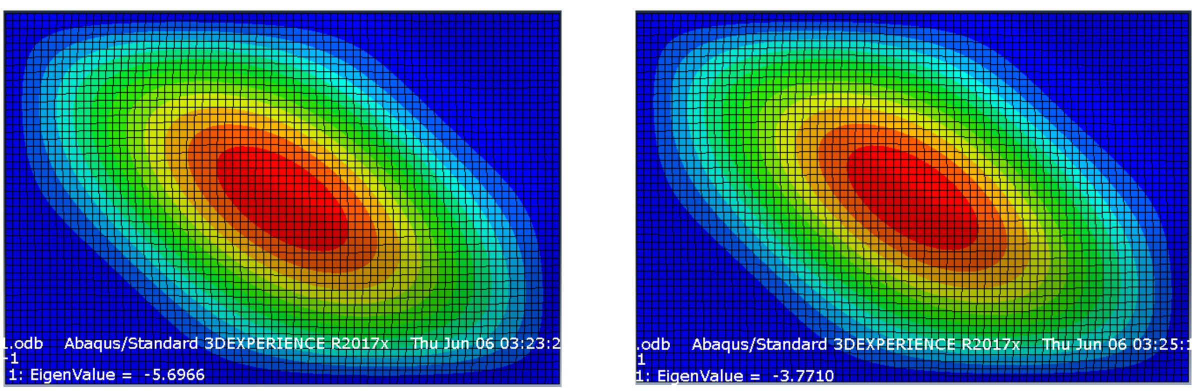


Figure 6. 18Case 3 Steel grades S235 and S355 Buckled shape of rectangular plate under shear stresses with simply support by ABAQUS

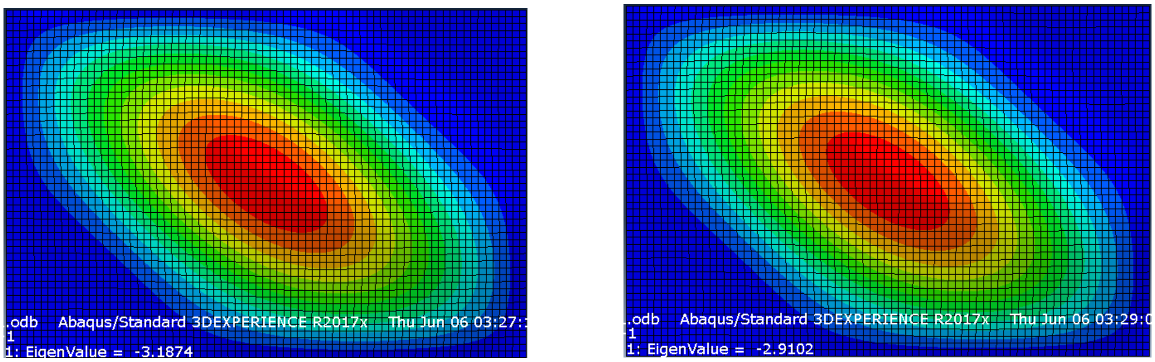


Figure 6. 19Case 3 Steel grades S420 and S460 Buckled shape of rectangular plate under shear stresses with simply support by ABAQUS

Table 6. 9Comparison of τcr case 2 analytical and ABAQUS results

Steel grade (MPa)	τ_{cr}		Differences %
	Abaqus	Analytic	
235	0.57	0.577	1.21
355	0.377	0.38	0.79
420	0.319	0.32	0.31
460	0.291	0.293	0.68

- **Case 4 Clamped Supported Square Plate (1000x1000 mm)**

- EBPLATE:

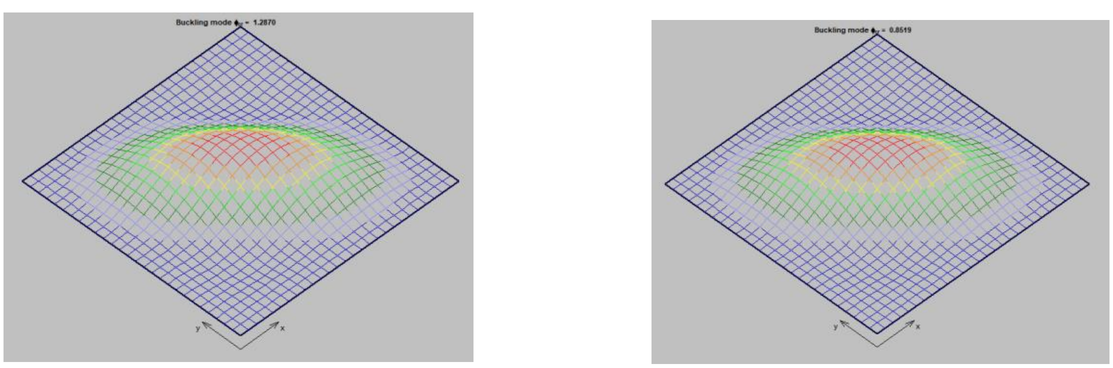


Figure 6. 20Case 4 Steel grades S235 and S355 Buckled shape of square clamped by EBPLATE

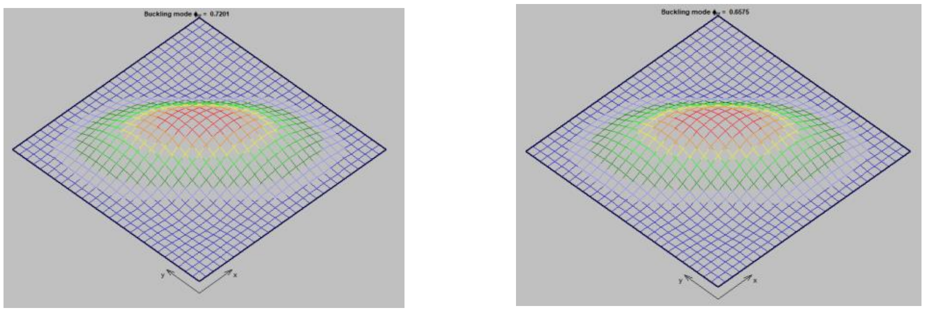


Figure 6. 21Case 4 Steel grades S420 and S460 Buckled shape of square clamped by

Table 6. 10Variation of τcr for case of square plates with Clamped supports

Steel grade (MPa)	Tcr		Difference %
	EBPLATE	Analytic	
235	1.29	1.285	0.39
355	0.852	0.85	0.235
420	0.72	0.72	0
460	0.66	0.656	0.61

Same remarks can be made for case 4.

