الجمهـوريـــة الجـــزائرية الديـــــقراطيـة الشـــــــعبيـة

Democratic and Popular Republic of Algeria

وزارة التـــعـليــم العـــالي والبــحث العـــلـمــي

Ministry of Higher Education and Scientific Research

جـــــــــــــامعة العربـــي التبســــي – تبســــــــــــة

Larbi Tébessi University – Tébessa –

Faculty of Science and Technology

Civil Engineering Department

A thesis submitted in partial fulfilment of the requirements for the degree of Master Research in Civil Engineering ''Pavement and Public Works Engineering''

By : DAMRA MOHAMED FAYEZ

A Formulation of a Steel-Concrete Mixed Beam Finite Element with Partial Shear Connection

Thesis defended on 29st june 2020

Thesis committee

President: Mr. HMIDANE H'mida

Thesis Advisor: Dr. BOUTAGOUGA Djamel

Examiner: Mr. BOULAARAS Said

Academic Year: 2019/2020

Acknowledgment

Thank God, Lord of the Worlds, this thesis has been completed after

Tired, hard and continuous work, I dedicate this humble work to my family, which I will not be entitled to, no matter how much I have spoken about it, and it has always been my support in strength and hope, and to my country and my land, Palestine. Also, thanks should be extended to all my colleagues and brothers who accompanied me on the path of long study and all my happy and sad days. In particular, my colleague Baker AbuTeir, for his great support in completing this work. I would like to express my sincere thanks and gratitude to my supervisor and professor Dr.BOUTAGOUGA Djamel. For his great patience, constant encouragement, kindness, and generosity in all stages of writing my research, he always found time to share his vast and profound knowledge with me in a very clear way. Major thanks to all professors working in the Civil Engineering Department for support and assistance at all levels during my university studies.

*Thank God first and foremost***.**

MOHAMAD FAYEZ DAMRA

ABSTRACT:

This dissertation presents the formulation of a mixed-beam finite element for the linear elastic analysis of steel-concrete mixed beams with partial interaction.

Bernoulli's kinematic assumptions are considered for both layers "steel and concrete", and a linear elastic behaviour is adopted for the mixed beam. Through considering the relationship between the interface longitudinal shear and the corresponding slip, the shear connection is modelled by considering two models of connection to the interface: discrete connection and continuous connection. The mixed-beam finite element stiffness matrix has been derived using the standard displacement based formulation. A Matlab program has been developed and used to investigate the behaviour of mixed steel-concrete beams through several numerical applications. Finally, A parametric analysis has been carried out to study the influence of partial interaction on the global behaviour of mixed beams.

Résumé :

Cette thèse présente la formulation d'un élément fini poutre-mixte pour l'analyse élastique linéaire de poutres mixtes acier-béton avec connexion partielle.

Les hypothèses cinématiques de Bernoulli sont prises en compte pour les deux couches « acier et béton », et un comportement élastique linéaire est adopté pour la poutre mixte. En considérant la relation entre le cisaillement longitudinal de l'interface et le glissement correspondant, la connexion est modélisée en considérant deux modèles de connexion: connexion discrète et connexion continue. La matrice de rigidité de l'élément fini poutre -mixte a été dérivée en utilisant la formulation standard basée sur le déplacement. Un programme Matlab a été développé et utilisé pour étudier le comportement des poutres mixtes acier-béton à travers plusieurs applications numériques. Enfin, une analyse paramétrique a été accomplie pour étudier l'influence de l'interaction partielle sur le comportement global des poutres mixtes.

ملخص:

تقدم هذه الرسالة صياغة عنصر محدد للعوارض المختلطة للتحليل المرن الخطي للعوارض المختلطة من الفولاذ والخرسانة مع اتصال جزئي.

تم أخذ افتراضات برنولي الحركية لكل من طبقات "الفولاذ والخرسانة" ، كما تم اعتماد سلوك مرن خطي للعارضة المختلطة. من خلال النظر في العلاقة بين القص الطولي للواجهة والانزلاق المقابل ، يتم تصميم اتصال القص من خلال النظر في نموذجين للاتصال بالواجهة: اتصال منفصل واتصال مستمر. تم اشتقاق مصفوفة صلابة العارضة المختلطة باستخدام الصيغة القياسية القائمة على الإزاحة. تم تطوير برنامج **(Matlab)** واستخدامه للتحقيق في سلوك العوارض المختلطة فولاذ-خرسانة من خلال العديد من التطبيقات العددية. وأخيرًا ، تم إجراء تحليل عددي لدراسة تأثير الاتصال الجزئي على السلوك العام للعارضة المختلطة.

Key words:

Steel, Concrete, Finite Element Method, Composite Beam, Bernoulli, Discrete Connection, Continues Connection, MATLAB.

Index ratings:

c	concrete index
s	index relating to the steel of the metal profile
sr	index relating to reinforcing steel
sc	index relating to the type of continuous connection to the interface
st	index relating to the type of discrete connection to the interface

Geometry notation:

A_c	Area of the concrete section of the slab [m2]
A_s	section area of the metal profile [m2]
A_{sr}	area of the longitudinal reinforcement section of the slab [m2]
H_c	distance between the reference axis of the slab and the interface [m]
H_s	distance between the reference axis of the metal profile and the interface [m]
L	length of a composite beam element [m]
I_c	quadratic moment of the concrete section of the slab with respect to the reference axis[m4]
I_s	quadratic moment of the section of the metal profile with respect to the reference axis[m4]
(a)	longitudinal connector spacing [m]

Notations relating to displacements :

u_c (x)	axial displacement at the reference axis of the concrete slab [m]
u_s (x)	axial displacement at the reference axis of the metal profile [m]
v (x)	transverse overall displacement of the section of the composite beam [m]
d_{sc} (x)	sliding of the metal profile in relation to the slab (at the interface)[m]

Notations relating to generalized strains and generalized stresses :

ε_c	axial deformation at the reference axis of the reinforced concrete slab [m / m]
ε_s	axial deformation at the reference axis of the metal profile [m / m]
κ	curvature [1 / m]
D_{sc}	shear force applied to the continuous connection process [N / m]
Q_{st}	shear force applied to the discrete connection process [N]
N_c	normal force in the slab [N]

N_s	normal force in the profile [N]
M_c	bending moment in the slab relative to the reference axis [N.m]
M_s	bending moment in the profile in relation to the reference axis [N.m]
T_c	shear force in the slab [N]
T_s	shear force in the profile [N]

Notations relating to vectors and matrices :

$\frac{d()}{dx}$	derivative of a variable with respect to x
{d}	element and structure nodal displacement matrix, both in global coordinates
{F}	global-coordinate structure force matrix
[K]	global-coordinate structure stiffness matrix
K_{xx}, Kyy	thermal conductivities (or permeabilities, for fluid mechanics) in the x and y directions, respectively
d (x)	vector of displacements
D (x)	vector of internal forces
e (x)	vector of generalized deformations
F	stiffness matrix of a finite element of the composite beam
K	stiffness matrix of a finite element of the composite beam
K_g	overall stiffness matrix
q	vector of the nodal displacements of a finite element
Q	vector of the nodal forces of a finite element

Notations relating to local behavior:

ε	total deformation [m / m]
σ	stress [N / m2]
f	load area

Material Parameter Notations :

E_c	elastic modulus of the concrete of the slab [N / m2]
E_s	elastic modulus of the profile steel [N / m2]
E_{sr}	elastic modulus of reinforcing steel [N / m2]
k_{sc}	linear stiffness of the connection process [N / m2]
k_{ss}	point stiffness of the connection process [N / m]

CONTENTS

Steel-Concrete mixed beams are widely used in civil engineering applications especially in high-rise buildings and long-span bridges. By the rational structural use of the right materials at the right places, mixed beams can efficiently offer high strength, high stiffness and high load capacity through the adequate coupling of the constructive materials into an integral cross-section. However, the overall performance of mixed beams largely depends on the type of connectors used to link the steel and concrete components where the Connectors help to transfer the stress from one component to the other to have a composite action.

Theoretically, if the shear connectors are having infinite stiffness, a full composite action can be achieved. In this case, the benefit of the composite beam can be fully exploited where no shear slip (i.e. Longitudinal displacement between the layers along the member length) develops at the interface between the two layers and a full shear interaction is achieved. In such a case a full interaction (i.e., no slip or no separation) can be achieved with a rigid connection. However, in practice, shear connectors are having finite stiffness, which results in the development of interfacial longitudinal slip between the two layers of mixed beams and a partial shear interaction is always found.

For real applications, a complete shear layer interaction is not obtained and the interlayer longitudinal slip influences the behaviour of the mixed beam and must be considered. However, the analysis and design of steel-concrete mixed beams is a complex task because of the interlayer slip between the sub-elements.

The main idea of this master thesis is to develop a mixed beam finite element using continuous and discrete connection models, and assess the capability of these finite elements models to predict the linear elastic behaviour of “steel-concrete” mixed beams with partial interaction.

A Matlab program for linear elastic beam and frame analysis is developed, in which, the partial shear interaction caused by the longitudinal interfacial slip due to the deformability of shear connectors is considered. The interaction between the two materials is modelled with nodal shear springs for the discrete connection model, and with distributed shear springs along the entire length of the beam for the continuous connection model.

1.1. Generalities

The most important and most frequently encountered combination of construction materials is that of steel and concrete, with applications in multi-story commercial buildings and factories, as well as in bridges. These materials can be used in mixed structural systems, for example concrete cores encircled by steel tubes, as well as in mixed structures where members consisting of steel and concrete act together compositely.

These essentially different materials are completely compatible and complementary to each other; they have almost the same thermal expansion; they have an ideal combination of strengths with the concrete efficient in compression and the steel in tension; concrete also gives corrosion protection and thermal insulation to the steel at elevated temperatures and additionally can restrain slender steel sections from local or lateral torsional buckling.

Buildings with steel and mixed elements experienced a renaissance during the 1980's, resulting in a profusion of new construction concepts and structural details. Single mixed elements, such as isolated beams, columns and slabs (Figure 1.1), whilst they are of high quality and resistance, they are also, in many cases, expensive. This is the case particularly for buildings with small column spacings, floor beam spans well below 9 m and low loadings. On the other hand, mixed floor construction is highly competitive if spans are increased to 12, 15 or even 20 m. There is, of course, a demand for larger column-free spans in buildings to facilitate open planning or greater flexibility in office layout [1].

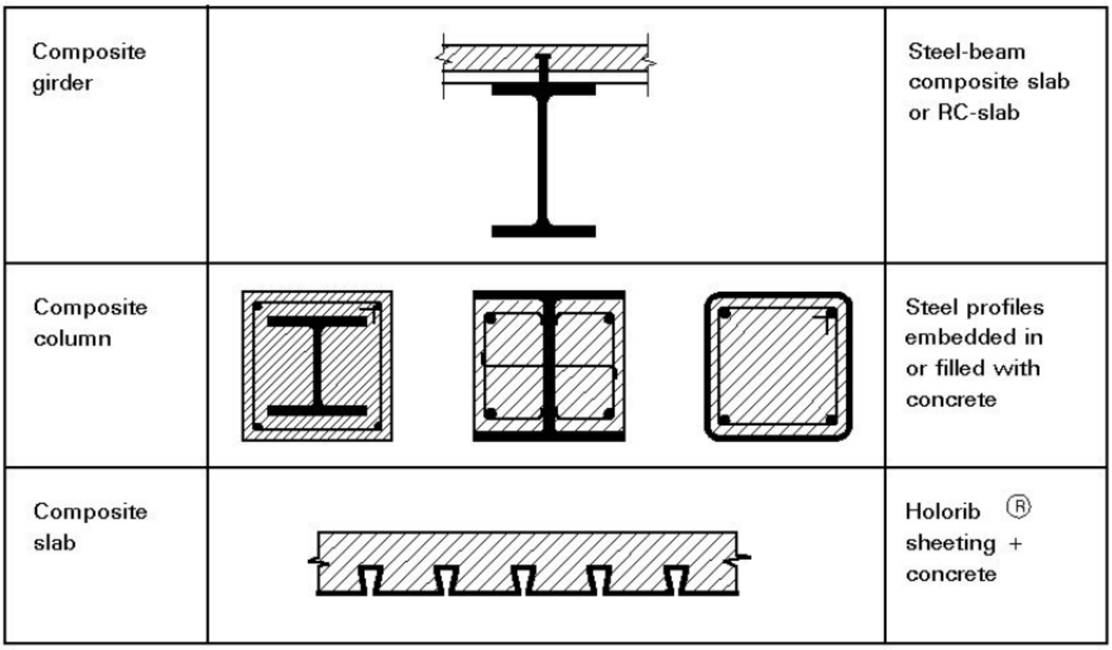


Figure 1.1 Mixed Structural Elements in Buildings.

1.2. MIXED BEAMS

Steel–concrete composite structural systems utilize the advantages of steel and concrete. The most general form of such a system is the mixed flexural members that are formed by connecting steel beams and a concrete slab by making use of shear connectors. A mixed beam consists of a steel section and a reinforced concrete slab interconnected by shear connectors. The efficiency of mixed beams stems from the basic fact that concrete is strong in compression while steel is strong in tension. Concrete can also provide support for compression steel against lateral-torsional and local buckling. Composite beams offer several advantages over non-composite sections; mainly, a reduction in size and deflection of the steel beam and a reduced floor vibration due to higher stiffness. An essential component of a mixed beam is the shear connection between the steel section and the concrete slab. This connection is provided by mechanical shear connectors, which allow the transfer of forces from the concrete to the steel and vice versa and also resist vertical uplift forces at the interface. The shear connectors are usually welded to the top flange of the steel beam before the slab is cast. These connectors ensure that the two different materials act as a single unit [2].

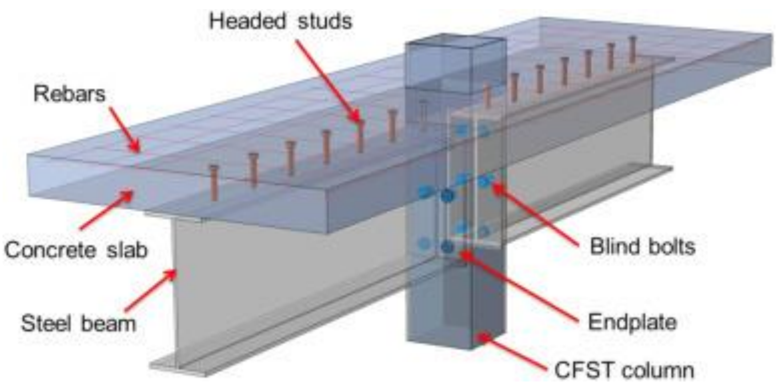


Figure 1.2 Mixed Beam

1.2.1. Shear transfer between the steel and concrete parts

To obtain an error-less structural performance of mixed beams, care should be taken to ensure an efficient shear transfer between the steel and concrete parts. The key point in the analysis and optimization of mixed structures is therefore the prediction of joint strength and its influence on global stability in most mixed beams, shear connection is provided by welding a steel member to the upper flange of the steel beam and surrounded in the reinforced concrete as shown in Fig. 1.3

These members transfer the forces between the steel girder and connector by shear and between the connector and concrete by bearing (Viest et al. 1997). The mechanical interlocking system in the deck profile provides resistance to the vertical separation and horizontal slippage between steel and concrete. A beneficial composite behavior can be achieved by minimizing the displacement of the concrete slab and steel beam at their edge. This composite action is usually assured by the shear connectors [3].

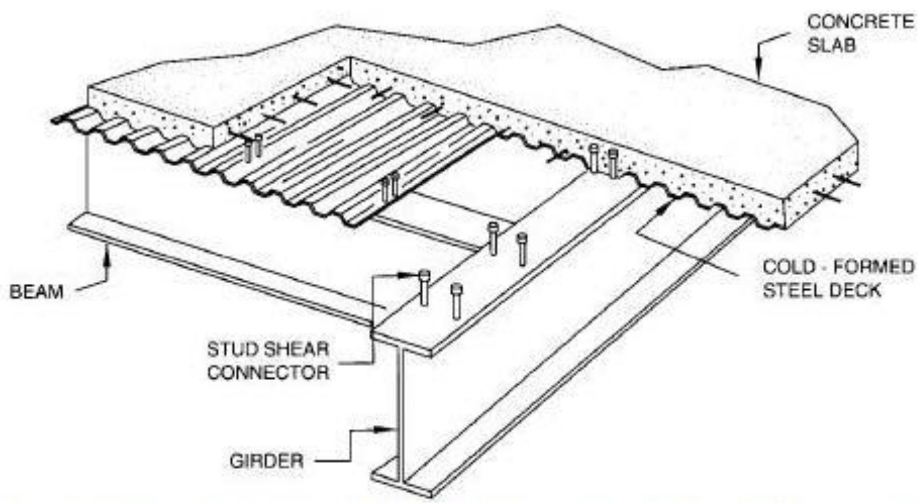


Figure.1.3 Typical shear connection in mixed structure

1.2.2. Mixed beams and slabs cross section

Mixed systems have seen widespread use in recent decades due to multiple benefits that occur by combining the individual mechanical properties of the main component materials, steel and concrete. The structural steel experiences high strength and ductility in tension and compression, while the concrete experiences high stiffness and robustness in compression. The composite action between the concrete slab and the steel beam is achieved through mechanical connectors such as shear studs. The resulted mixed beam-column provides an increase of the rigidity, strength and the ultimate moment capacity of the mixed element, compared with the independent use of each material. An important aspect in the structural behavior of steel-concrete mixed beams is represented by the level of shear connection between the concrete slab and steel beam, which is defined as the ratio between the shear connection capacity provided by the studs and the weakest component capacity (concrete slab or steel beam). If the disposed number of shear connectors at the steel concrete interface of the mixed beam is lower than the number that provides full shear connection, then the stiffness and ultimate capacity decreases, while the ductility of the mixed beam may be enhanced [4].