- **Abaqus 3D:**

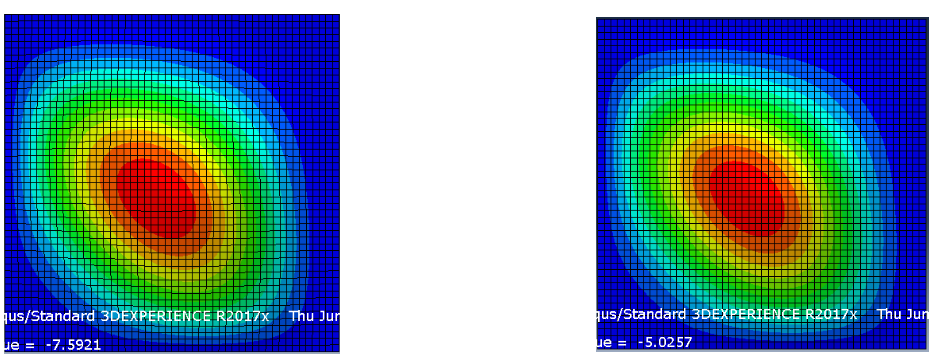


Figure 6. 22Case 4 Steel grades S235 and S355 Buckled shape of square clamped by ABAQUS

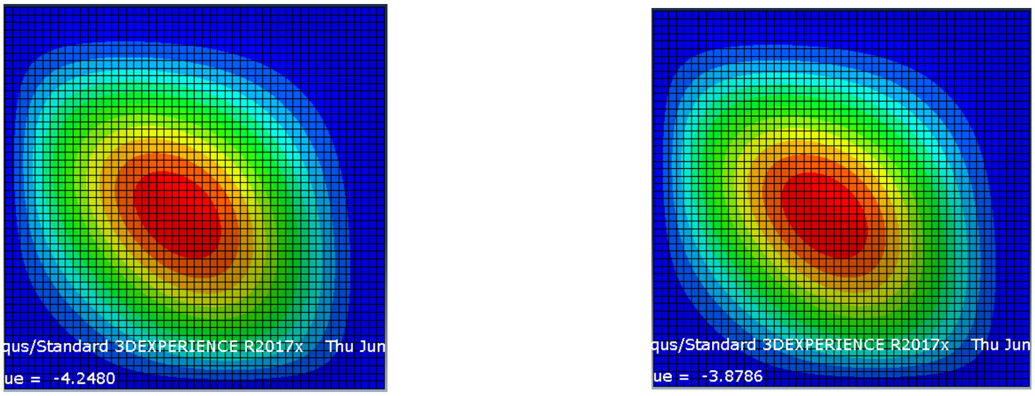


Figure 6. 23Case 4 Steel grades S420 and S460 Buckled shape of square clamped by ABAQUS

Table 6. 11Variation of τcr for case 4 of square plates with Clamped supports

Steel grade (MPa)	**τcr**		**Difference %**
	EBPLATE	**Analytic**	
235	**1.29**	**1.285**	**0.39**
355	**0.852**	**0.85**	**0.235**
420	**0.72**	**0.72**	**0**
460	**0.66**	**0.656**	**0.6**

Same remarks.

Table 6. 12Comparison of the τcr value for case of Square Plates with hinged supports between EC3 EBPLATE and ABAQUS

Cases	Steel Grade	τ_{cr} (MPa)			Differences in %		
		Abaqus	EBPLATE	analytical	Aba/EBPLATE	ABAQUS/ Ana	EBP/Ana
1000x1000x10	S235	0.75	0.75	0.754	0	0.53	0.53

	S355	0.496	0.498	0.5	0.4	0.8	0.4
	S420	0.42	0.42	0.422	0	0.47	0.47
	S460	0.383	0.385	0.385	0.52	0.52	0
1000x1200x10	S235	0.642	0.645	0.655	0.31	1.98	1
	S355	0.425	0.427	0.434	0.47	2.07	1.61
	S420	0.359	0.36	0.367	0.27	2.17	1.9
	S460	0.328	0.33	0.335	0.6	2.09	1.49
1000x1500x10	S235	0.57	0.57	0.577	0	1.21	1.21
	S355	0.377	0.38	0.38	0.79	0.79	0
	S420	0.319	0.32	0.32	0.31	0.31	0
	S460	0.291	0.29	0.293	0.31	0.68	1.02

Table 6. 13Comparison of the τcr values for case of Square Plates with fixed supports between Analytical, EBPLATE and ABAQUS

Cases	**Steel grade**				**Differences in%**		
		Aba	**EBP**	**Analytical**	**Aba/EBP**	**Aba/ana**	**EBP/Ana**
1000x1000x10	**S235**	**1.31**	**1.29**	**1.285**	**1.55**	**1.94**	**0.39**
	S355	**0.867**	**0.852**	**0.85**	**1.76**	**2**	**0.23**
	S420	**0.733**	**0.72**	**0.72**	**1.8**	**1.8**	**0**
	S460	**0.67**	**0.66**	**0.656**	**1.51**	**2.13**	**0.4**

General Discussion

Broadly speaking, results of the linear elastic buckling were as expected. The parametric study has shown once again the importance of the grade of the steel in the evaluation of the critical stress. In fact, as the steel grades grows, the smaller stress we get, showing that the mild steel has a better resistance against the elastic shear buckling than HSS cases with figures as twice as much for both the case of square or even rectangular plates simply supported. Also, same remarks can be drawn for the case of clamped plates. The aspect ratio plays an important role in elastic buckling under pure shear as it modifies the value of τ_{cr} by decreasing them as the ratio become larger. The square plate shows better behaviour compared to the rectangular ones.

Changing the support conditions from simply supported to clamped one's has shown its importance namely by increasing the value of τ_{cr} for the particular case of square plate roughly twice as much for each studied case.

As can be seen from the precedent sections, which represents the parametric study of elastic linear buckling of thin plates that the values of the critical shear buckling stress given by the three means of investigations are close to each other's with very slight differences.

The numerical outcomes from ABAQUS can be then trusted for a more complicated analyse: inelastic buckling behaviour of the same cases.

It can be concluded that the results of this section can be considered as a kind of validation of the ABAQUS models can be approved.

Additionally, the following remarks

- The differences between ABAQUS and EBPLATE are minimal, often 0 or close to 0, indicating consistency between the simulation and the Elastic Buckling Program.
- Differences between the ABAQUS/EBPLATE results and the analytical results are also small but slightly higher than ABAQUS/EBPLATELATE differences.
- Throughout different dimensions, the percentage differences are consistently low, reinforcing the reliability of both ABAQUS and EBPLATELATE methods when compared to the analytical approach. 6.3 RESULTS AND DISCUSSION OF THE INELASTIC BUCKLING ANALYSIS:

6.3.1 Introduction

In this chapter, an introduction to the inelastic buckling analysis background is given. A full second order analysis takes into account the material non-linearity and geometric deformation. This second order analysis is essential when the buckling behaviour is influenced by the modified geometry of the structure under load. However, the axial strain in the arch members will cause the arch to flatten, which increases the axial forces and strains. The inelastic buckling models are built-up in Abaqus to investigate the impact of lateral torsional buckling on the carrying capacity of slender sections is provide and discussed.

6.3.2 Inelastic buckling analysis (GMNL)

It is vital to first develop a reliable and efficient FE model capable of producing realistic and accurate results particularly for elastic buckling, inelastic buckling and even post-buckling models. The validation of the linear FEA model has been successfully done with the data available in literature, which gives a more confidence in the models which will be used as initial condition for more complex buckling analysis of plate with and without cut-out. When studying the behaviour of thin plates instability, a geometrical and material non-linear imperfection analysis (GMNL) is carried out. To determine the inelastic buckling resistance of the plate as it considered to give most true buckling resistance of beam. Also, as explained in the previous section, the first order buckling analysis would only give eigenvectors for buckling modes related to the original geometry. The buckling instability, the load-displacement response shows a negative stiffness and the structure must release strain energy to remain in equilibrium Figure 6.1. Therefore, it is important that a solution method is chosen that can predict the load-displacement response after lateral torsional buckling has occurred.