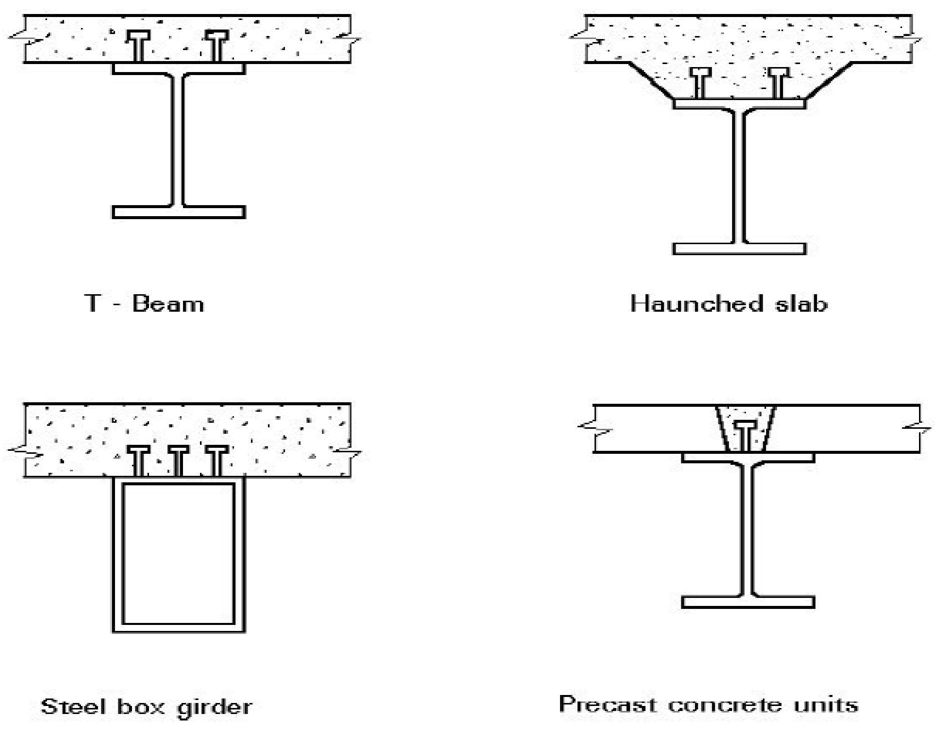


Figure 1.4 Typical Beam Cross Sections

These materials combine the strength of steel with the compressive strength and the stiffness of concrete, producing a highly economical and interesting structural system. From the beginning, the most common type of mixed beam in use has been an I-steel profile connected to the concrete slab or profiled steel-concrete mixed slab. Given its importance, this traditional mixed beam (Fig. 1.5(a)), the composite action between the concrete and steel profile can be achieved by means of mechanical shear connectors as headed studs, proving to be an efficient shear connector. However, in several situations, it can be interesting to reduce the overall depth of the floor using the beams contained within the depth of the floor (see Fig. 1.5(b)). The concrete between the flanges of the beam results in several advantages, such as high fire-resistance and load capacity, as well as a significant increase in the bending stiffness compared to a steel beam. The local buckling strength also increases in relation to the steel section, and the overall height of both mixed beam and mixed floor is reduced. In addition, lower construction cost compared to reinforced concrete (RC) or steel frame system and shorter construction time compared to RC can be obtained using encased beams. Therefore, the concrete cast in the flanges of the steel beam is an innovative and interesting alternative. Despite the advantages in terms of structural behavior and costs, the encased beam is a constructive solution not totally understood yet, especially in relation to the headed stud's contribution to load capacity and composite behavior. Comparing traditional mixed beams (Fig. 1.5(a)) and partially encased beams (Fig. 1.5(e)), we note that the reinforced concrete between flanges increases the bending stiffness and reduces the vertical displacements [5].

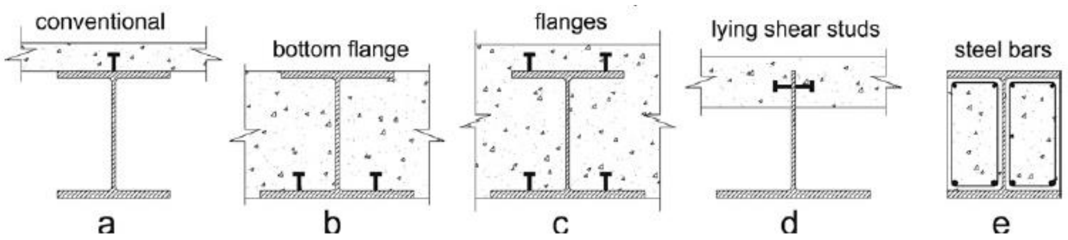


Fig 1.5 Examples of mixed beams [5]

1.3. Steel-concrete composite behavior in mixed beams

The behavior of a mixed steel-concrete cross-section is bounded by two extreme limits. The upper stiffness limit (Figure 1.7) is that of "composite action." In this case the cross-section has a single neutral axis and the two material strains are identical at the material interface. The method of transformed sections applies for analysis. When the layers are not rigidly connected, relative motion (termed "slip") occurs at the interface of the materials. The single neutral axis splits and when the slip between the layers propagates, the now two neutral axes move farther and farther apart. When some interlayer shear resisting force is present the composite action is referred to as "partially connected". The lower limit is that of "non-composite action" interaction, the condition of no shear transfer between the two layers (Figure 1.6). The material layers have individual neutral axes and discontinuous strains at the material interface. There is neither mechanical bond nor friction between the two layers. Independent action of the layers results in both layers experiencing tensile and compressive strains and stresses about their individual neutral axes. The non- composite limit is the least stiff and results in the largest deflection [6].

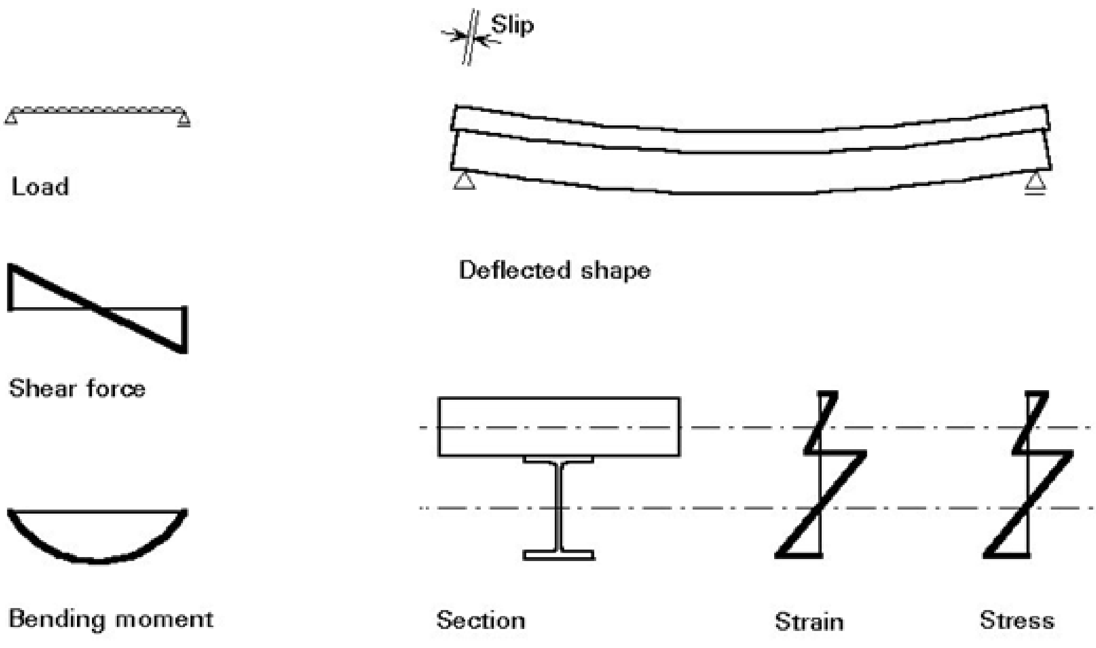


Figure 1.6 Non-composite beam

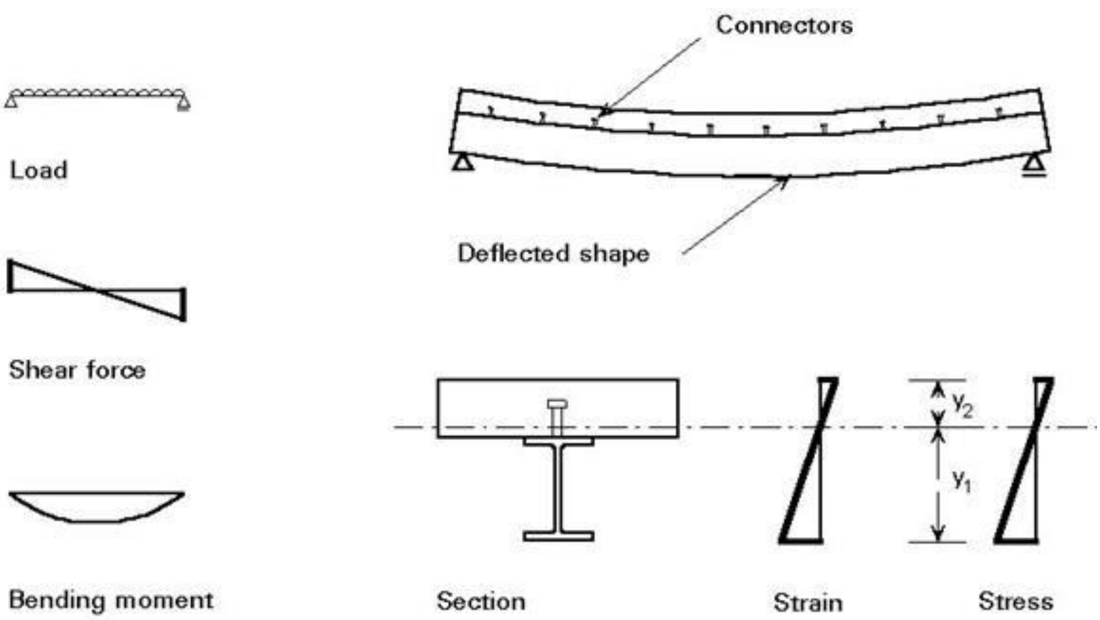


Fig 1.7 Composite beam

The following definitions are used to make clear the differences between resistance (strength) and stiffness properties:

– With regard to resistance, distinction is made between complete and partial shear connection. The connection is considered to be complete if the resistance of the mixed beam is decided by the bending resistance, not the horizontal shear resistance.

– Complete or incomplete interaction between the concrete slab and the steel section results in a more or less stiff mixed beam. Such incomplete interaction arises when flexible connectors such as headed studs are used and slip (relative displacement) occurs at the steel-concrete interface.

– The use of composite action has certain advantages. In particular, a mixed beam has greater stiffness and usually a higher load resistance than its non-composite counterpart. Consequently, a smaller steel section is usually required. The result is a saving of material and depth of construction. In turn, the latter leads to lower story heights in buildings and lower embankments for bridges.

1.4. Shear Connectors

The shear connectors are commonly used to ensure composite action in a steel–concrete mixed beams. Their main function is to resist longitudinal shear forces at the steel–concrete interface, and to prevent vertical separation between the concrete slab and the supporting steel beam, connection is provided by mechanical shear connectors, which allow the transfer of forces from the concrete to the steel and vice versa and also resist vertical uplift forces at the interface. The shear connectors are usually welded to the top flange of the steel beam before the slab is cast. These connectors ensure that the two different materials act as a single unit.

A mixed flexural member will have higher strength and stiffness compared to a bare steel member, resulting in reduced deflection and floor vibration in the structure. Mixed flexural members can be used as girders in bridges or primary and secondary beams in building systems. Mechanical connectors for shear transfer must be used in these members to achieve the desired composite behavior. The shear connectors are placed at the interface between steel beam and concrete slab, and they are responsible for transferring the horizontal shear forces that are formed due to flexural action. The need for mechanical shear connectors also arises to transfer earthquake forces between concrete slab and steel beams that are part of the lateral load resisting system of the structure. Besides, these elements function under axial loads to resist vertical upward forces and prevent the premature separation of steel beams and concrete slab in the vertical direction [7].

1.4.1 Types of Shear Connectors

Many types of shear connectors have been developed and used in the past. The most widely used shear connector in practice is the welded stud (Figure 1.8(a)) with a suitable head that contributes to the shear transfer and prevents the uplift. Nevertheless, due to the small load carrying capacity of stud connectors and also due to the fatigue problems caused by live loads on mixed bridges, some other alternative shear connectors are proposed such as the angle connector with anti-uplift bar (Figure 1.8(b)) and the channel connector (Figure 1.8(c)) which are frequently used in Algeria and in some other countries

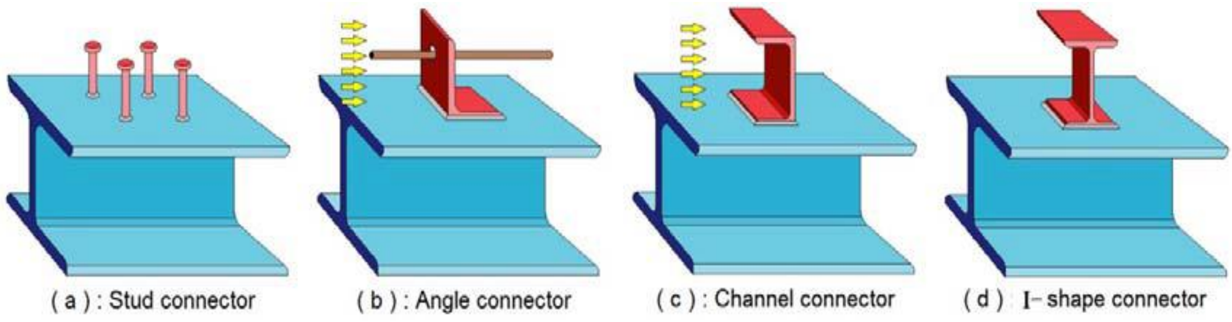


Figure 1.8 Types of Shear Connectors

The economic considerations continue to motivate the development of new systems to ensure the load transfer between steel and concrete components in mixed structures. Recently, several authors have proposed new types of shear connectors, such as Y-type perfobond rib connector, J-hook connector, Bolted connector, Rubber-sleeved stud, and V-shaped angle connector [8].

1.4.2 Effectiveness the shear connectors

The headed stud is the most widely used shear connector in mixed construction. Its popularity stems from proven performance and the ease of installation using a welding gun and fast welding, good anchor in concrete, the arrangement of reinforcement through the slab is easy, production of large scale size is easy, the standard dimensioned head is a resistance factor for slab uplift and they are practical for use in steel deck slabs. Nonetheless, due to the small load carrying capacity of stud connectors, they have to be installed in large numbers. This usually produces a cluttering effect and an unsafe working place and severe crushing of the concrete occurs at the front of the connector's root seriously decreasing the modulus of the concrete. Breakdown of the shear connection can occur either by the stud shearing failure or by the crushing of concrete. The stress developed by the applied load on the shattering restraint of the lower surface of the concrete slab in contact with the steel flanges and the limitation of the concrete expansion due to the transverse reinforce-mint determine the strength of the connector.

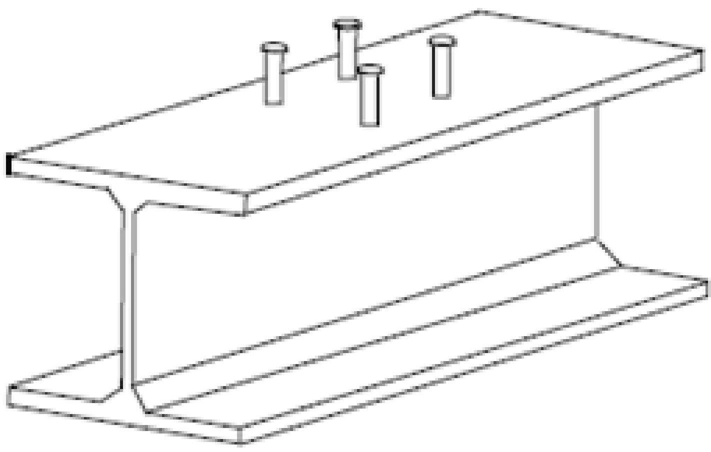


Figure 1.9 Head stud shear connector

Therefore, using C-shaped connectors could be a good alternative are a type of shear connectors which have been increasingly used in construction projects. These connectors are appropriate not only at transferring horizontal shear forces but also at controlling uplift forces. In addition, they can be easily installed in practice and no special construction equipment is required C-shaped shear connectors are classified into two major categories including channel and angle connectors. The only difference between these connectors lies in the lower flange of the profiles. C-shaped angle connectors do not have the lower flange of the channel connectors and the web of the connector is directly connected to the upper flange of the beam. Therefore, C-shaped angle connectors use fewer materials than channel connectors do, while they can show competitive performance with channel connectors.

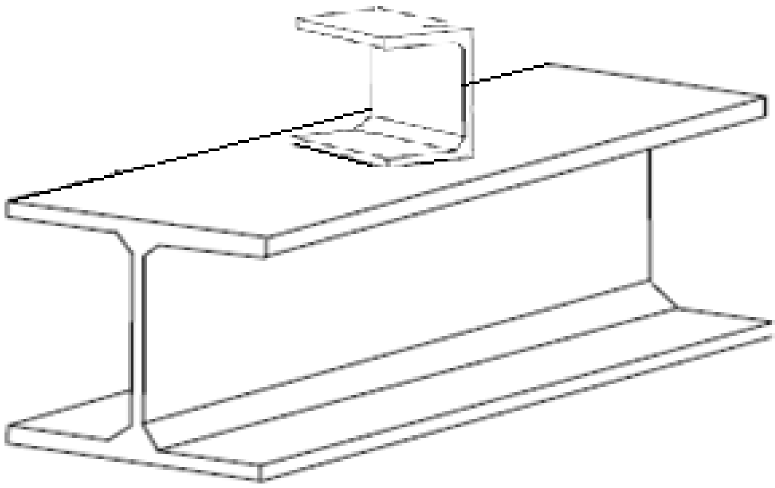


Figure 1.10 Channel shear connector

As shown in Figure 1.8, the shape of I-shape connector is appropriate to resist shear forces and prevent vertical separation between the steel beam and the concrete slab. In addition, angle and channel connectors are limited to shear transfer in the recommended direction only, while the I-shape connector can resist and transfer shear in the two directions with same quantity, making it the more useful shear connector in mixed beams subjected to seismic loading. Moreover, the facility of producing the I-shape connectors by their cutting from the ordinary laminated I profiles is another advantage. The welding task has the same characteristics as referred for angle and channel connectors [9].

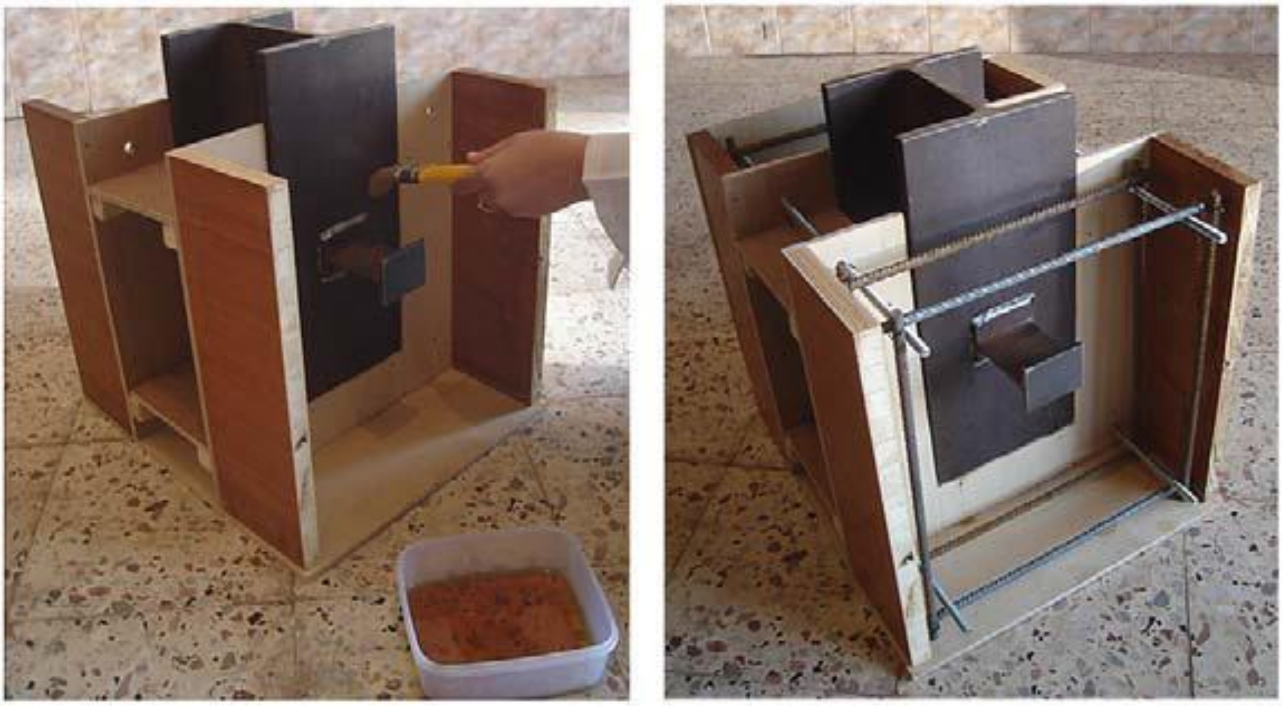


Figure 1.11 I-shape Shear Connectors

2.1. General:

The finite element analysis is a numerical technique. In this method all the complexities of the problems, like varying shape, boundary conditions and loads are maintained as they are but the solutions obtained are approximate. Because of its diversity and flexibility as an analysis tool, it is receiving much attention in engineering. The fast improvements in computer hardware technology and slashing of cost of computers have boosted this method, since the computer is the basic need for the application of this method. A number of popular brand of finite element analysis packages are now available commercially. Some of the popular packages are STAAD-PRO, GT-STRUDEL, NASTRAN, NISA and ANSYS. Using these packages one can analyses several complex structures.

The finite element analysis originated as a method of stress analysis in the design of aircrafts. It started as an extension of matrix method of structural analysis. Today this method is used not only for the analysis in solid mechanics, but even in the analysis of fluid flow, heat transfer, electric and magnetic fields and many others. Civil engineers use this method extensively for the analysis of beams, space frames, plates, shells, folded plates, foundations, rock mechanics problems and seepage analysis of fluid through porous media. Both static and dynamic problems can be handled by finite element analysis. This method is used extensively for the analysis and design of ships, aircrafts, space crafts, electric motors and heat engines.

2.2 General Description Of The Method:

In engineering problems there are some basic unknowns. If they are found, the behavior of the entire structure can be predicted. The **basic unknowns** or the **Field variables** which are encountered in the engineering problems are displacements in solid mechanics, velocities in fluid mechanics, electric, and magnetic potentials in electrical engineering and temperatures in heat flow problems.

In a continuum, these unknowns are infinite. The finite element procedure reduces such unknowns to a finite number by dividing the solution region into small parts called **elements** and by expressing the unknown field variables in terms of assumed **approximating functions** (Interpolating functions/Shape functions) within each element. The approximating functions are defined in terms of field variables of specified points called **nodes** or **nodal points**. Thus in the finite element analysis the unknowns are the field variables of the nodal points. Once these are found the field variables at any point can be found by using interpolation functions.

After selecting elements and nodal unknowns next step in finite element analysis is to assemble **element properties** for each element. For example, in solid mechanics, we have to find the force-displacement i.e. stiffness characteristics of each individual element. Mathematically this relationship is of the form $[k]e\ \{\delta\}e = \{F\}e$ where $[k]e$ is element stiffness matrix, $\{\delta\}e$ is nodal displacement vector of the element and $\{F\}e$ is nodal force vector. The element of stiffness matrix kij represent the force in coordinate direction 'i' due to a unit displacement in coordinate direction 'j'. Four methods are available for formulating these element properties viz. direct approach, variational approach, weighted residual approach and energy balance approach. Any one of these methods can be used for assembling element properties. In solid mechanics variational approach is commonly employed to assemble stiffness matrix and nodal force vector (consistent loads).

Element properties are used to assemble global properties/structure properties to get system equations $[k]\ \{\delta\} = \{F\}$. Then the boundary conditions are imposed. The solutions of these simultaneous equations give the nodal unknowns. Using these nodal values additional calculations are made to get the required values e.g. stresses, strains, moments, etc. in solid mechanics problems [10].

2.3 The advantages of finite element method:

The finite element method is now the most powerful numerical method that can be used to analyze any complicated structure in engineering and though the finite element method is originally developed to solve structural problems, due to a lot of researches, it is now available to solve various field problems such as temperature fields, flow fields, and electromagnetic fields.

The finite element method possesses the following superiorities:

1. The original continuous medium is provided with infinite degrees of freedom and is impossible to be solved on computers. After being divided into finite elements, the nodal displacements are taken as unknown quantities. The number of unknown quantities is finite and it may be solved on computers.
2. It may be used to analyze various types of structures with complicated shapes in actual engineering.

3. Different materials may be used for different elements.
4. The nonlinear problems may be calculated, including material nonlinearity and geometrical nonlinearity (great deformation) problems [11].

2.4 General Steps of the Finite Element Method

The finite element method involves modeling the structure using small interconnected elements called finite elements. A displacement function is associated with each finite element. Every interconnected element is linked, directly or indirectly, to every other element through common (or shared) interfaces, including nodes and/or boundary lines and/or surfaces. By using known stress/strain properties for the material making up the structure, one can determine the behavior of a given node in terms of the properties of every other element in the structure. The total set of equations describing the behavior of each node results in a series of algebraic equations best expressed in matrix notation. We now present the steps, along with explanations necessary at this time, used in the finite element method formulation and solution of a structural problem. The purpose of setting forth these general steps now is to expose you to the procedure generally followed in a finite element formulation of a problem. We suggest that you review this section periodically as we develop the specific element equations. Keep in mind that the analyst must make decisions regarding dividing the structure or continuum into finite elements and selecting the element type or types to be used in the analysis (step 1), the kinds of loads to be applied, and the types of boundary conditions or supports to be applied. The other steps, 2 through 7, are carried out automatically by a computer program.

Step 1 Discretize and Select the Element Types:

Step 1 involves dividing the body into an equivalent system of finite elements with associated nodes and choosing the most appropriate element type to model most closely the actual physical behavior. The total number of elements used and their variation in size and type within a given body are primarily matters of engineering judgment. The elements must be made small enough to give usable results and yet large enough to reduce computational effort. Small elements (and possibly higher-order elements) are generally desirable where the results are changing rapidly, such as where changes in geometry occur; large elements can be used where results are relatively constant. Where the concept becomes quite significant. The discretized body or mesh is often created with mesh-generation programs or preprocessor programs available to the user.

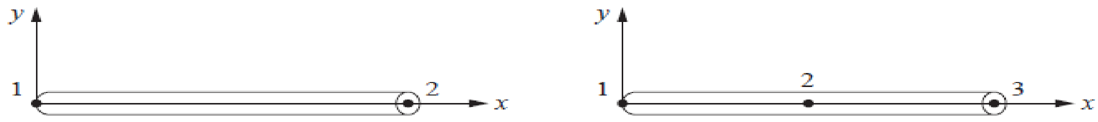
The choice of elements used in a finite element analysis depends on the physical makeup of the body under actual loading conditions and on how close to the actual behavior the analyst wants the results to be. Judgment concerning the appropriateness of one, two, or three dimensional idealizations is necessary. Moreover, the choice of the most appropriate element for a particular problem is one of the major tasks that must be carried out by the designer/analyst. Elements that are commonly employed in practice most of which are considered in this text are shown in Figure 2.1.

The primary line elements [Figure 2.1(a)] consist of bar (or truss) and beam elements. They have a cross-sectional area but are usually represented by line segments. In general, the cross-sectional area within the element can vary, but throughout this text it will be considered to be constant. These elements are often used to model trusses and frame structures. The simplest line element (called a linear element) has two nodes, one at each end, although higher-order elements having three nodes [Figure 2.1(a)] or more (called quadratic, cubic, etc., elements) also exist.

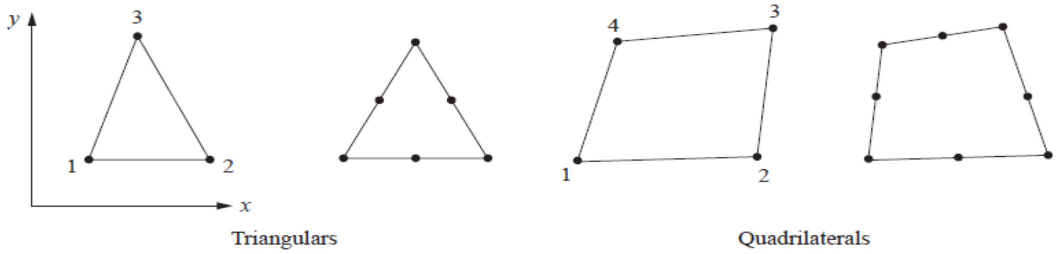
The basic two-dimensional (or plane) elements [Figure 2.1(b)] are loaded by forces in their own plane (plane stress or plane strain conditions). They are triangular or quadrilateral elements. The simplest two-dimensional elements have corner nodes only (linear elements) with straight sides or boundaries, although there are also higher-order elements, typically with midside nodes [Figure 2.1(b)] (called quadratic elements) and curved sides. The elements can have variable thicknesses throughout or be constant. They are often used to model a wide range of engineering problems.

The most common three-dimensional elements [Figure 2.1(c)] are tetrahedral and hexahedral (or brick) elements; they are used when it becomes necessary to perform a three-dimensional stress analysis. The basic three-dimensional elements have corner nodes only and straight sides, whereas higher-order elements with midedge nodes (and possible midface nodes) have curved surfaces for their sides [Figure 2.1(c)].

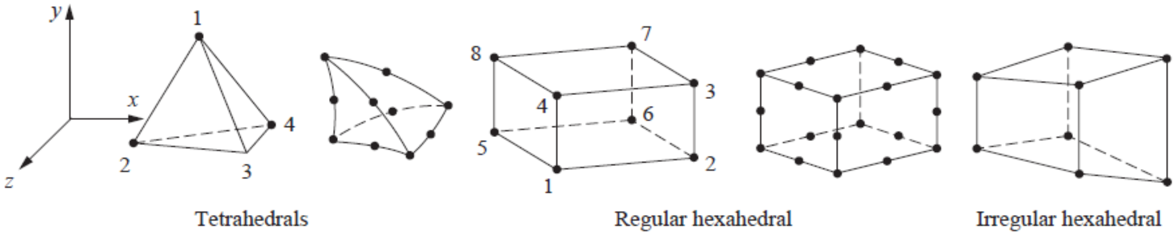
The axisymmetric element [Figure 2.1(d)] is developed by rotating a triangle or quadrilateral about a fixed axis located in the plane of the element through 360. This element can be used when the geometry and loading of the problem are axisymmetric.



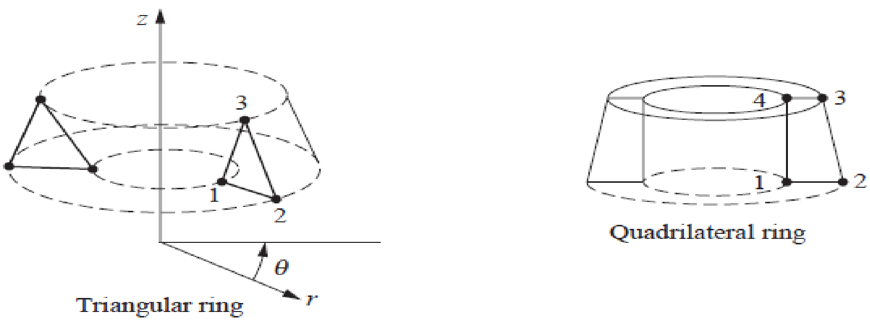
(a) Simple two-noded line element (typically used to represent a bar or beam element) and the higher-order line element.



(b) Simple two-dimensional elements with corner nodes (typically used to represent plane stress/ strain) and higher-order two-dimensional elements with intermediate nodes along the sides.



(c) Simple three-dimensional elements (typically used to represent three-dimensional stress state) and higher-order three-dimensional elements with intermediate nodes along edges.



(d) Simple axisymmetric triangular and quadrilateral elements used for axisymmetric problems.

Figure 2.1 Various types of simple lowest-order finite elements with corner nodes only and higher-order elements with intermediate nodes.

Step 2 Select a Displacement Function:

Step 2 involves choosing a displacement function within each element. The function is defined within the element using the nodal values of the element. Linear, quadratic, and cubic polynomials are frequently used functions because they are simple to work with in finite element formulation. However, trigonometric series can also be used. For a two-dimensional element, the displacement function is a function of the coordinates in its plane (say, the x-y plane). The functions are expressed in terms of the nodal unknowns (in the two-dimensional problem, in terms of an x and a y component). The same general displacement function can be used repeatedly for each element. Hence the finite element method is one in which a continuous quantity, such as the displacement throughout the body, is approximated by a discrete model composed of a set of piecewise-continuous functions defined within each finite domain or finite element.

Step 3 Define the Strain/ Displacement and Stress/Strain Relationships:

Strain/displacement and stress/strain relationships are necessary for deriving the equations for each finite element. In the case of one-dimensional deformation, say, in the x direction, we have strain ex related to displacement u by

$$\varepsilon_x = \frac{du}{dx} \tag{2.1}$$

For small strains, In addition, the stresses must be related to the strains through the stress/strain law generally called the constitutive law. The ability to define the material behavior accurately is most important in obtaining acceptable results. The simplest of stress/strain laws, Hooke’s law, which is often used in stress analysis, is given by

$$\sigma_x = E\varepsilon_x \tag{2.2}$$

Where $\sigma_x =$ stress in the x direction and E= modulus of elasticity.

Step 4 Derive the Element Stiffness Matrix and Equations:

Initially, the development of element stiffness matrices and element equations was based on the concept of stiffness influence coefficients, which presupposes a background in structural analysis. We now present alternative methods used in this text that do not require this special background.

Step 5 Assemble the Element Equations to Obtain the Global or Total Equations and Introduce Boundary Conditions:

In this step the individual element nodal equilibrium equations generated in step 4 are assembled into the global nodal equilibrium equations. Another more direct method of superposition (called the direct stiffness method), whose basis is nodal force equilibrium, can be used to obtain the global equations for the whole structure. Implicit in the direct stiffness method is the concept of continuity, or compatibility, which requires that the structure remain together and that no tears occur anywhere within the structure.

The final assembled or global equation written in matrix form is

$$\{F\} = [K]\,\{d\} \tag{2.3}$$

Where {F} is the vector of global nodal forces, [K] is the structure global or total stiffness matrix, (for most problems, the global stiffness matrix is square and symmetric) and {d} is now the vector of known and unknown structure nodal degrees of freedom or generalized displacements. It can be shown that at this stage, the global stiffness matrix [K] is a singular matrix because its determinant is equal to zero. To remove this singularity problem, we must invoke certain boundary conditions (or constraints or supports) so that the structure remains in place instead of moving as a rigid body. At this time it is sufficient to note that invoking boundary or support conditions results in a modification of the global Eq. (2.3). We also emphasize that the applied known loads have been accounted for in the global force matrix {F}.

Step 6 Solve for the Unknown Degrees of Freedom (or Generalized Displacements):

Equation (2.3), modified to account for the boundary conditions, is a set of simultaneous Algebraic equations that can be written in expanded matrix form as:

$$\begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1n} \\ K_{21} & K_{22} & \dots & K_{2n} \\ \vdots & & & \vdots \\ K_{n1} & K_{n2} & \dots & K_{nn} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{Bmatrix} \tag{2.4}$$

Where now n is the structure total number of unknown nodal degrees of freedom. These equations can be solved for the ds by using an elimination method (such as Gauss's method) or an iterative method (such as the Gauss–Seidel method). The ds are called the primary unknowns; because they are the first quantities determined using the stiffness (or displacement) finite element method.