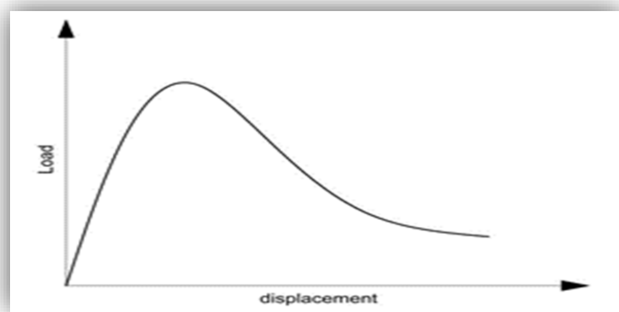


Figure 6. 24Possible non-linear buckling load-displacement behaviour

6.3.3 Modelling the nonlinear behaviour using ABAQUS

The theoretical background of such solution can be the modified Riks method or arc-length method. In ABAQUS, this is an algorithm which provides effective solutions for such cases. The modified Riks method uses a tangent line of a function to intersect with an arc, situated at the end of every step. From this point on, the curve will converge over the arc-length until it reaches an intersection of the arc with the function. At this point, the step is completed and the process will continue with the next step (Figure 6.2).

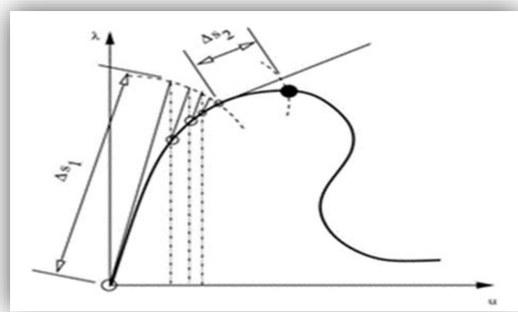


Figure 6. 25Graphical example of the modified RIKS method

In addition to the linear elastic model, the non-linear plastic model also includes the plastic material properties.

6.3.4 Material properties

(1) Material properties should be taken as characteristic values.

(2) Depending on the accuracy and the allowable strain required for the analysis the following assumptions for the material behaviour may be used, see Figure 6.3:

a) elastic-plastic without strain hardening

b) elastic-plastic with a nominal plateau slope

c) elastic-plastic with linear strain hardening

d) true stress-strain curve modified from the test results as follows

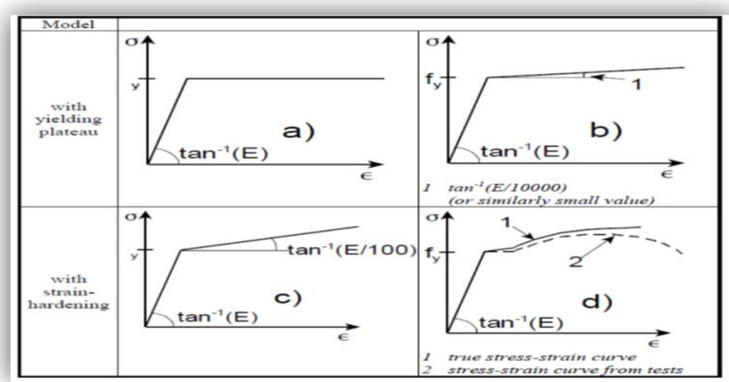


Figure 6. 26Modelling of material behaviour [Eurocode 3: Design of steel structures]

$$\sigma_{true} = \sigma(1+\varepsilon)$$
$$\varepsilon_{true} = \ell n(1+\varepsilon)$$

6.3.5 Results and discussion

Buckling refers to the sudden collapse of a structural member, subjected to high shear stress loading. This collapse takes the form of a sudden lateral deflection of the structural member. Therefore, the structure's load bearing ability is compromised under buckling. A structure's behaviour under loading is usually studied with the use of load-displacement plots. This applies also to studying the structural behaviour in the post-buckling region (past the bifurcation point). The results of the present investigation will be presented and discussed in the terms of loading curve history: Buckling loads vs. out-plane displacement curves, the second part of the discussion is devoted to the stress and deformation pattern analyses by mean of available failure criteria implanted in ABAQUS: the Von -Mises yield criterion which is by far the most common yield criteria in metals.

Obtained Results for Simply supported plates

Case 1: Simply Supported Square Plate (1000x1000x10 mm):

- **Load-out of plane deflections curves:**

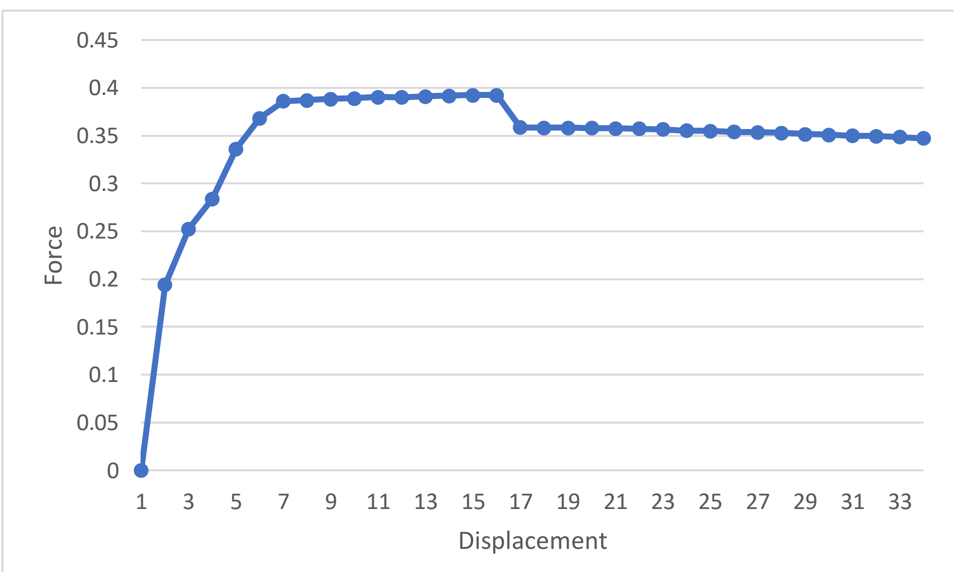


Figure 6. 27Elastic and inelastic load- out of in plane deflection of plate square with steel grade S235

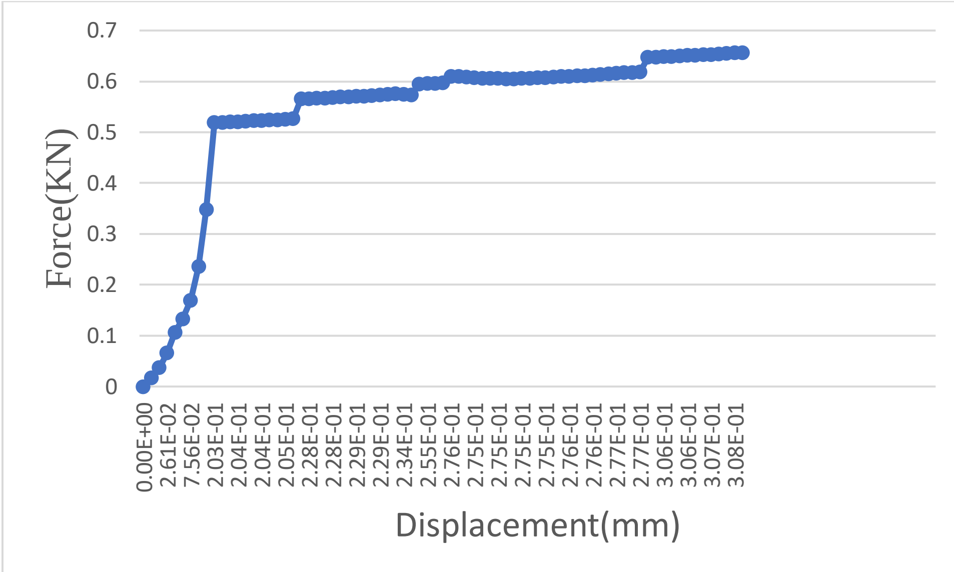


Figure 6. 28Elastic and inelastic load- in plane deflection of intact plate square with steel grade S355