Step 7 Solve for the Element Strains and Stresses

For the structural stress-analysis problem, important secondary quantities of strain and stress (or moment and shear force) can be obtained because they can be directly expressed in terms of the displacements determined in step 6.

Typical relationships between strain and displacement and between stress and strain such as Eqs. (2.1) and (2.2) for one-dimensional stress given in step 3 can be used.

Step 8 Interpret the Results

The final goal is to interpret and analyze the results for use in the design/analysis process.

Determination of locations in the structure where large deformations and large stresses occur is generally important in making design/analysis decisions. Postprocessor computer programs help the user to interpret the results by displaying them in graphical form [12].

3.1. Generalities:

The term beam has a very specific meaning in engineering mechanics: it is a component that is designed to support transverse loads, that is, loads that act perpendicular to the longitudinal axis of the beam, Fig 3.1. The beam supports the load by bending only.

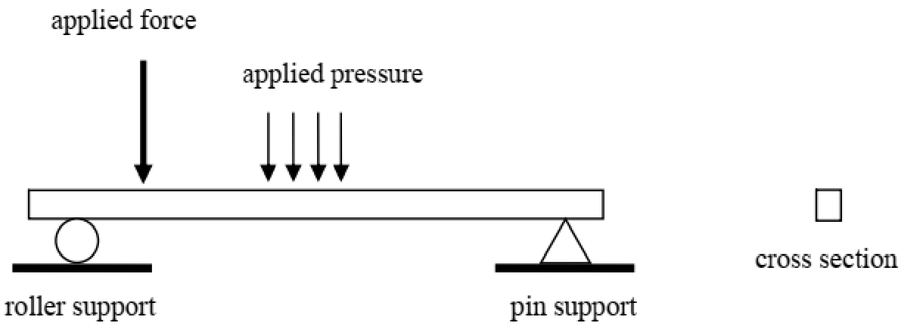


Figure 3.1 A supported beam loaded by a force and a distribution of pressure

The beam theory is used in the design and analysis of a wide range of structures, from buildings to bridges to automobile and ships structures. The beam can be supported in various ways, for example by roller supports or pin supports. The cross section of a beam can be any of many possible shapes. It is assumed that the beam has a longitudinal plane of symmetry, with the cross section symmetric about this plane. Further, it will be assumed that the loading and supports are also symmetric about this plane. With these conditions, the beam has no tendency to twist and will undergo bending only.

3.2. Moments and Forces in a Beam

Normal and shear stresses act over any cross section of a beam, as shown in Fig 3.2. The normal and shear stresses acting on each side of the cross section are equal and opposite for equilibrium. The normal stresses σ will vary over a section during bending. Over one part of the section the stress will be tensile, leading to extension of material fibers, whereas over the other part the stresses will be compressive, leading to contraction of material fibers. This distribution of normal stress results in a moment M acting on the section, as illustrated in Fig 3.3. Similarly, shear stresses τ act over a section and these result in a shear force V.

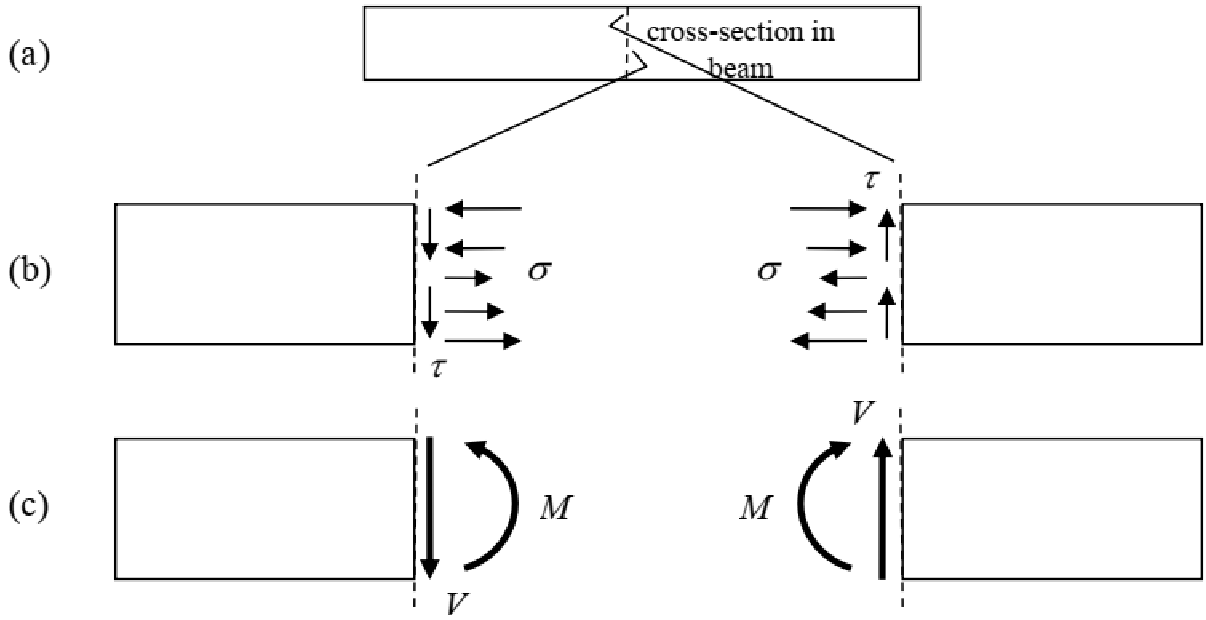


Figure 3.2 stresses and moments acting over a cross-section of a beam; (a) a cross section, (b) normal and shear stresses acting over the cross-section, (c) the moment and shear force resultant of the normal and shear stresses

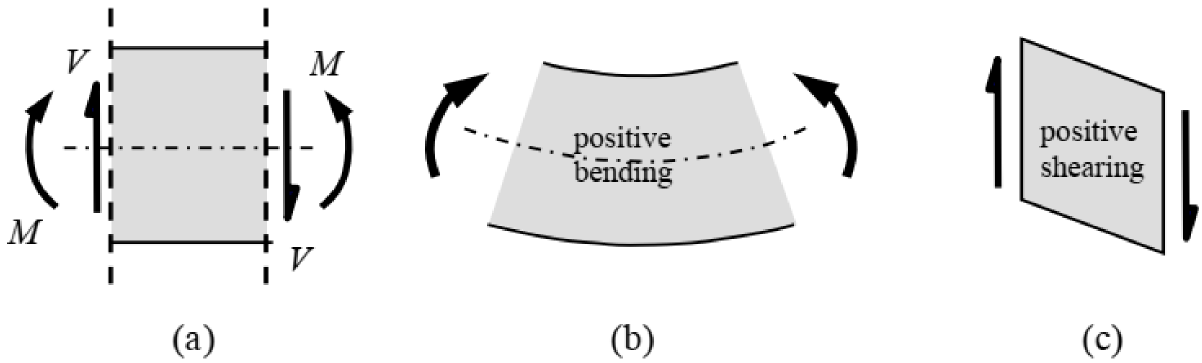


Figure 3.3 sign convention for moments and shear forces

3.3. Beam Theories:

There are a number of beam theories that are used to represent the kinematics of deformation. To describe the various beam theories, we introduce the following coordinate system:

The *x*-coordinate is taken along the length of the beam, *x*-coordinate along the thickness (the height) of the beam, and the *y*-coordinate is taken along the width of the beam. In a general beam theory, all applied loads and geometry are such that the displacements (*u*, *v*, *w*) along the coordinates (x, y, z) are only functions of the x and z coordinates. Here it is further assumed that the displacement v is identically zero.

Two mathematical models, namely the shear-deformable "Timoshenko Beam Theory" model and the shear-in deformable "Euler-Bernoulli Beam Theory" model, are currently used. In Euler Bernoulli beam theory, shear deformations are neglected, and plane sections remain plane and normal to the longitudinal axis. In the Timoshenko beam theory, plane sections still remain plane but are no longer normal to the longitudinal axis.

In the Euler - Bernoulli beam the deformation at a section, dw/dx, is just the rotation due to bending only, since the plane section remains normal to the longitudinal axis. However, in the Timoshenko beam the section deformation is the sum of two contributions: one is due to bending, dw/dx, and the other is the shear deformation, du/dy [13]. Since the Timoshenko beam theory is higher order than the Euler-Bernoulli theory, it is known to be superior in predicting the transient response of the beam. The superiority of the Timoshenko model is more pronounced for beams with a low aspect ratio.

3.3.1. Euler-Bernoulli beam:

The simplest beam theory is the Euler-Bernoulli beam theory (EBT). It is effectively a model for how beams behave under axial forces and bending. It was developed around 1750 and is still the method that we most often use to analyses the behavior of bending elements. This assumption is generally relatively valid for bending beams unless the beam experiences significant shear or torsional stresses relative to the bending (axial) stresses. Shear stresses in beams may become large relative to the bending stresses in cases where a beam is very deep and short in length.

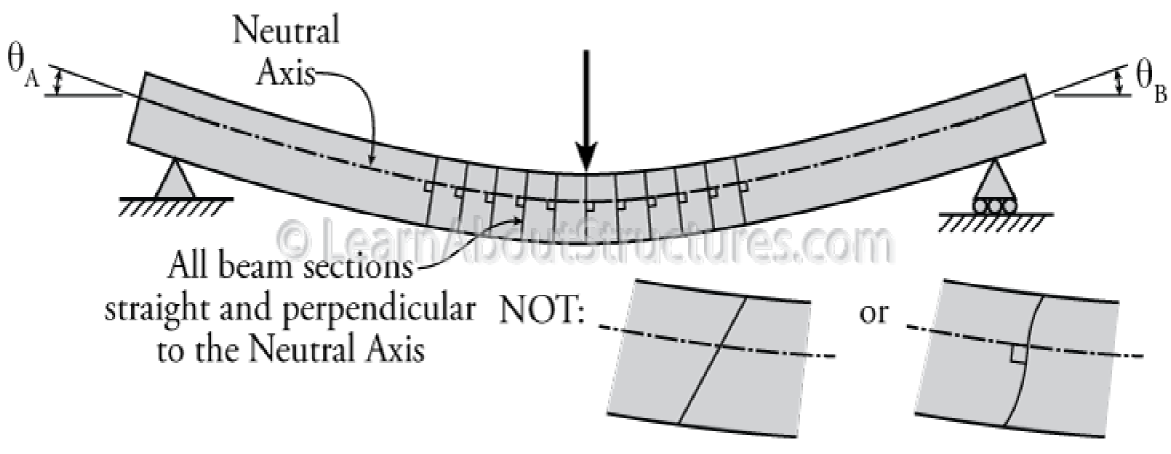


Fig 3.4 Euler-Bernoulli beam theory (EBT) is based on the displacement field.

$$u^E(x,z) = -z\frac{dw_0^E}{dx} \tag{3.1a}$$

$$w^E(x,z) = w_0^E(x) \tag{3.1b}$$

Where w^E is the transverse deflection of the point (x, 0), a point on the mid-plane (i.e., z = 0) of the beam and the superscript 'E" denotes the quantities in the Euler-Bernoulli beam theory. The displacement field in Eq. (3.1) implies that straight lines normal to the mid-plane before deformation remain straight and normal to the mid-plane after deformation, as shown in Figure (3.5a). These assumptions amount to neglecting both transverse shear and transverse normal strains [14].

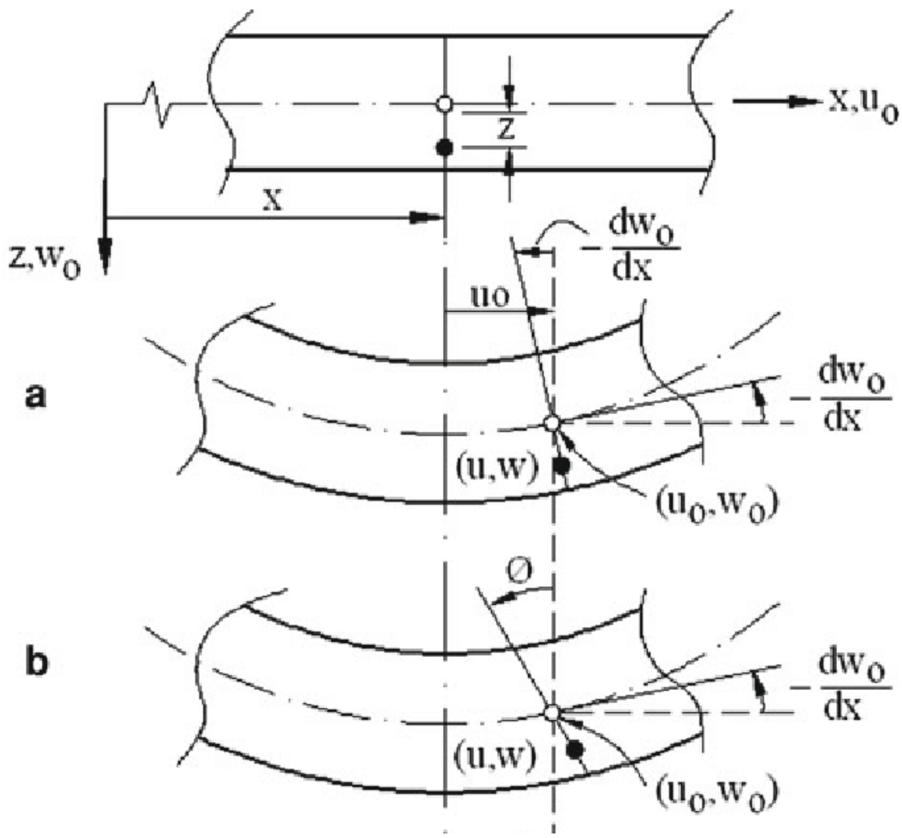


Fig 3.5 Deformation of a typical transverse normal line in various beam theories (u_0 denotes displacement due to in-plane stretching, which is not considered here).

3.3.2. Timoshenko Beam Theory:

The Timoshenko beam theory was developed by early in the 20th century. The model takes into account shear deformation and rotational bending effects, making it suitable for describing the behavior of thick beams.

In static Timoshenko beam theory without axial effects, the displacements of the beam are assumed to be given by

$$u_x(x, y, z) = -z\,\varphi(x) \tag{3.2}$$

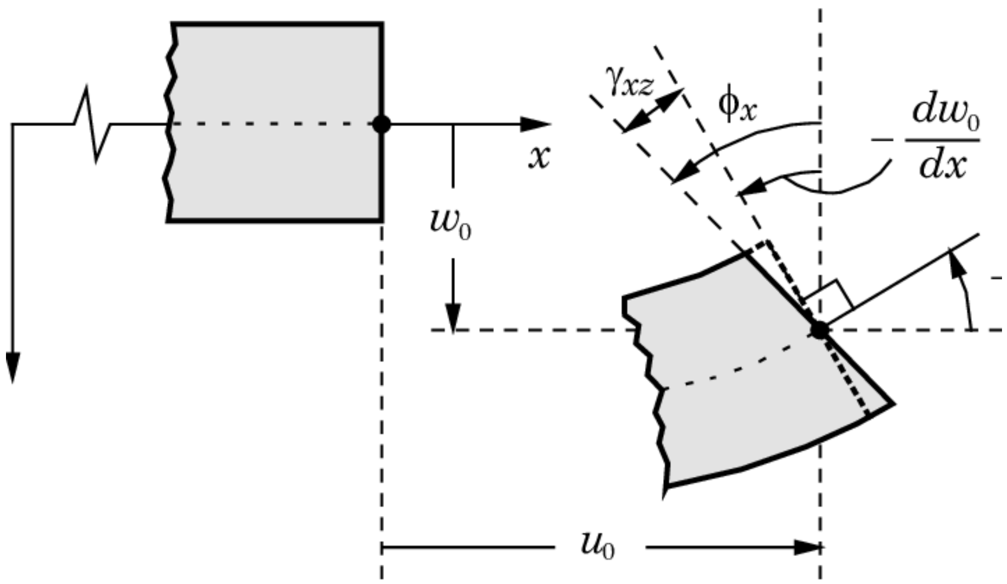


Fig 3.6 Timoshenko Beam Theory

The governing equations are the following coupled system of ordinary differential equations:

$$\frac{d^2}{dx^2}\left(EI\,\frac{d\varphi}{dx}\right) = q(x) \tag{3.3}$$

$$\frac{dw}{dx} = \varphi - \frac{1}{\kappa AG}\frac{d}{dx}\left(EI\,\frac{d\varphi}{dx}\right) \tag{3.4}$$

The Timoshenko beam theory for the static case is equivalent to the Euler-Bernoulli theory when the last term above is neglected.

3.4. CLASSICAL EULER-BERNOULLI BEAM THEORY:

3.4.1. Basic assumptions:

The classical Euler-Bernoulli plane beam theory is based on the following hypotheses [15]:

1. The vertical displacement (deflection) w of the points contained on a cross-section is small and equal to the deflection of the beam axis.
2. The lateral displacement w (along the y axis in Figure 3.7) is zero.
3. Cross-sections normal to the beam axis remain plane and orthogonal to the beam axis after deformation (normal orthogonally condition).

3.4.2. Relation between curvature and beam deflection

Let P be a point on the neutral surface of the beam at a distance (x) from the origin of the coordinate system. The slope of the beam is approximately equal to the angle made by the neutral surface with the x-axis for the small angles encountered in beam theory. Therefore, with this approximation,

$$\boxed{\theta = \frac{dw}{dx}} \tag{3.5}$$

Therefore, for an infinitesimal element, the relation can be written as:

$$\boxed{\kappa = \frac{d^2 w}{dx^2}} \tag{3.6}$$

3.4.3. Displacement field:

Following the above hypotheses the displacement field is written as

$$\begin{aligned} u(x,y,z) &= -z\theta(x) \\ u(x,y,z) &= 0 \\ u(x,y,z) &= w(x) \end{aligned} \tag{3.7}$$

Hypothesis 3 implies that the rotation is equal to the slope of the beam axis (Figure 3.7); i.e.

$$\boxed{\theta = \frac{dw}{dx}} \quad \text{And} \quad \boxed{u = -z\frac{dw}{dx}} \tag{3.8}$$

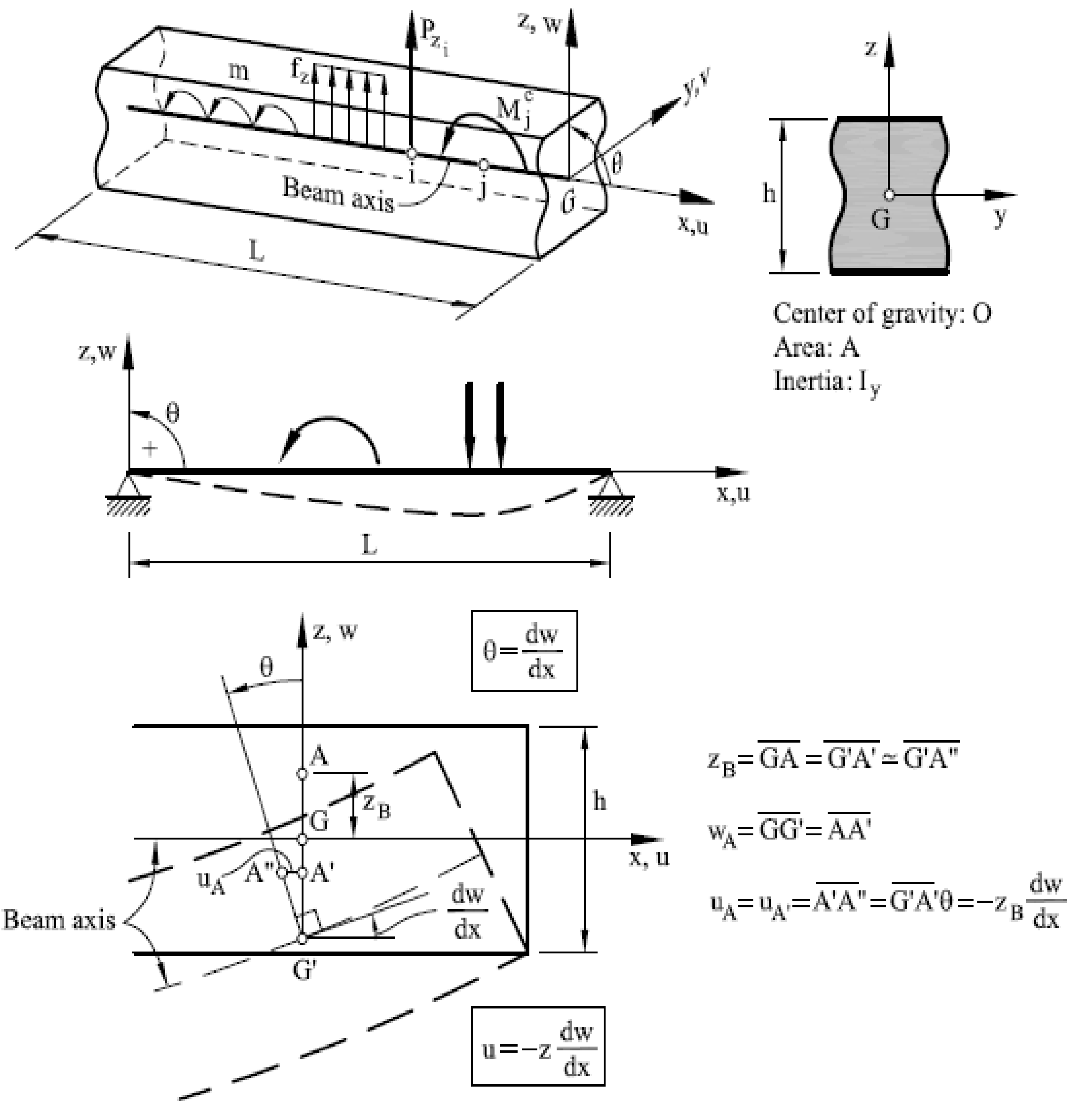


Fig. 3.7 Euler-Bernoulli plane beams. Definition of loads and displacements

3.4.4. Stress-strain relations

For a homogeneous isotropic linear elastic material, the stress is related to the strain by $\sigma = E \cdot \varepsilon$, where E is the Young's modulus. Hence the stress in an Euler–Bernoulli beam is given by

$$\sigma = -z \cdot E \frac{d^2 w}{dx^2} \qquad (3.9)$$

Note that the above relation, when compared with the relation between the axial stress and the bending moment, leads to

$$M = -EI \frac{d^2 w}{dx^2} \qquad (3.10)$$

Since the shear force is given by $Q = \frac{dM}{dx}$, we also have

$$Q = -EI \frac{d^3 w}{dx^3} \qquad (3.11)$$

3.4.5. Strain and stress fields:

Starting from the strain field for a 3D solid [16] we find:

$$\varepsilon_x = \frac{du}{dx} = -z \frac{d^2 w}{dx^2} \quad , \qquad \varepsilon_y = \varepsilon_z = \gamma_{xy} = \gamma_{xz} = \gamma_{yz} = 0 \qquad (3.12)$$

i.e. the beam is under a pure axial strain (ε_x) state. The axial stress σ_x (Figure 3.8) is related to ε_x by Hook law [15] as

$$\sigma_X = E\varepsilon_x = -zE \frac{d^2 w}{dx^2} \qquad (3.13)$$

Where E is the Young modulus.

3.4.6. Bending moment-curvature relationship:

The bending moment for a cross section is defined as (Figure 3.8)

$$M = -\iint_A z\sigma_x dA = \left(\iint_A z^2 dA\right) E\frac{d^2w}{dx^2} = EI_y\frac{d^2w}{dx^2} = EI_y\kappa \quad (3.14)$$

Where $I_y = \iint z^2 dA$ is the moment of inertia (or inertia modulus) of the section with respect to the y axis and $k = \frac{d^2w}{dx^2}$ is the curvature of the beam axis [17].

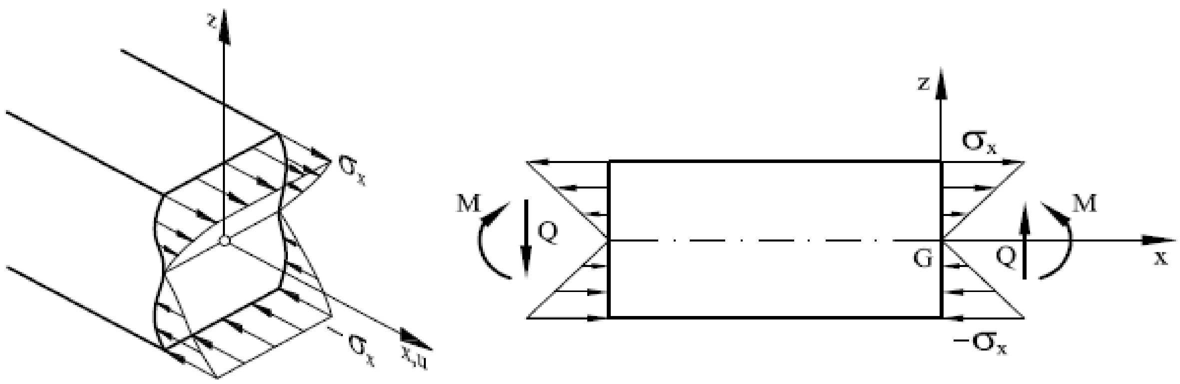


Fig. 3.8 Sign criteria for axial stress σ_x, bending moment M and shear force Q [17].

3.5. Static beam equation:

The Euler–Bernoulli equation describes the relationship between the beam's deflection and the applied load:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2w}{dx^2}\right) = q \quad (3.15)$$

***q* :** is a distributed load

The moment I must be calculated with respect to the axis which passes through the centroid of the cross-section and which is perpendicular to the applied loading. Explicitly, for a beam whose axis is oriented along *x* with a loading along *z*, the beam's cross-section is in the *yz* plane, and the relevant second moment of area is

$$I = \iint z^2 \, dydz \tag{3.16}$$

Where it is assumed that the centroid of the cross-section occurs at y = z = 0.

Often, the product *EI* (known as the flexural rigidity) is a constant, so that

$$EI \frac{d^4w}{dx^4} = q \tag{3.17}$$

This equation, describing the deflection of a uniform, static beam, is used widely in engineering practice.

4.1 Introduction:

The purpose of the connection between the concrete slab and the steel profile is to make the two materials work to achieve a mixed action. Given their different behavior, this connection is made by mechanical means called "connectors", which have the role of preventing or at least reducing the relative slip between the two materials to be assembled, as well as their separation by the possible lifting of the slab. According to the behavior and the resistance of the connection, expressed by the relation between the force requesting the connectors and the sliding measured at the contact interface.

In this chapter, we approach the modeling of the mechanical behavior of beams mixed steel-concrete. The fundamental equations are developed here in detail. In particular, two models are presented; the first relates to continuous modeling of the connection, for the second, the discrete connection is taken into account, as it appears in reality. Equilibrium equations as well as kinematic relationships are developed for these two models of the connection.

4.2 Effect of the connection (steel-concrete) in composite beams:

Mixed beams are bent load-bearing elements made up of a steel beam supporting a concrete slab. It is preferable to involve each of these materials optimally taking into account their physical behavior. Knowing that the concrete has good compressive strength, but its tensile strength is very low and it may be negligible. Steel performs well under compression than under tension, but the often high slenderness of the elements makes it sensitive to phenomena of instability in compressed areas [18].

Based on these considerations, it can be seen that the bent beam composed of a steel beam and concrete slab, without connection between the two, is not a good solution, because each element is flexed independently (Figure4.1 (a)). The part tension of the concrete slab cracks and does not participate in the flexural strength longitudinal. The strength of the steel beam will probably be limited by phenomenon instability (spillage, buckling of the core, or the compressed sole).

With a connection between the two materials (Figure 4.1 (b)), the distribution of deformations specific (ε) shows that the use of materials is significantly different and above all better. The whole slab, or a significant part of it (it depends on the position of the neutral axis), is compressed. The upper sole of the metal beam is held, laterally and in torsion, by the slab. Also, steel is used almost exclusively in traction (this also depends on the position of the neutral axis). Concerning the behavior at the steel-concrete interface, without connection, there is sliding between the two materials, which translate into a discontinuity in the distribution of specific deformations (ε) at the steel-concrete interface. However, with connection, the slipping is prevented; the section then behaves monolithically and the distribution deformation is continuous.

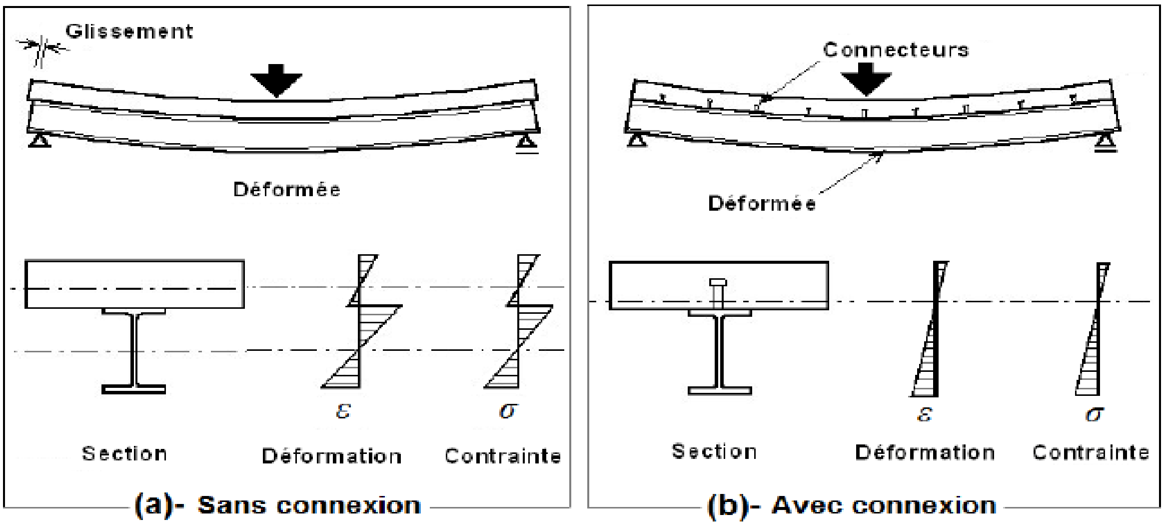


Figure 4.1: Effect of the steel-concrete connection [19].

After this comparison, it follows that the presence of the connection increases, at the same time, the strength and rigidity of the composite beam and, in practice, leads to the reduction of its dimensions and often reducing its cost. The connection is, therefore, the most technically sound.

4.3. Interaction modes and degree of connection:

There are three modes of interaction:

- **The complete interaction** is defined by the absence of sliding between steel and concrete. In this case, the composite beam behaves monolithically and this case has an upper limit in resistance and a lower limit in deformation and deflection.
- **The absence of interaction** is characterized by a free sliding at the interface (steel-concrete). This case corresponds to the absence of connectors. It has a lower limit in strength and an upper limit in deformation and deflection.
- **The partial interaction** is located between the two previous interactions. She is obtained in the case of small composite beams with ductile connectors and lowers than the total number of connectors ensuring complete interaction. In this case, we will have always a slip between the concrete slab and the metal beam involving a deformation discontinuity at the steel-concrete interface.

Figure 4.2 shows the distribution of the strains for the three modes of interaction.

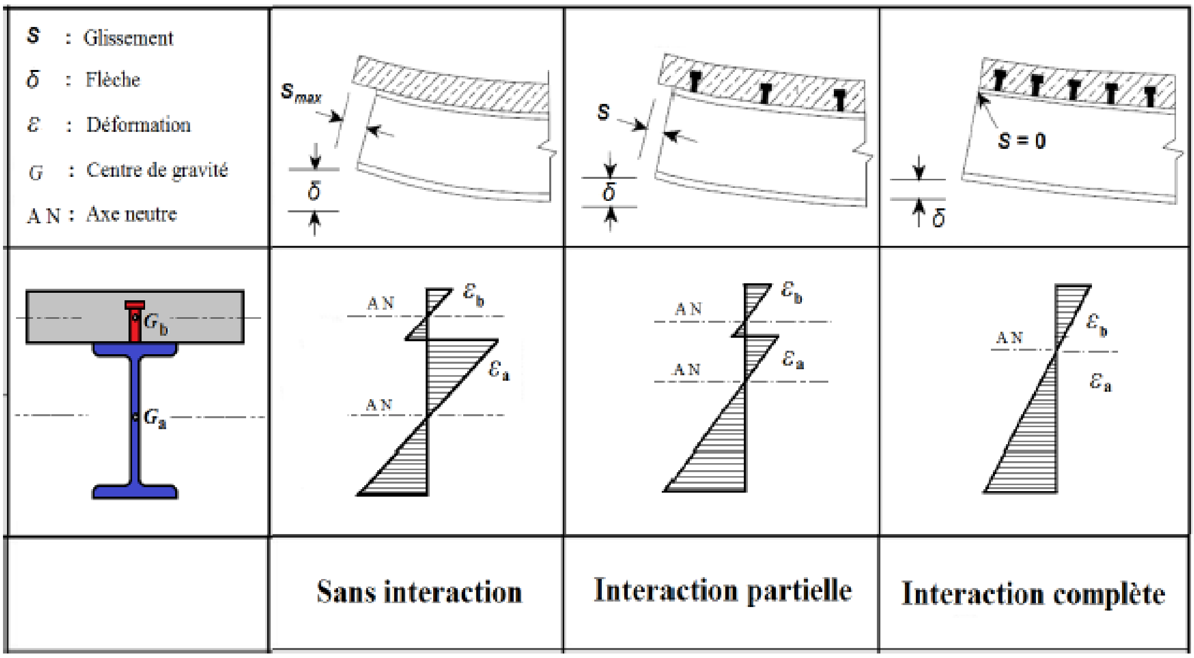


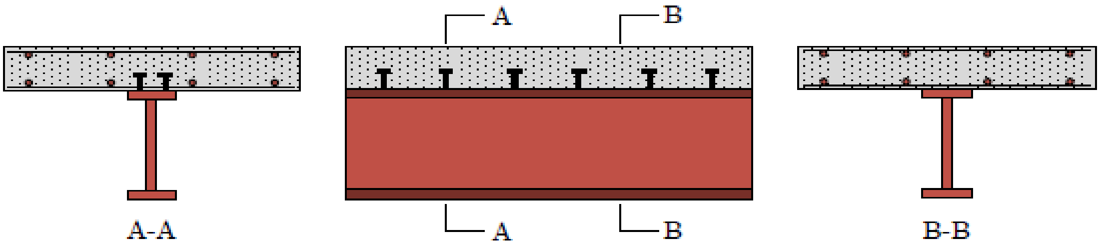
Figure 4.2: Interaction modes [19].

Regarding the resistance of the composite beam, a distinction is made between the complete connection and the partial connection. The two connection modes are defined as follows:

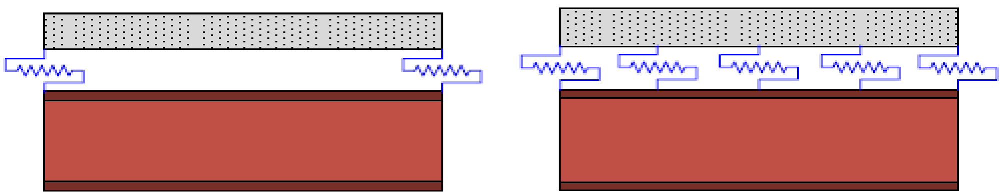
- **The connection** is complete when an increase in the number of connectors no longer increases the flexural strength of the composite beam. In this case, enough connectors to take up the interaction force between the slab concrete and the metal beam in each span between two sections adjacent reviews.
- **The connection** is partial when the number of connectors is lower than that of the full connection [19].