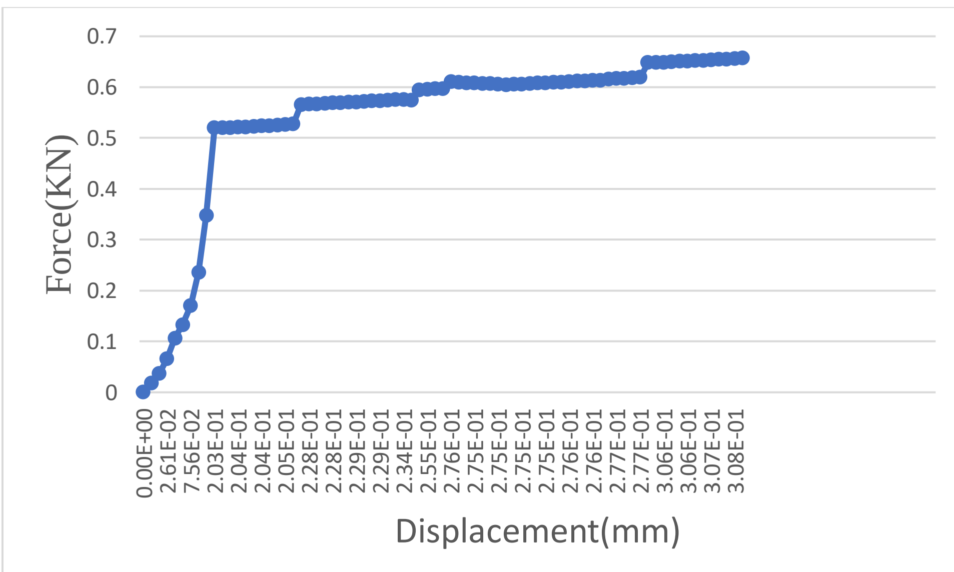


Figure 6. 29Elastic and inelastic load- in plane deflection of plate square with steel grade S420

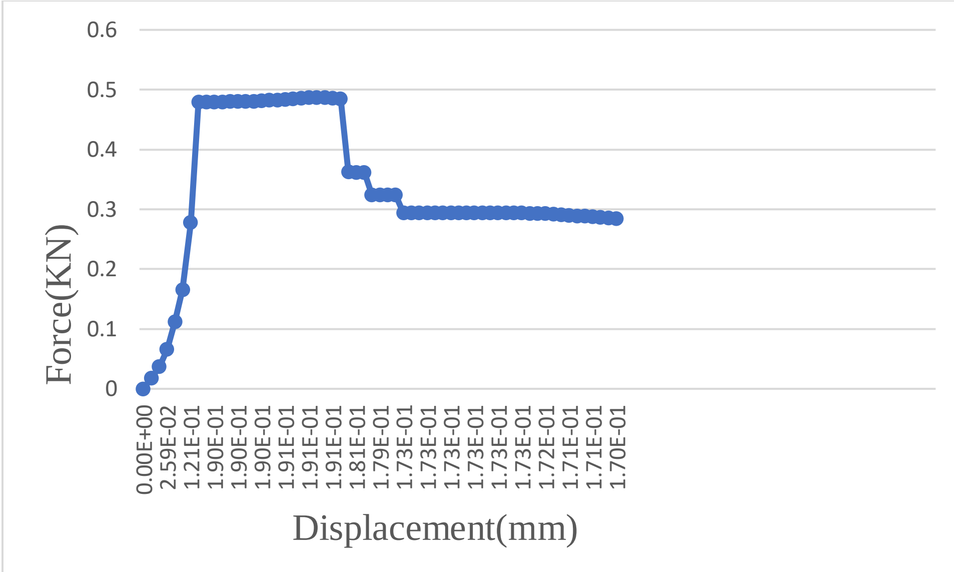


Figure 6. 30Elastic and inelastic load- in plane deflection of plate square with steel grade S460

- **Load- out-plane deflections curves**

Load-deflections curves are chosen to be better retrace the loading history for studied different cases. All load-deflection curves show two distinct parts: linear and nonlinear branches. The following Figures represent the skeleton curves of the FE-models of Load- out of plane deflection are depicted for models having the same features and to highlight the effect of steel grades and of the other parameters considered in this study.

For Case 1: Simply Supported Square Plate (1000x1000x10 mm): Figure 6.27 to Figure 6.30. As can be seen from these figures the inelastic behaviour of the plate is better represented for S235 grade with three distinct branches representing the linear and nonlinear behaviour while the third shows a decrease of the stiffness of the plate and then a relatively stable behaviour for the remaining. However, this "perfect" situation becomes more and more as the steel grades becomes larger which suggests that the implanted of the adequate initial imperfection needs to be done. This will need more effort and long time to be well done. Also, a better refining model has to be built-up and the curve must shows the post-buckling behaviour. .

As can be deduced from the above figures, the increase of steel grade does not strengthen the plate.

- **Stresses and deformations results**

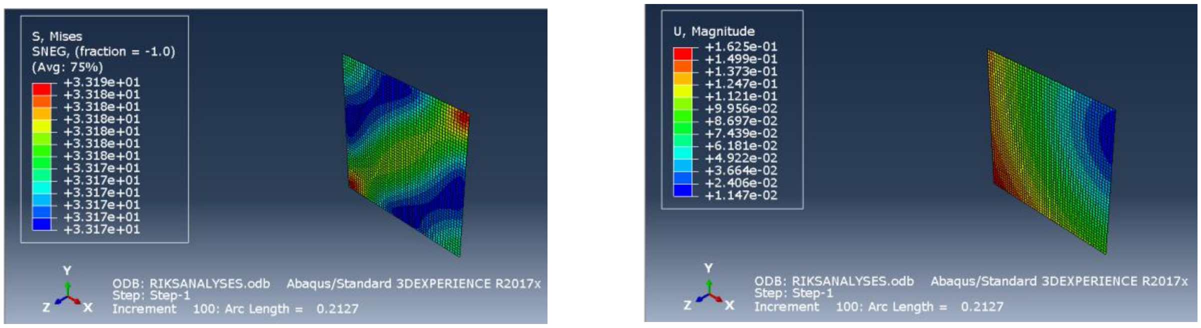


Figure 6. 31Case of Steel grade S235 Contours of Von-Mises stress distribution and deformed shape

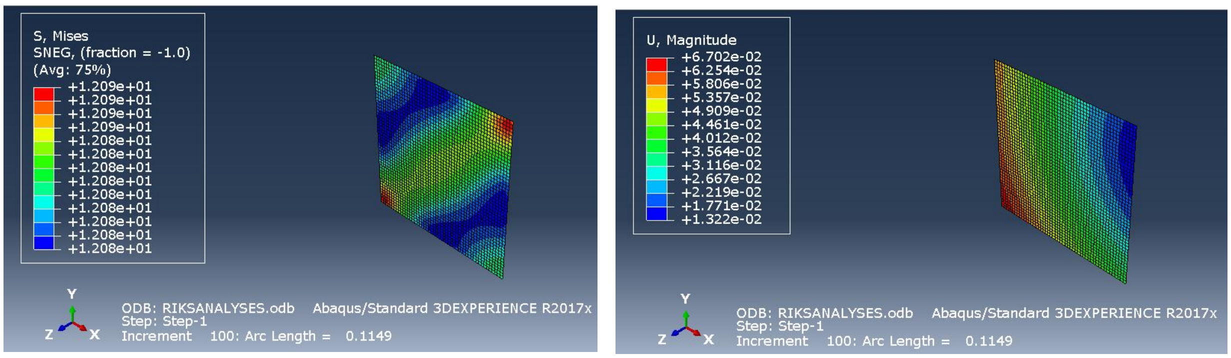


Figure 6. 32Case of Steel grade S355 Contours of Von -Mises stress distribution and deformed shape