4.4 Assumptions and simplifications:

The object of this thesis is the study of composite steel-concrete beams subjected to a static loading. We consider that the transformations are small, that is to say, that the deformations and displacements are small. So the different configurations can be confused with the initial configuration. Kinematically, the relative vertical displacement between the concrete slab and the steel girder which, in all rigor, could occur, will be neglected in this model. The presence of sliding at the steel-concrete interface does not adopt the Navier-Bernoulli kinematic hypothesis for the whole of the section. However, it is permissible to consider that the concrete slab and the metal profile both behave like a bending beam whose kinematics are defined by the Navier-Bernoulli hypothesis. In other words, that in each part of the composite beam, the planar sections normal to the medium fiber before deformation remain flat and normal to medium fiber after deformation. Note that in the presence of reinforcement, cracking of the slab, especially in negative and weak moment areas degree of connection, does not challenge this assumption. Furthermore, the dimension's usual composite beams have longitudinal slenderness (ratio of the length of span on the transverse sectional height) greater than 10, which generally allows neglecting the shear deformations due to the shearing force. As part of these hypotheses, the behavior of the composite beam is then dominated by the deformation axial, the curvature of the section of each part of the composite beam, and the deformation of connectors.



(a) Real composite beam



(b) Discrete connection model (c) distributed connection model

Figure 4.3 Connection models

One of the objectives of our work is the realization of a computer code allowing to dimension mixed beams with the two connection models, namely the discrete connection and the distributed connection (see Figure 4.3) The shear force at the interface will be punctual to the right of the connector for discreet connection and distributed for the continuous connection.

4.5 Mixed steel-concrete beam: fundamental equations:

First, the slab-beam connection is assumed to be distributed. The case of the discreet connection is dealt with in section(**4.5.1.2**). From a mechanical point of view, the behavior of a deformable body is governed by three groups of equations:

1. Equilibrium equations.
2. Kinematic relationships.
3. The constitutive equations.

These equations are developed in detail, for the two connection models, in the paragraphs below. Only the equilibrium equations change for the case of the discrete connection.

4.5.1 Equations of equilibrium:

The discrete connection introduces discontinuities of the fields of forces of which it will be appropriate to take into account in equilibrium equations. Kinematics, treated in section **4.5.2** is not affected by the connection mode.

4.5.1.1 Case of the continuous connection:

Equilibrium equations are obtained by considering the equilibrium of a mixed beam element of infinitesimal length dx subjected to a distributed load p_z as indicated in figure 4.4. The equilibrium of the composite beam element is therefore written in the non-deformed configuration. To do this, we consider the equilibrium of the concrete slab and the steel beam, taken separately. The quantities with index **C** are relative to the concrete slab. Those with an index ⊲ are relative to the steel beam. The **SC** index relates to connection.

The resulting balance and moment (around the midpoint) of the concrete slab leads to:

$$dN_c(x) + D_{sc}(x)dx = 0 \tag{4.1}$$

$$dT_c(x) + (V_{sc}(x) + p_z)\, dx = 0 \tag{4.2}$$

$$dM_c(x) - T_c(x)dx + H_c D_{sc}(x)dx - \frac{1}{2} dT_c(x)\, dx = 0 \tag{4.3}$$

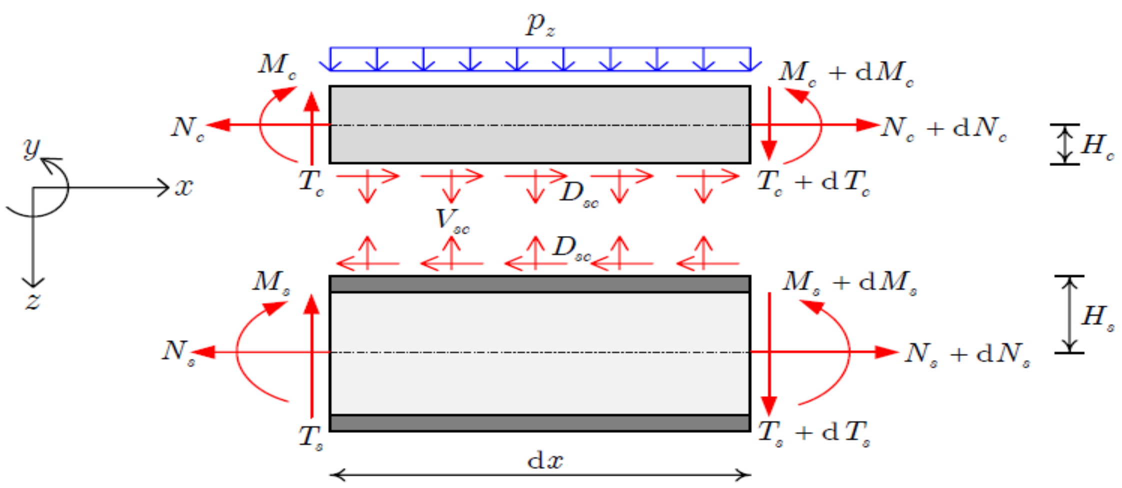


Figure 4.4 - Infinitesimal element of a composite beam

Where p_z denotes the uniformly distributed load applied along with the element. By neglecting the terms of the second-order, we obtain, for the concrete slab, the following system of equations:

$$\frac{dN_c(x)}{dx} + D_{sc}(x) = 0 \tag{4.4}$$

$$\frac{dT_c(x)}{dx} + V_{sc}(x) = 0 \tag{4.5}$$

$$\frac{dM_c(x)}{dx} - T_c(x) + H_c D_{sc}(x) = 0 \tag{4.6}$$

Similarly, the resulting equilibrium and moment (around the midpoint) of the steel beam leads to:

$$\mathrm{d}N_s(x) - D_{sc}(x)\mathrm{d}x = 0 \tag{4.7}$$

$$\mathrm{d}T_s(x) - V_{sc}(x)\mathrm{d}x = 0 \tag{4.8}$$

$$\mathrm{d}M_s(x) - T_s(x)\mathrm{d}x + H_s D_{sc}(x)\mathrm{d}x - \frac{1}{2}\mathrm{d}T_s(x)\,\mathrm{d}x = 0 \tag{4.9}$$

Which is simplified, after having neglected the terms of the second order:

$$\frac{dN_s(x)}{dx} - D_{sc}(x) = 0 \tag{4.10}$$

$$\frac{dT_s(x)}{dx} - V_{sc}(x) = 0 \tag{4.11}$$

$$\frac{dM_s(x)}{dx} - T_s(x) + H_s D_{sc}(x) = 0 \tag{4.12}$$

By combining equations **4.5** and **4.11**, we find the classical relation linking the charge distributed p_z to the total shear force T = Tc + Ts:

$$\frac{\mathrm{d}T(x)}{\mathrm{d}x} + p_z = 0 \tag{4.13}$$

Similarly, by combining equations **4.6** and **4.12**, we obtain the following relation:

$$\frac{\mathrm{d}M(x)}{\mathrm{d}x} - T(x) + H\,D_{sc}(x) = 0 \tag{4.14}$$

Where we denote by:

* $H = H_c + H_s$: the distance between the reference axes of the slab and the profile.

* $M(x) = M_c(x) + M_s(x)$.

$M(x)$ should not be confused with the total bending moment of the mixed section $Mt(x) = M_c(x) + M_s(x) + H \times Ns(x)$.

Furthermore, we observe in relation **4.14** that the effect of the connection appears clearly. The shearing force is then eliminated from the equation **4.14**, which leads to:

$$\frac{\mathrm{d}^2\mathrm{M}(x)}{\mathrm{d}x^2} + H\frac{\mathrm{dD_{sc}}(x)}{\mathrm{d}x} + p_z = 0 \tag{4.15}$$

This last equation completed with relations **4.4** and **4.10** constitutes the system of equations independent which govern the equilibrium of a mixed beam with an interaction force $\mathrm{D_{sc}}(x)$ (which does not exclude the existence of a slip). We will notice, since the shearing force is obtained by deriving the total bending moment that we have four unknown independent forces: $N_c(x)$, $N_s(x)$, $\mathrm{D_{sc}}(x)$ and $M(x)$.

The equilibrium equations (**4.4**), **(4.10)** and **(4.15)** can also be written in the form matrix like:

$$\partial\mathrm{D}(x) - \partial_{sc}\mathrm{D_{sc}}(x) - \mathrm{P_e} = 0 \tag{4.16}$$

Where:

$\mathrm{D}(x) = \left[\ \mathrm{N_s}(x)\quad \mathrm{N_c}(x)\quad \mathrm{M}(x)\ \right]^T$, denotes the vector of the internal forces of a composite section;

$\mathrm{P}_e = \left[\ 0\quad 0\quad p_z\ \right]^T$, denotes the vector of external forces applied along the element.

∂ and ∂_{sc} are two differential operators defined by:

$$\partial = \begin{bmatrix} \frac{d}{dx} & 0 & 0 \\ 0 & \frac{d}{dx} & 0 \\ 0 & 0 & -\frac{d^2}{dx^2} \end{bmatrix} \quad , \quad \partial_{sc} = \begin{bmatrix} 1 & -1 & H\frac{d}{dx} \end{bmatrix}^T \tag{4.17}$$

4.5.1.2 Discrete connection case:

For this type of connection, two zones should be distinguished: that between connectors and the one right to the connector (**see Figure 4.5**). In the first area, the equations of equilibrium are identical to those developed in the previous paragraph by taking $\mathrm{D_{sc}} = 0$, which leads to:

$$\frac{dN_c(x)}{dx} = 0 \tag{4.18}$$

$$\frac{dN_s(x)}{dx} = 0 \tag{4.19}$$

$$\frac{d^2M(x)}{dx^2} + p_z = 0 \tag{4.20}$$

At the level of the connectors, the normal forces in the slab and the metal profile are discontinuous. Assuming that the transmission of forces between the concrete slab and the metal profile only takes place at one point of the steel-concrete interface, the forces in line with the connector are calculated in the following (considering an element of length Δx:

$$N_c = N_c^+ - N_c^- = -Q_{st} \quad (4.21)$$

$$N_s = N_s^+ - N_s^- = Q_{st} \quad (4.22)$$

$$M_c = M_c^+ - M_c^- = -H_c Q_{st} \quad (4.23)$$

$$M_s = M_s^+ - M_s^- = -H_s Q_{st} \quad (4.24)$$

Thus, the discontinuity of the normal force in the concrete slab and the metal profile is equal to the force in the connector Q_{st}, taken in absolute value. This relationship is written in matrix form:

$$\begin{bmatrix} N_c \\ N_s \\ M \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -H \end{bmatrix} Q_{st} \quad (4.25)$$

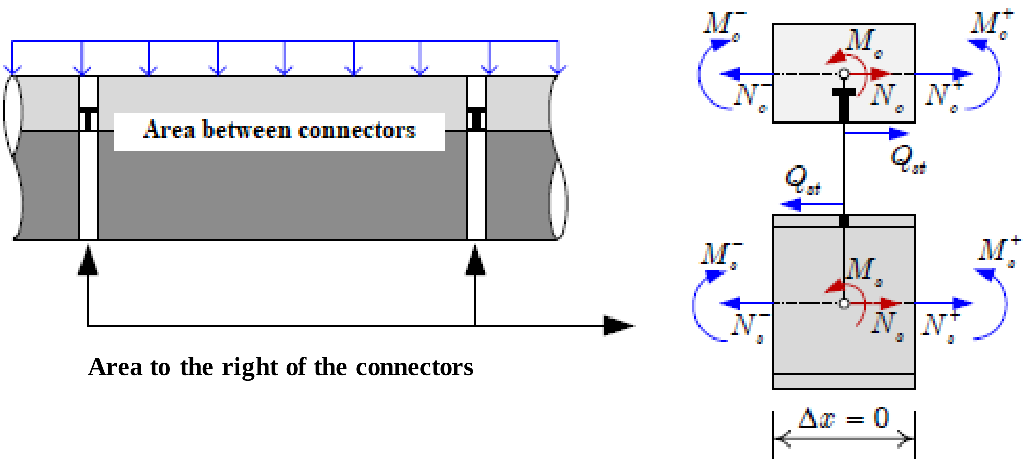


Fig. 4.5 connecter element.

4.5.2 Kinematic relationships:

In this paragraph, the relations which connect the generalized deformations (curvature and strains) to displacements are given within the framework of small displacements. In considering that the straight sections remain straight and normal to the neutral axis for each component of the composite beam, we obtain the following relationships:

- **Steel beam:**

$$\varepsilon_s(x) = \frac{du_s(x)}{dx} \tag{4.26}$$

$$\theta_s(x) = -\frac{d\upsilon_s(x)}{dx} \tag{4.27}$$

$$\kappa_s(x) = -\frac{d\theta_s(x)}{dx} = -\frac{d^2\upsilon_s(x)}{dx^2} \tag{4.28}$$

- **Concrete slab:**

$$\varepsilon_c(x) = \frac{du_c(x)}{dx} \tag{4.29}$$

$$\theta_c(x) = -\frac{d\upsilon_c(x)}{dx} \tag{4.30}$$

$$\kappa_c(x) = -\frac{d\theta_c(x)}{dx} = -\frac{d^2\upsilon_c(x)}{dx^2} \tag{4.31}$$

Where ε_i represents the deformation of the reference axis of component (i); θ_i represents the section rotation of component (i) and κ_i represents the curvature of component (i).

- Steel-concrete interface: A slip d_{sc} occurs at the steel-concrete interface which resulting from a relative displacement. The rotations being small, which provides:

$$d_{sc}(x) = u_s(x) - H_s\theta_s(x) - u_c(x) - H_c\theta_c(x) \tag{4.32}$$

$$\upsilon_{sc}(x) = \upsilon_s(x) - \upsilon_c(x) \tag{4.33}$$

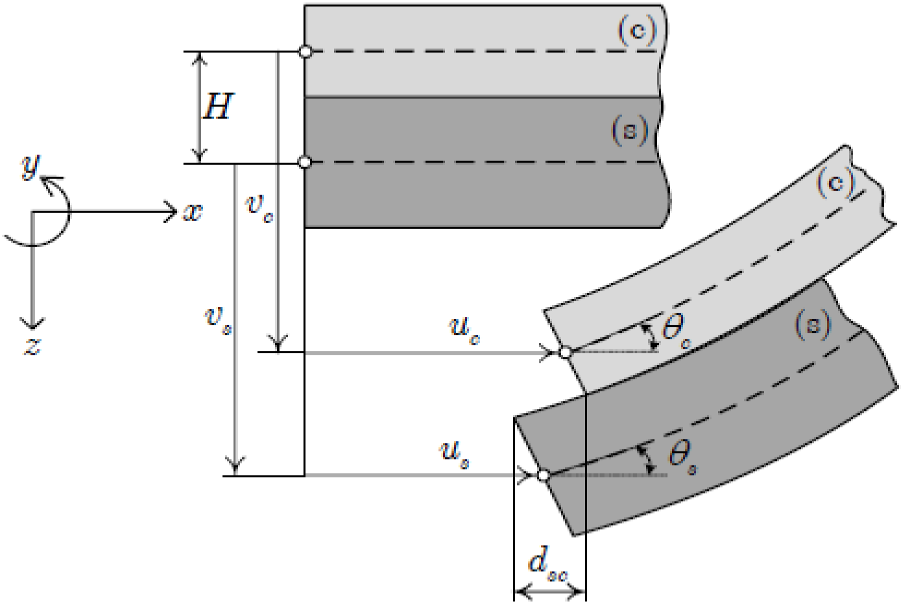


Figure 4.6 Kinematics of the composite beam

Several studies of a theoretical and experimental nature show that in the most of cases, the lifting of the concrete slab relative to the steel beam is very small and its effects on the overall behavior of the composite beam are negligible. It is therefore possible to neglect the lifting phenomenon of the concrete slab relative to the metal profile. Thus, in the absence of lifting (detachment), the field of transverse displacement is identical for the steel beam and the concrete slab:

$$\mathrm{v}(x) = \mathrm{v_s}(x) = \mathrm{v_c}(x) \tag{4.34}$$

As a result, the rotations and curvatures are identical for the two components:

$$\theta(x) = \theta_{\mathrm{s}}(x) = \theta_{\mathrm{c}}(x) \tag{4.35}$$

$$\kappa(x) = \kappa_{\mathrm{s}}(x) = \kappa_{\mathrm{c}}(x) \tag{4.36}$$

Thus the independent kinematic variables are, $\mathrm{v}(x)$, $\mathrm{u_s}(x)$ and $\mathrm{u_c}(x)$, which leads to the following kinematic relationships:

$$\varepsilon_c(x) = \frac{du_c(x)}{dx} \tag{4.37}$$

$$\varepsilon_s(x) = \frac{du_s(x)}{dx} \tag{4.38}$$

$$\theta(x) = -\frac{dv(x)}{dx} \tag{4.39}$$

$$\kappa(x) = -\frac{d^2v(x)}{dx^2} \tag{4.40}$$

$$\mathrm{d_{sc}}(x) = \mathrm{u_s}(x) - \mathrm{u_c}(x) + \mathrm{H}\frac{dv(x)}{dx} \tag{4.41}$$

Or, in matrix form:

$$\partial \mathrm{d}(x) - \mathrm{e}(x) = 0 \tag{4.42}$$

$$\partial_{sc}^T \mathrm{d}(x) - \mathrm{d_{sc}}(x) = 0 \tag{4.43}$$

Where:

$\mathrm{d}(x) = \left[\mathrm{u_s}(x)\ \mathrm{u_c}(x)\ v(x)\right]^T$, is the vector of displacements.

$\mathrm{e}(x) = \left[\varepsilon_s(x)\ \varepsilon_c(x)\ \kappa(x)\right]^T$, is the vector of the generalized strains associated with vector of internal forces D (x).

4.5.3 Constitutive behavior:

The purpose of this section is to establish generalized behavioral relationships from uniaxial stress/strain relationships of each of the components.