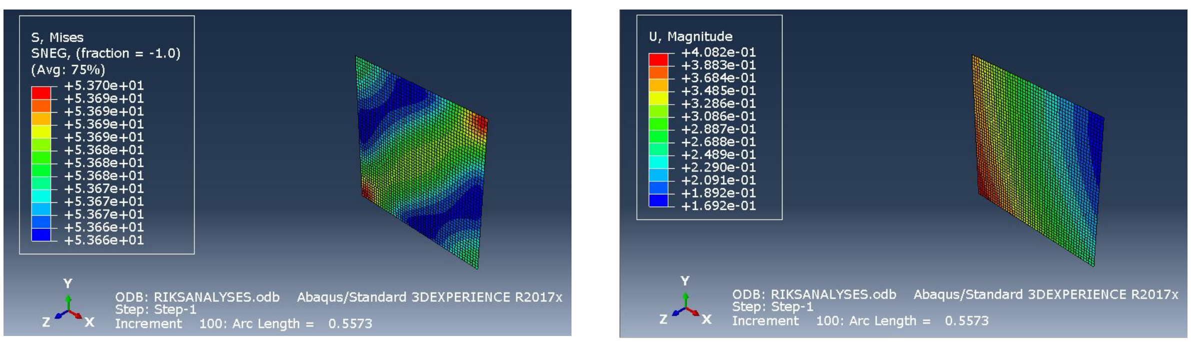


Figure 6. 33Case of Steel grade S420 Contours of Von -Mises stress distribution and deformed shape

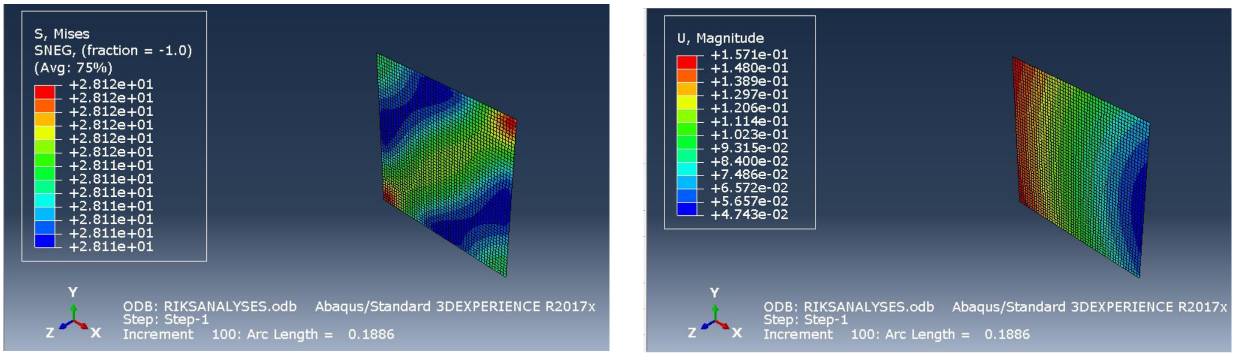


Figure 6. 34Case of Steel grade S460 Contours of Von -Mises stress distribution and deformed shape

-Discussion of the patterns of Von -Mises yielding criteria and maximum deformation

The above figures are extracted from the inelastic finite element by Abaqus. They are of two kinds: Stress and deformation contours at the ultimate deformation step of analysis obtained from inelastic analysis. As can be seen from Figures 6.31 to 6.34, same pattern representing the stresses states for all studied cases can be seen with different values of the equivalent stress for geometric nonlinear analysis with higher values for superior grades. It was also observed that during the extraction of results that Von -Mises stress distribution stared, as demonstrated for the classical theory, from the corners moving to the centre of plate this mean that the effective stresses are in the corners so the plate is generally depend in the most on the edges and corners. Once again, the ultimate magnitude deformation is shown to be situated at one edge of the plate for all studied cases despite their steel grades. As the steel is more ductile as the magnitude of deformation is greater.

When the plate buckling strength exceeds the steel plate yield strength, the material plasticity leads to plastic deformation that redistributes the stresses, and paves the way towards extending the plastic stress region in a manner that enables the plate section to develop larger internal force resultant that balances the incremental applied loading. In this case, the development of plastic deformation region offered the desired ample warning prior to failure when steel Von -Mises (σe) stresses fell on the failure surface. It can also noticeable, that imperfection sensitivity of the plate plays an important role different steel grades square plates by resisting to the out-plane deformation as can be seen from relative figures despite the configuration of the square plate.

CASE 2: Square Plate (1000x1000x10mm) with fixed support:

- **Load-out-plane deflections curves:**

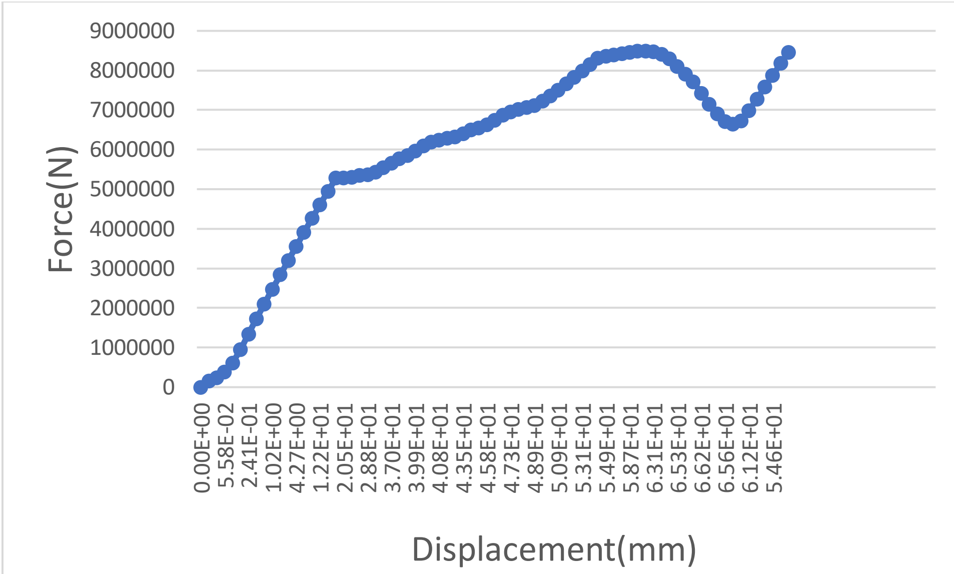


Figure 6. 35Elastic and inelastic load- in plane deflection of intact plate square with steel grade S235

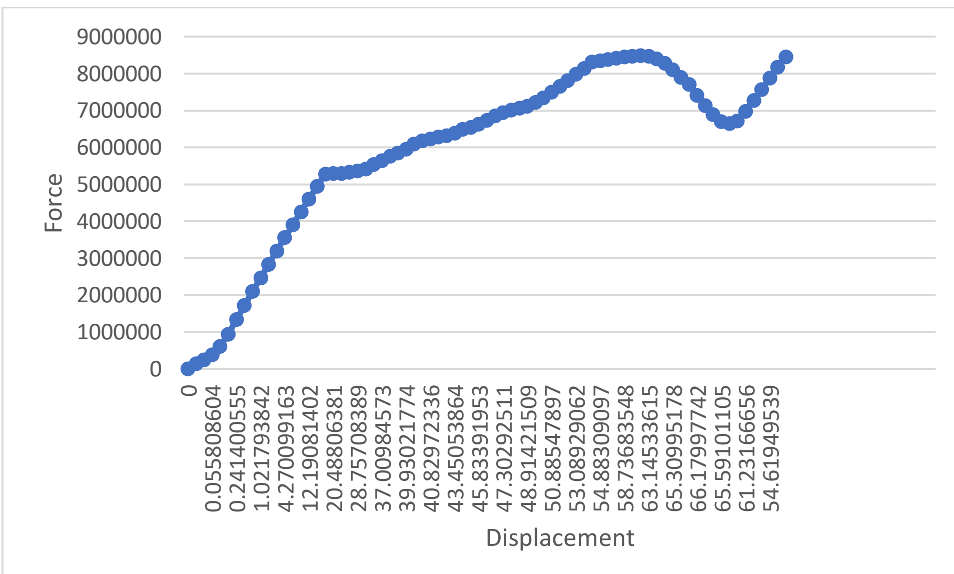


Figure 6. 36Elastic and inelastic load- in plane deflection of intact plate square with steel grade S355

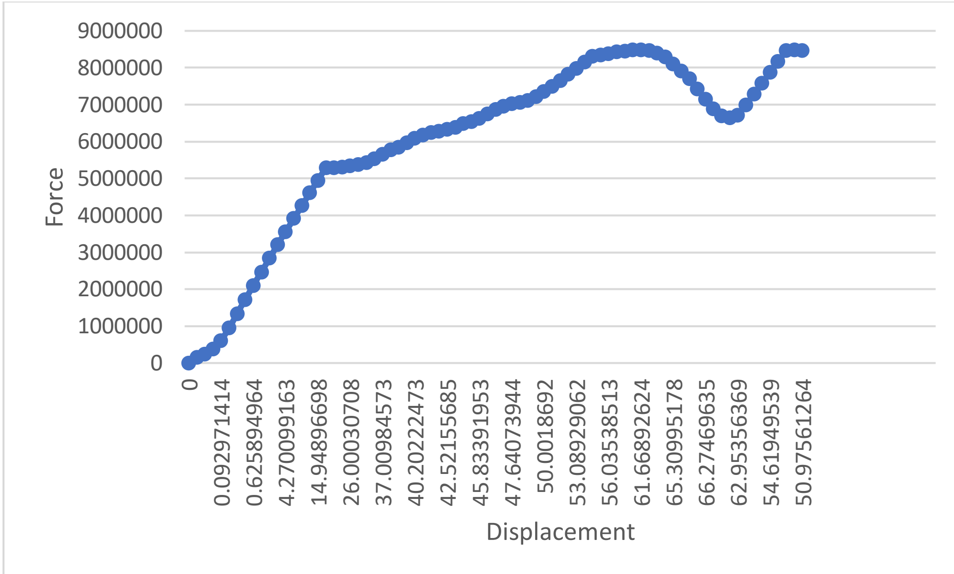


Figure 6. 37Elastic and inelastic load- in plane deflection of intact plate square with steel grade S420

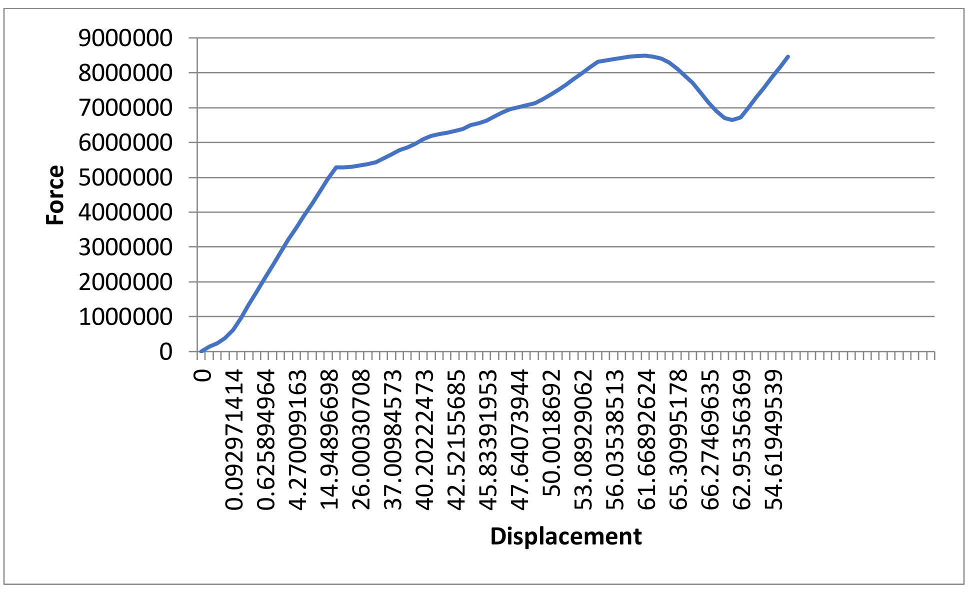


Figure 6. 38Elastic and inelastic load- in plane deflection of intact plate square with steel grade S460

- **Stresses and deformations results**

For case 2 representing the clamped plate, Figures 6.33 to 6.38 depict the general behaviour of square plate with different steel grades from ductile (s235) to HSS (S420 and S460). Compared to the previous case, these figures show better results and conformed to the theoretical predictions. The whole shape of the load- deflection curves are similar with, different values of course. In fact, an elastic and elasto-plastic behaviour is clearly seen for all cases with specific post-buckling behaviour for higher values of the out-plane displacements as it is suggested in RIKS Method. It can be said, as an overall conclusion that the model represents roughly well the actual behaviour of the clamped plate.

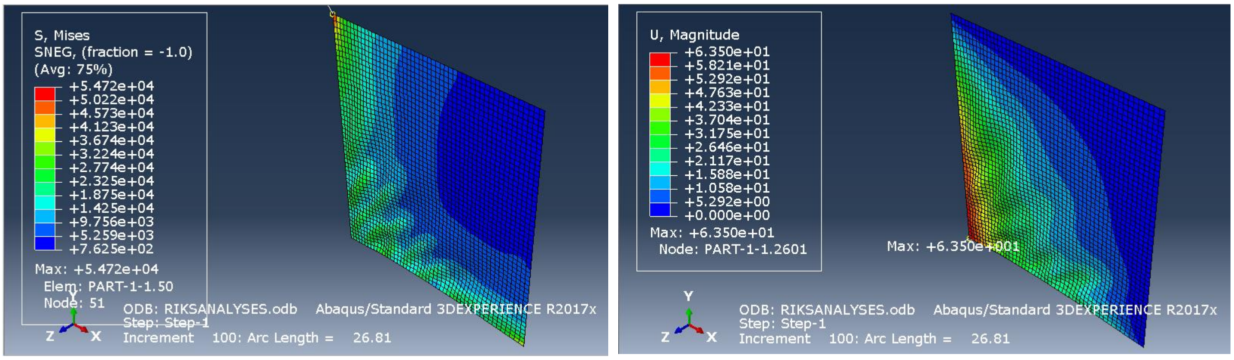


Figure 6. 39Case of Steel grade S235 Contours of Von -Mises stress distribution and deformed shape