4.5.3.1 Section behavior: Relations between forces and generalized deformations:

Based on the assumption that the cross-sections of each sub-beam (concrete-slab and steel-section) remain plane after deformation, the normal deformation field is written:

In the steel section: $$\varepsilon_s(x, z_s) = \varepsilon_s(x) + z_s\kappa(x) \tag{4.44}$$

In the concrete slab: $$\varepsilon_c(x, z_c) = \varepsilon_c(x) + z_c\kappa(x) \tag{4.45}$$

The normal stress field is deduced from the deformation field using the elasticity equations. By definition, internal forces result from the integration of the stress field over the cross-section:

$$N_s(x) = \int \sigma_c(x, z)\, \mathrm{d}A \tag{4.46}$$

$$N_c(x) = \int_{A_c} \sigma_c(x, z)\, \mathrm{d}A + \sum_{n_{sr}} \sigma_{sr}(x, z_{sr})\, A_{sr} \tag{4.47}$$

$$M(x) = \int_{A_c} z_c\,\sigma_c(x, z_c)\,\mathrm{d}A + \sum_{n_{sr}} z_{sr}\,\sigma_{sr}(x, z_{sr})\,A_{sr} + \int_{A_s} z_s\,\sigma_s(x, z_s)\,\mathrm{d}A \tag{4.48}$$

Where *As*, *Ac* and A_{sr} are respectively the areas of the steel cross section, the slab cross section and reinforcement; n_{sr} is the number of rebars. These equations show that the cross section behavior depends on the material properties and the geometry of the section. We will note by:

$$\mathbf{D}(x) = \hat{\mathbf{D}}\,(\mathbf{e}(x)) \quad \text{ou} \quad \mathbf{e}(x) = \hat{\mathbf{e}}\,(\mathbf{D}(x)) \tag{4.49}$$

In this work, we consider a linear elastic behavior for steel and concrete. Thus, the introduction of relations (4.59) and (4.44) in relations (4.47 - 4.48) gives:

$$N_s(x) = (EA)_s\,\varepsilon_s(x) + (ES)_s\,\kappa(x) \tag{4.50}$$

$$N_c(x) = (EA)_c\,\varepsilon_c(x) + (ES)_c\,\kappa(x) \tag{4.51}$$

$$M(x) = (ES)_c\,\varepsilon_c(x) + (ES)_s\,\varepsilon_s(x) + (EI)\,\kappa(x) \tag{4.52}$$

Where:

$$(ES)_c = E_c S_c + \sum_{n_{sr}} E_{sr} A_{sr} z_{sr} \qquad (ES)_s = E_s S_s$$

$$(EA)_c = E_c A_c + \sum_{n_{sr}} E_{sr} A_{sr} \qquad (EA)_s = E_s A_s$$

$$(EI) = E_c I_c + E_s I_s + \sum_{n_{sr}} E_{sr} A_{sr} z_{sr}^2$$

Relations (4.50), (4.51) and (4.52) can be written in matrix form in the following way:

$$\mathrm{D}(x) = \mathrm{k}\,\partial \mathrm{d}(x) \tag{4.53}$$

Where *k* denotes the stiffness matrix of the section:

$$\mathrm{k} = \begin{bmatrix} (\mathrm{EA})_s & 0 & (\mathrm{ES})_s \\ 0 & (\mathrm{EA})_c & (\mathrm{ES})_c \\ 0 & 0 & (\mathrm{EI}) \end{bmatrix} \tag{4.54}$$

By choosing as reference axis of each section (slab and steel section) the axis passing the center of gravity of the section (Sc = Ss = 0) and making use of the kinematic relations (4.37 - 4.40), we can write the law of behavior by making the displacements appear explicitly:

$$N_s = (EA)_s \frac{du_s}{dx} \tag{4.55}$$

$$N_c = (EA)_c \frac{du_c}{dx} \tag{4.56}$$

$$M = -(EA) \frac{d^2u}{dx^2} \tag{4.57}$$

4.5.3.2 Behavior low of the connection:

The connection can be characterized by push-out tests. Several pushes- tests out have been made in the literature which shows that the force-slip relationship is strongly non-linear beyond a slip of 1 *mm*. So, generally speaking, the shear force is related to the interfacial slip by a nonlinear constitutive law which is written in the form:

Discreet connection: $Q_{st} = Q^{\wedge}(d_{sc})$ (4.58)

Continuous connection: $D_{sc}(x) = D^{\wedge}_{sc}(d_{sc}(x))$ (4.59)

If we suppose that the connection has a linear elastic behavior, the relations (4.58) and (4.59) become:

Discreet connection: $Q_{st} = k_{st} d_{sc}$ (4.60)

Continuous connection: $D_{ss}(x) = k_{sc} d_{sc}(x)$ (4.61)

Where k_{st} [*N/m*] denotes the rigidity of the discrete connection and k_{sc} [*N/m*^{2}] denotes the rigidity of the continuous connection. If the beam is discreetly connected (type connector studs, angles...) and we want to model it as a distributed connection, the stiffness k_{sc} is then given by:

$$k_{sc} = \frac{k_{st}}{a} \tag{4.62}$$

Where (*a*) is the longitudinal spacing between the connectors. If the spacing is not regular along the beam, the stiffness k_{sc} varies along the beam.

4.6 Mixed beam stiffness matrix:

In this part, we develop the stiffness matrix for a mixed beam by considering the two connection modes: discrete connection and continuous connection. We will assume that the behavior of the materials and the connection is linear elastic.

In order to build the stiffness matrix of a mixed beam element, we separate this element into three sub-elements (cf. Figure 4.7): an unconnected composite beam element (upper layer and lower layer) and the connection elements which provide longitudinal shear stiffness. The stiffness matrix is obtained by assembling the stiffness matrices of these elements.

4.6.1 Unconnected composite beam stiffness:

The objective of this section is to establish the stiffness matrix of unconnected mixed beam. We will suppose that the loading is described by the function $p_z(x)$, and the two ends of the beam constitute nodes in the sense of finite elements.

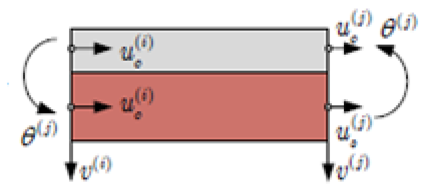


Figure 4.7 Unconnected mixed beam element

By combining equilibrium equations (4.18 - 4.20) and behavioral relationships expressed as a function of displacements (4.55 - 4.57), we obtain equilibrium equations expressed as a function of displacements:

$$\partial_x^4 \mathrm{v}(x) = \frac{p_z(x)}{(EI)} \quad (4.63)$$

$$\partial_x^2 \mathrm{u}_c(x) = 0 \quad (4.64)$$

$$\partial_x^2 \mathrm{u}_s(x) = 0 \quad (4.65)$$

Where we denote $\partial_x^2 \bullet = d^n \bullet / dx^n$. Lack of connection leads to a decoupled system whose solution is:

$$v(x) = C_1 x^3 + C_2 x^2 + C_3 x + C_4 \quad (4.66)$$

$$u_c(x) = C_5 x + C_6 \quad (4.67)$$

$$u_s(x) = C_7 x + C_8 \quad (4.68)$$

The constants C_i, i = 1…8, are determined by imposing that the displacements at the ends are equal to the nodal displacements, which are the main unknowns:

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{bmatrix} = \begin{bmatrix} u_c(0) \\ u_s(0) \\ v(0) \\ \theta(0) \\ u_c(L) \\ u_c(L) \\ v(L) \\ \theta(L) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L & 1 \\ L^3 & L^2 & L & 1 & 0 & 0 & 0 & 0 \\ 3L^2 & 2L & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \end{bmatrix} \quad (4.69)$$

Hence, in matrix form:

$$q = XC \quad (4.70)$$

The displacements being independent, the matrix **X** is invertible. We thus obtain the constants C_i according to the nodal displacements q_i:

$$C = X^{-1}\,(q\,) \quad (4.71)$$

This is equivalent to defining the interpolation functions for the Finite elements of beam / column type. Internal forces along the mixed beam section are obtained by substituting the relations (4.66 – 4.68) in (4.55 - 4.57):

$$M(x) = -(EI)\,(6C_1 x + 2C_2) \quad (4.72)$$

$$N_c(x) = (EA)_c\; C_5 \quad (4.73)$$

$$N_s(x) = (EA)s\; C7 \quad (4.74)$$

The stiffness matrix translates the link between the nodal forces Q_i and the displacements nodal q_i. It is first obtained by imposing the conditions on static limits:

$$Q = YC \tag{4.75}$$

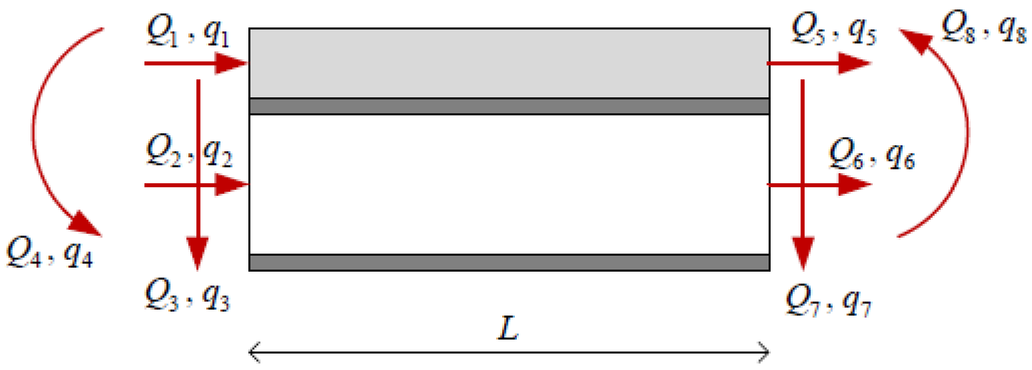


Figure 4.8 Nodal forces and nodal displacements of a composite beam element

Where:

$$Y=\begin{bmatrix} 0 & 0 & 0 & 0 & -(EA)_c & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -(EA)_s & 0 \\ 6(EI) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2(EI) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (EA)_c & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & (EA)_s & 0 \\ -6(EI) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6L(EI) & 2(EI) & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.76}$$

By introducing in the second step the equation (4.71) in (4.75), we obtain:

$$K_e^{(nc)} q = Q \tag{4.77}$$

Where:

$$K_e^{(nc)} = YX^{-1} \tag{4.78}$$

Represents the stiffness matrix of the unconnected composite beam element.

4.6.2 Mixed beam finite element with discrete connection:

We develop in this section the stiffness matrix of a mixed beam finite element with discrete connection model in the case of the linear elastic behavior of the studs. For the case of discrete connection, the studs are modeled by a specific element without physical length but for which the shear stiffness exists (shear spring) located at the beam nodes.

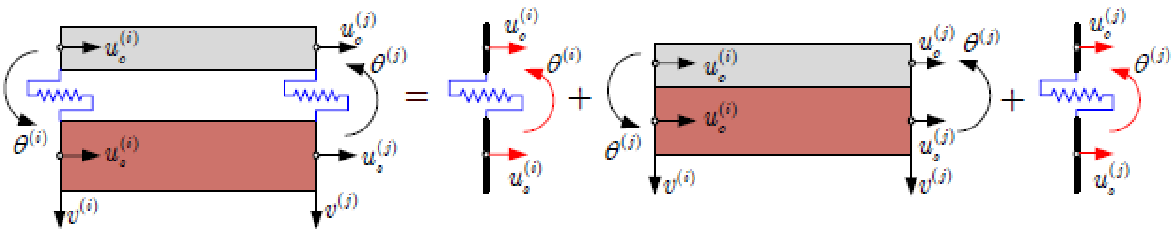


Figure 4.9 Mixed beam element with discrete connection

By introducing the kinematic slip relation (4.41) in the equation (4.60), we obtain the relation which is then introduced in the equilibrium equation (4.25) in order to link the forces to corresponding locations:

$$Q_{st} = k_{st}\begin{bmatrix}-1 & 1 & -H\end{bmatrix}\begin{bmatrix}u_c \\ u_s \\ \theta\end{bmatrix} \quad (4.79)$$

$$\begin{bmatrix}N_c \\ N_s \\ M\end{bmatrix} = \begin{bmatrix}-1 \\ 1 \\ -H\end{bmatrix} k_{st}\begin{bmatrix}-1 & 1 & -H\end{bmatrix}\begin{bmatrix}u_c \\ u_s \\ \theta\end{bmatrix} \quad (4.80)$$

Hence the expression of the stiffness matrix and the connection element:

$$K_e^{(st)} = k_{st}\begin{bmatrix}1 & -1 & H \\ -1 & 1 & -H \\ H & -H & H^2\end{bmatrix} \quad (4.81)$$

The assembly of the elementary stiffness matrices is done taking into account the conditions kinematics imposed on the nodes. **Figure 4.9** shows diagrammatically how is built the stiffness matrix K_e of an element of mixed beam connected discretely at both ends, from the coefficients of $K_e^{(nc)}$, of $K_i^{(st)}$ and of $K_j^{(st)}$.

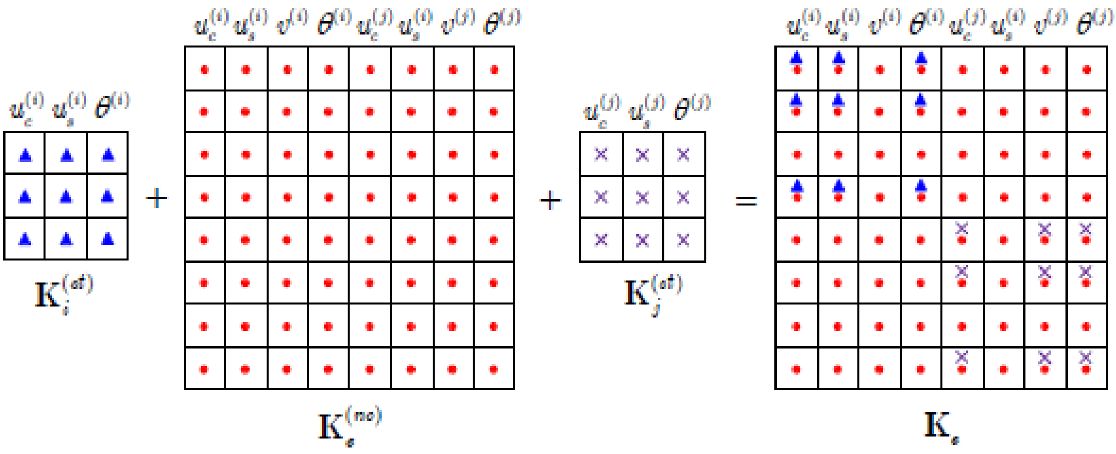


Figure 4.10 Assembly of steel, concrete and connector matrixes

4.6.3 Mixed beam finite element with continuous connection:

We develop in this section the stiffness matrix of a mixed beam finite element with continuous connection model in the case of the linear elastic behavior of the studs. For the case of continuous connection, the connection is modeled by shear springs continuously distributed along the beam interface.

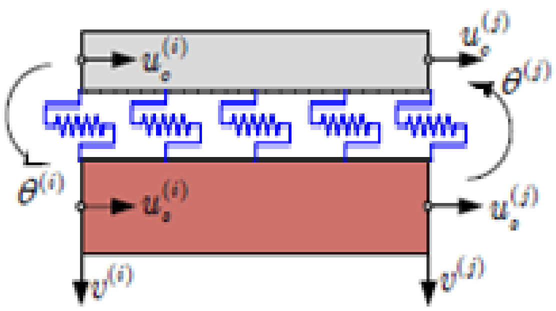


Figure 4.11 Mixed beam element with continuous connection

The strain energy of the continuous connection (shear springs) is:

$$U_e^{(sc)} = \frac{1}{2}\int_0^L \left[K_c\right]^T K_{sc}\left[K_c\right]dx \qquad (4.82)$$

The continuous connection stiffness matrix can be computed as:

$$K_e^{(sc)} = \int_0^L \left[K_c\right]^T K_{sc}\left[K_c\right]dx \qquad (4.83)$$

The continuous connection stiffness matrix can be computed by numerical integration as follow

$$K_e^{(sc)} = \sum_{i=1}^{n_g} \left[K_c\right]^T K_{sc} \left[K_c\right] W \det J \tag{4.84}$$

Where:

$$K_c = \left[N_1 \quad -N_1 \quad H*dndx(1) \quad H*dndx(2) \quad N_2 \quad -N_2 \quad H*dndx(3) \quad H*dndx(4)\right] \tag{4.83}$$

Ni: Linear shape functions.

dndx: Bernoulli beam element shape functions derivatives in Cartesian coordinates.

$$H = \frac{h_c}{2} + \frac{h_s}{2} \tag{4.85}$$

W: Wight Gauss, and det*J* : is the Jacobian determinant.

5.1 Introduction:

The main purpose of this section is to assess the capability of the proposed finite element model to predict the elastic structural behavior of mixed steel-concrete beams with partial connection. For that purpose, the stiffness matrices developed in chapter four was implemented in a MATLAB program developed during this work in order to compare the two connection models (discrete and continuous).

In this chapter, three steel-concrete mixed beams are considered. Besides, one frame with steel-concrete mixed beam is studied.

5.2 Two-span continuous composite beam (B1):

We analyze in this paragraph a mixed beam with two equal spans resting on three simple supports. It has a total length of 5 *m*. The beam consists of an IPE 200 steel section connected to a concrete slab whose section is 1200 × 120 mm^2, reinforced by two layers of 5 HA12 reinforcement. The first span is subjected to a concentrated load of 30 *KN* while the second is subjected to a concentrated load of 20 *KN*.