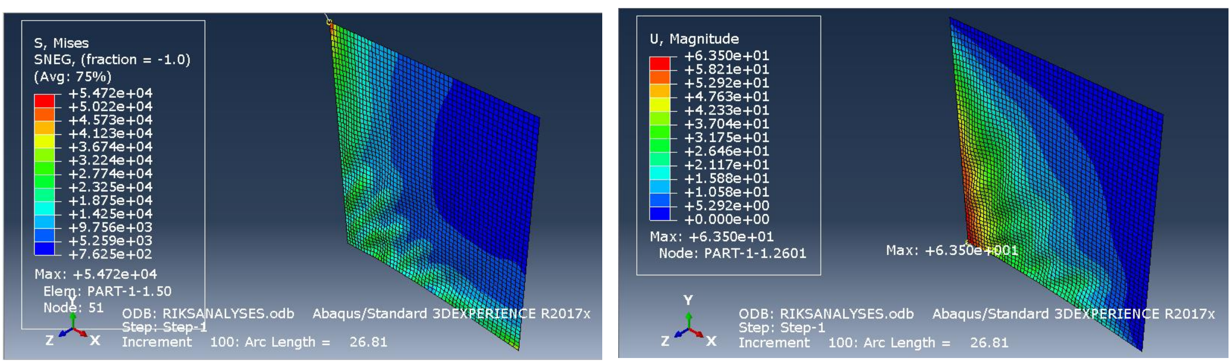


Figure 6. 40Case of Steel grade S355Contours of Von -Mises stress distribution and deformed shape

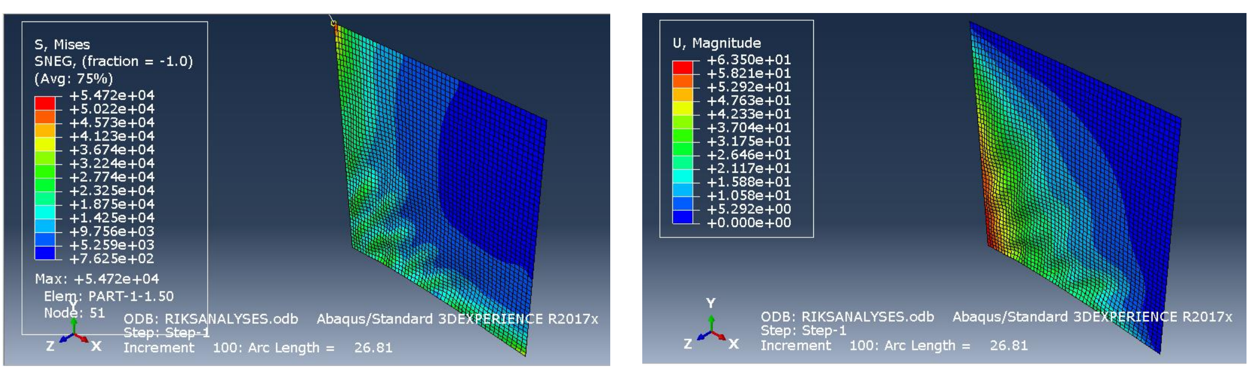


Figure 6. 41Case of Steel grade S420: Contours of Von -Mises stress distribution and deformed shape

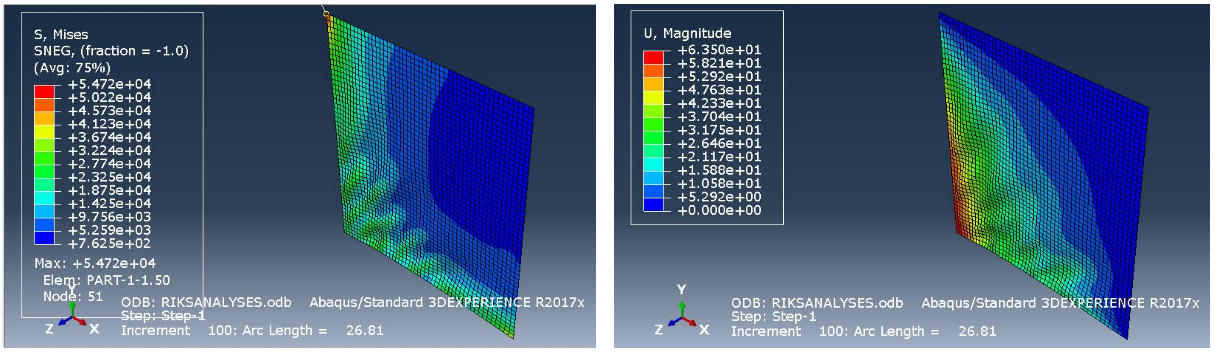


Figure 6. 42Case of Steel grade S460 Contours of Von -Mises stress distribution and deformed shape

Discussion of the patterns of Von -Mises yielding criteria and maximum deformation

Once again, as in the previous section the above figures are extracted from the inelastic finite element by Abaqus. They are of two kinds: Stress and deformation contours at the ultimate deformation step of analysis obtained from inelastic analysis. As can be seen from Figures 6.39 to 6.42, same pattern representing the stresses states for all studied cases can be seen with almost same values of the equivalent stress for geometric nonlinear analysis with higher values for superior grades. This is due essentially to the support conditions: clamped supports. It was also observed that during the extraction of results that Von -Mises stress distribution stared, as demonstrated for the classical theory, from the corners moving to the centre of plate this mean that the effective stresses are in the corners so the plate is generally depend in the most on the edges and corners. Once again, the ultimate magnitude deformation is shown to be situated at partially one edge of the plate for all studied cases despite their steel grades. As the steel is more ductile as the magnitude of deformation is greater.

General remarks

Nonlinear analysis was possible by a step realistic behaviour of the structure, which is programmed in ABAQUS/Standard. Incremental procedure based on RIKS algorithm is used to solve system of nonlinear equations is considered when non-linearity of the material, such as plasticity is present, or post-buckling behaviour is of interest this was possible by a step-by-step loading process, which simulates a more realistic behaviour of the structure, which is programmed in ABAQUS/Standard. Incremental RIKS algorithm. The basic RIKS algorithm is essentially Newton's method with load magnitude as an additional unknown to solve simultaneously for loads and displacements, thus, it can provide solutions even in cases of complex and unstable response.

Generally speaking, thin steel plates are rather sensitive for the influence of initial geometrical imperfections. The accuracy and stability of solutions is a difficult consideration in nonlinear analysis. As this study pertains to steel plates under shear forces, it was very difficult to find-out the true value of the initial geometric imperfection, especially for the simply supported plate. It was decided to attribute different values, depending of the steel grade, of initial geometric imperfections. After long and comprehensive tries, it was at the end possible to have some acceptable values for imperfection especially for clamped plates.

When the plate buckling strength exceeds the steel plate yield strength, the material plasticity leads to plastic deformation that redistributes the stresses, and paves the way towards extending the plastic stress region in a manner that enables the plate section to develop larger internal force resultant that balances the incremental applied loading. In this case, the development of plastic deformation region offered the desired ample warning prior to failure when steel Von -Mises (σ_{equ}) stresses fell on the failure surface. It can also noticeable, that imperfection sensitivity of the plate plays an important role higher steel grade in both simply and clamped square plates by resisting to the out-plane deformation as can be seen from relative figures. Same remarks can be made, as it was for the elastic buckling analysis, the value of the critical buckling load depends mainly on the thickness of the square plate and the buckling factor increases. The above figures indicate that the shape of the buckling behaviour for a given plate depends on the support conditions being very efficient for clamped ones and have maintained a similar pattern for Von -Mises stress distribution contour or deformed shapes.

As demonstrated, the RIKS algorithm is a powerful tool for assessing a structure's behaviour in the post-buckling region for the clamped plates. By introducing imperfections (either by mesh perturbation or by scaled mode shapes-as performed here-) we achieve a smoother transition to the post-buckling region. The cases shown above can be applied when investigating the behaviour of imperfection-sensitive structures. The results are showing the importance of the steel grade of plate along with the imperfection sensitive and the effect of the shapes and sizes of plates along with support conditions and steel grades

CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORKS

CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORKS

This present Master's dissertation aims to confine the advanced topic study of the elastic and inelastic shear buckling behaviour of thin plates made from high strength steel (HSS) and that trigger initial geometrical nonlinearities only. Also, to evaluate the impact of instability behaviours on the carrying capacity of steel plates with different support conditions. It is vital to first develop a reliable and efficient FE model capable of producing realistic and accurate results particularly for elastic buckling, inelastic buckling and even post-buckling models. The subject of this dissertation. Few researches work on the elastic and inelastic of shear buckling behaviour of HSS plate are available in literature. The non-linear finite element method with initial geometric imperfection is compulsory to capture the shear buckling behaviour of the HSS Thin Steel Plate. These initial geometry imperfections can come from the slender structure that cannot maintain its perfect shape or useless quality during the assembly process.

A-CONCLUSIONS:

For the elastic linear shear buckling analysis

As far as the elastic linear shear buckling analysis is concerned Results of the linear elastic buckling were as expected.

The undertaken parametric study has shown:

- The importance of the grade of the steel in the evaluation of the critical stress. In fact, as the steel grades grows, the smaller stress is given, showing that the mild steel has a better resistance against the elastic shear buckling than HSS cases with values as twice as much for both the case of square or even rectangular plates simply supported.
- Same remarks can be drawn for the case of clamped plates. The aspect ratio plays an important role in elastic buckling under pure shear as it modifies the value of τ_{cr} by decreasing them as the ratio become larger. The square plate shows better behaviour compared to the rectangular ones.
- Changing the support conditions from simply supported to clamped one's has shown its importance namely by increasing the value of τ_{cr} for the particular case of square plate roughly twice as much for each studied case.
- As can be seen from the chapter 5 (first part), which represents the parametric study of elastic linear buckling of thin plates that the values of the critical shear buckling stress

given by the three means of investigations are close to each other's with very slight differences.

- The numerical outcomes from ABAQUS can be then trusted for a more complicated analyse: inelastic buckling behaviour of the same cases.
- It can be concluded that the results of this section can be considered as a kind of validation of the ABAQUS models can be approved.

Additionally, the following remarks can be made:

- The differences between ABAQUS and EBPLATE are minimal, often 0 or close to 0, indicating consistency between the simulation and the Elastic Buckling Program.
- Differences between the ABAQUS/EBPLATE results and the analytical results are also small but slightly higher than ABAQUS/EBPLATELATE differences.
- Throughout different dimensions, the percentage differences are consistently low, reinforcing the reliability of both ABAQUS and EBPLATELATE methods when compared to the analytical approach.

For the inelastic shear buckling analysis

As few researches work on shear buckling are available, several difficulties have risen during the achievement of this work especially the 3D modelling of boundary conditions supports for clamped plates and mainly the built-up of the inelastic FEA models which has been principally very hard task and time consuming for evaluating the imperfection sensitivity of the plates.

Thin steel plates are rather sensitive for the influence of initial geometrical imperfections. The accuracy and stability of solutions is a difficult consideration in nonlinear analysis. As this study pertains to steel plates under shear forces, it was very difficult to find-out the true value of the initial geometric imperfection, especially for the simply supported plate. It was decided to attribute different values, depending of the steel grade, of initial geometric imperfections. After long and comprehensive tries, it was at the end possible to have some acceptable values for imperfection especially for clamped plates.

The results of the present investigation are presented and discussed in the terms of loading curve history: Buckling loads vs. out-plane displacement curves, the second part of the discussion is devoted to the stress and deformation pattern analyses by mean of available failure criteria implanted in ABAQUS: the Von Mises yield criterion which is by far the most common yield criteria in metals.

Some conclusions can be drawn:

Load-deflections curves are chosen to be better retrace the loading history for studied different cases. All load-deflection curves show two distinct parts: linear and nonlinear branches. Figures in chapter 6 represent the skeleton curves of the FE-models of Load- out of plane deflection are depicted for models having the same features and to highlight the effect of steel grades and of the other parameters considered in this study.

For Case 1: Simply Supported Square Plate (1000x1000x10 mm): Figure 6.27 to Figure 6.30.

- The inelastic behaviour of the plate is better represented for S235 grade with three distinct branches representing the linear and nonlinear behaviour while the third shows a decrease of the stiffness of the plate and then a relatively stable behaviour for the remaining.
- This "perfect" situation becomes more and more poor as the steel grades becomes larger which suggests that the implanted of the adequate initial imperfection needs to be done. This will need, certainly, more effort and long time to be well done. Also, a better refining model has to be built-up.
- It can be said, as an overall conclusion that the model represents poorly the actual behaviour of simply supported plate.
- From all curves no post-buckling behaviour is remarkable which leads to build-up a more refined with a more dense meshing especially in areas in the vicinity of the supports.

For case 2 representing the clamped plate, Figures 6.33 to 6.38 depict the general behaviour of square plate with different steel grades from ductile (S235) to HSS (S420 and S460).

- Compared to the previous case, these figures show better results and conformed to the theoretical predictions.
- The whole shape of the load- deflection curves are similar with, different values of course. In fact, an elastic and elasto-plastic behaviour is clearly seen for all cases with specific post-

buckling behaviour for higher values of the out-plane displacements as it is suggested in RIKS Method.

- It can be said, as an overall conclusion that the model represents roughly well the actual behaviour of the clamped plate.
- In all curves, a post-buckling behaviour, as it is in the classical theory is noticeable which indicates

Patterns of Von -Mises yielding criteria and maximum deformation

From the inelastic finite element by Abaqus. They are of two kinds: Stress and deformation contours at the ultimate deformation step of analysis obtained from inelastic analysis.

Case 1 Simply supported plate

- Same pattern representing the stresses states for all studied cases can be seen with different values of the equivalent stress for geometric nonlinear analysis with higher values for superior grades.
- It was also observed that during the extraction of results that Von -Mises stress distribution stared, as demonstrated for the classical theory, from the corners moving to the centre of plate this mean that the effective stresses are in the corners so the plate is generally depend in the most on the edges and corners.
- Once again, the ultimate magnitude deformation is shown to be situated at one edge of the plate for all studied cases despite their steel grades. As the steel is more ductile as the magnitude of deformation is greater.

Case 2 representing the clamped plate,

- The general behaviour of square plate with different steel grades from ductile (s235) to HSS (S420 and S460). Compared to the previous case, these figures show better results and conformed to the theoretical predictions.
- The whole shape of the load- deflection curves are similar with, different values of course. In fact, an elastic and elasto-plastic behaviour is clearly seen for all cases with specific post-buckling behaviour for higher values of the out-plane displacements as it is suggested in RIKS Method.
- It can be said, as an overall conclusion that the model represents roughly well the actual behaviour of the clamped plate.

B- SUGGESTIONS FOR FURTHER WORKS

It should be noted that the work in this dissertation is at an initial stage and possible avenues of future research should be carried out. Some of the possible avenues of future work are

1- Further verification is required by considering an advanced research the adequate imperfection with some experimental data;

2- Extend the study to the scenarios in which out-of-plane stability effects with imperfections, which consider the combined influence of both geometric imperfections and residual stresses, of steel and high strength stainless steel plates by GMNIA.

3- Equivalent imperfections for second-order inelastic analysis of structural members for local buckling.

4- Carry out an inelastic analysis taking into account for imperfections including imperfections by material nonlinearity, geometric imperfection, and residual stresses.

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