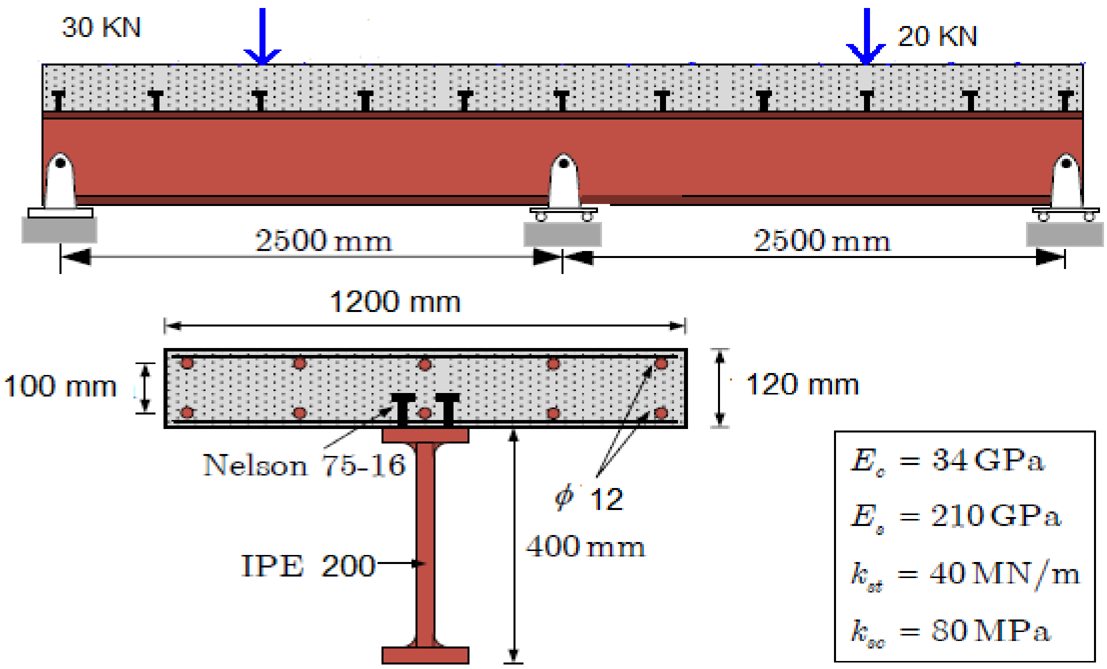


Figure 5.1 Description of the reference beam B1

The connection is made by 11 pairs of Nelson studs, h = 75 mm, ϕ = 16 mm, evenly spaced along the length of the beam. In other words, the longitudinal spacing of the studs, (a), is equal to 0.5 m. The stiffness of each stud will be taken equal to 20 MN/m. The stiffness "k_{st}" of the connection used in the simulation with the discrete connection model, is therefore equal to 2×20 = 40 MN/m. The equivalent linear stiffness of connection "k_{sc}", used in the simulation with the continuous connection model is then:

$$k_{sc} = \frac{k_{st}}{a} = \frac{40}{0.5} = 80\ MPa \qquad (5.1)$$

The kinematic boundary conditions are: $v\ (x = 0) = 0$, $v\ (x = 2.5\ m) = 0$, $v\ (x = 5m) = 0$ and $us\ (x = 0) = 0$.

Ec= 34 GPa, Es= 210 GPa, Esr= 210 GPa.

The beam is simulated with 10 elements of mixed beam with discrete connection at first, and then with 10 elements of mixed beam with continuous connection.

5.2.1 Comparison of connection types:

Looking at Figures 5.2 to 5.9, we see that the distributed connection model gives results almost identical to those obtained with the discrete connection.

Figures 5.2 and 5.9 respectively illustrate the distribution of the deflection and the slip along the beam length. Note that the continuous connection model overestimates the deflection. This id due to the fact that the normal force being greater with a discrete connection model while total moment is the same in both models.

Moreover, we observe in figure 5.9 that the continuous connection model overestimates the slip. On the other hand, it underestimates the axial displacement of the slab (see Figure 5.3) and the steel section (see Figure 5.4).

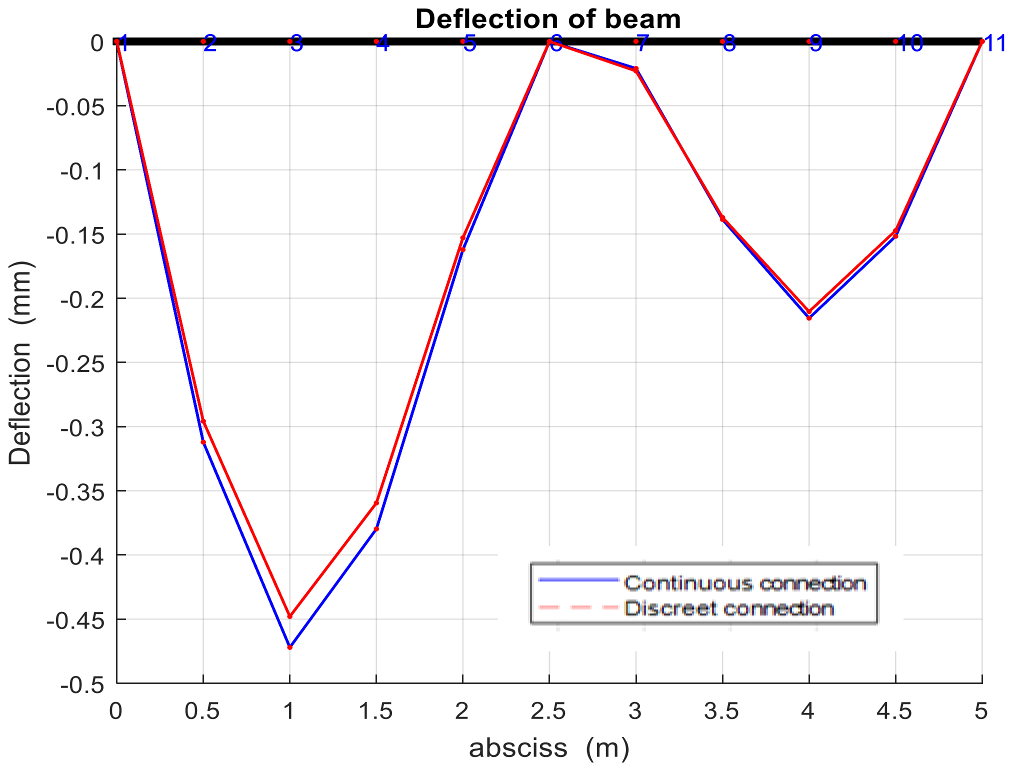


Figure 5.2 Beam B1: Distribution of the deflection along the beam length

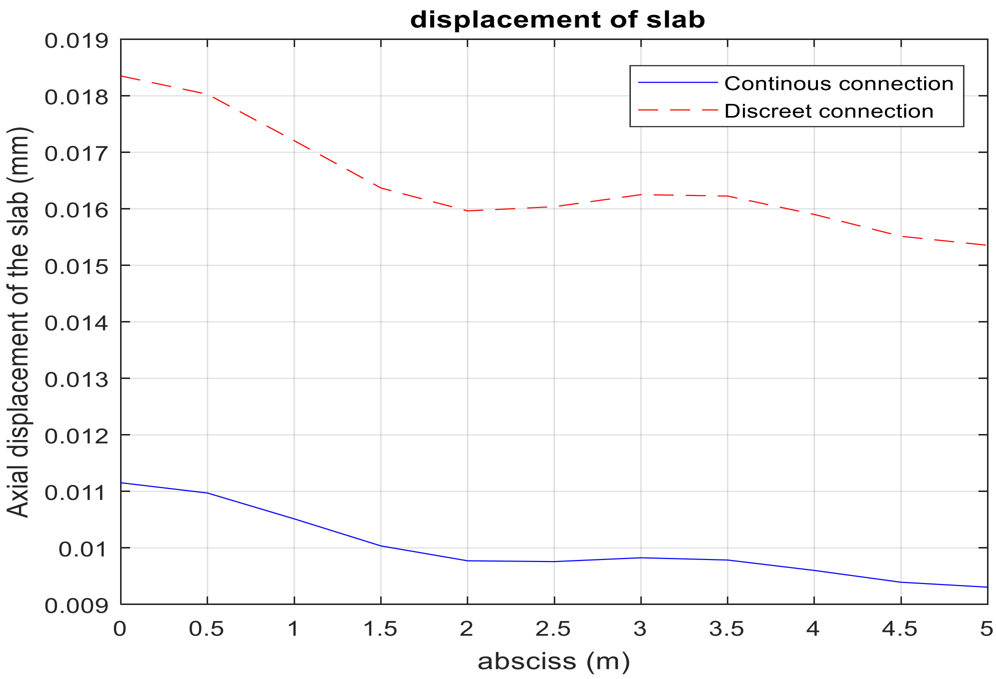


Figure 5.3 Beam B1: Distribution of the axial displacement in the slab along the beam length

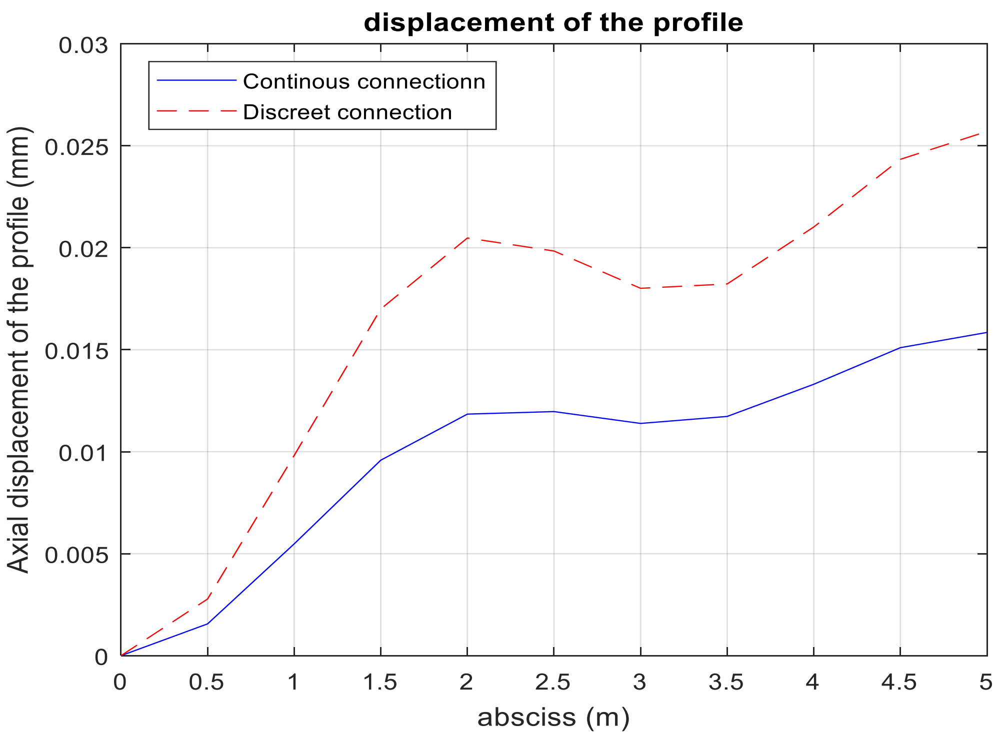


Figure 5.4 Beam B1: Distribution of the axial displacement in the steel section along the beam length

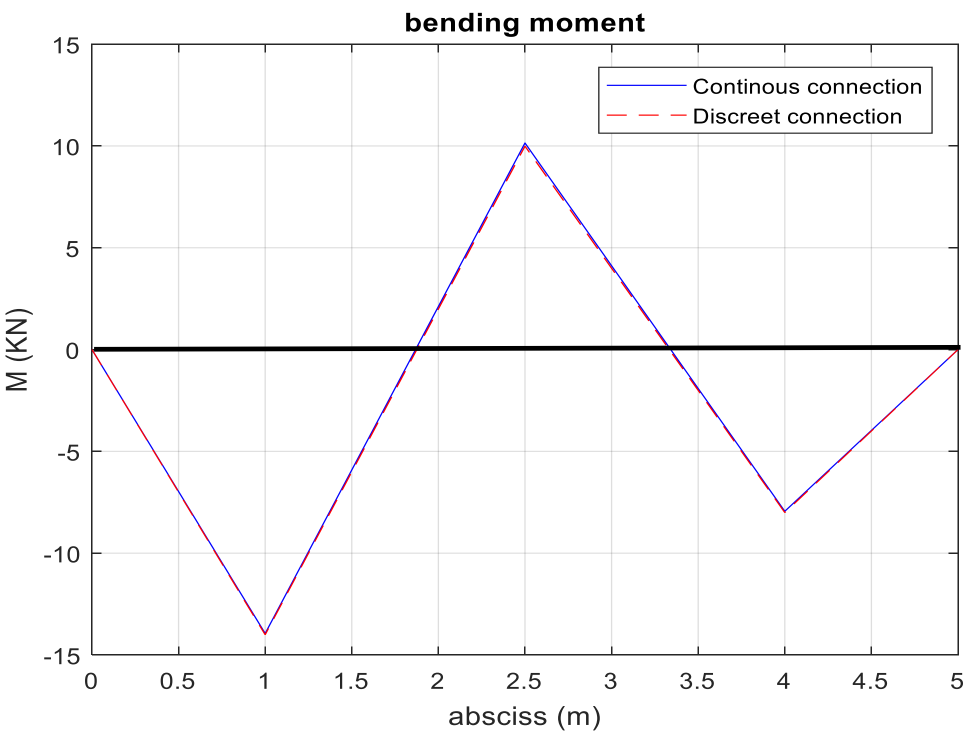


Figure 5.5 Beam B1: Distribution of the total bending moment along the beam length

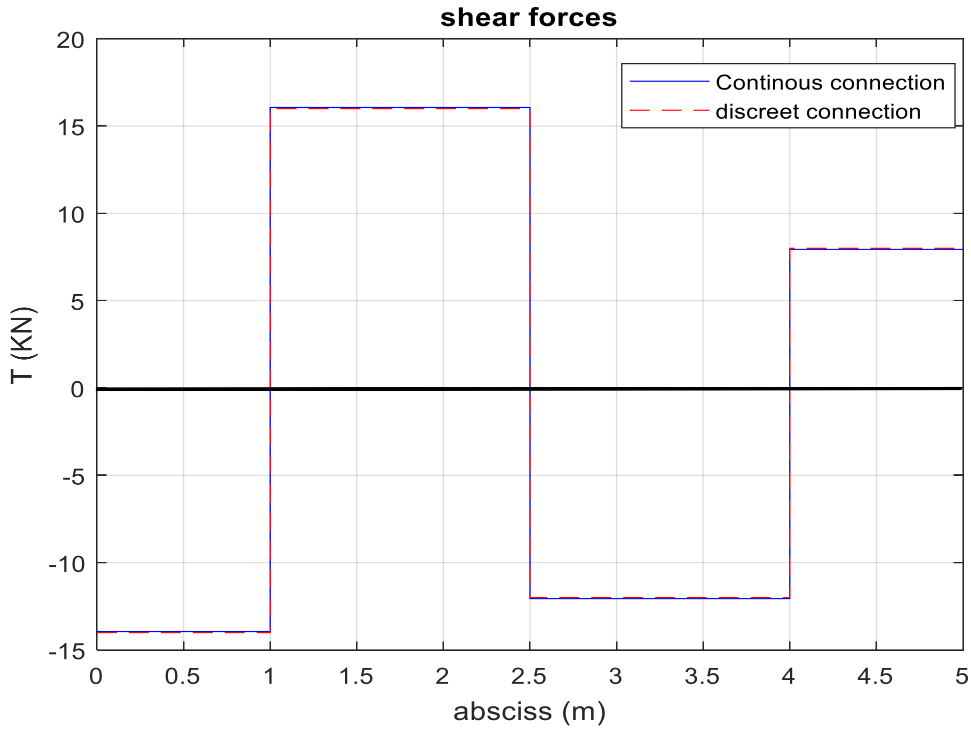


Figure 5.6 Beam B1: Distribution of the total shear force along the beam length

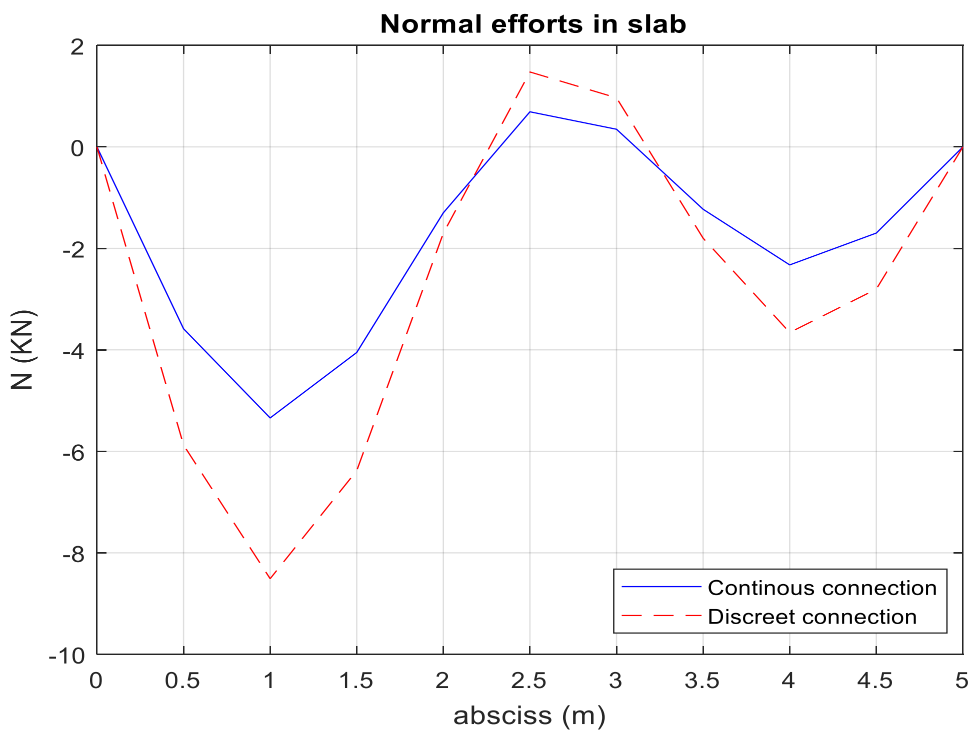


Figure 5.7 Beam B1: Distribution of the normal force in the slab along the beam length

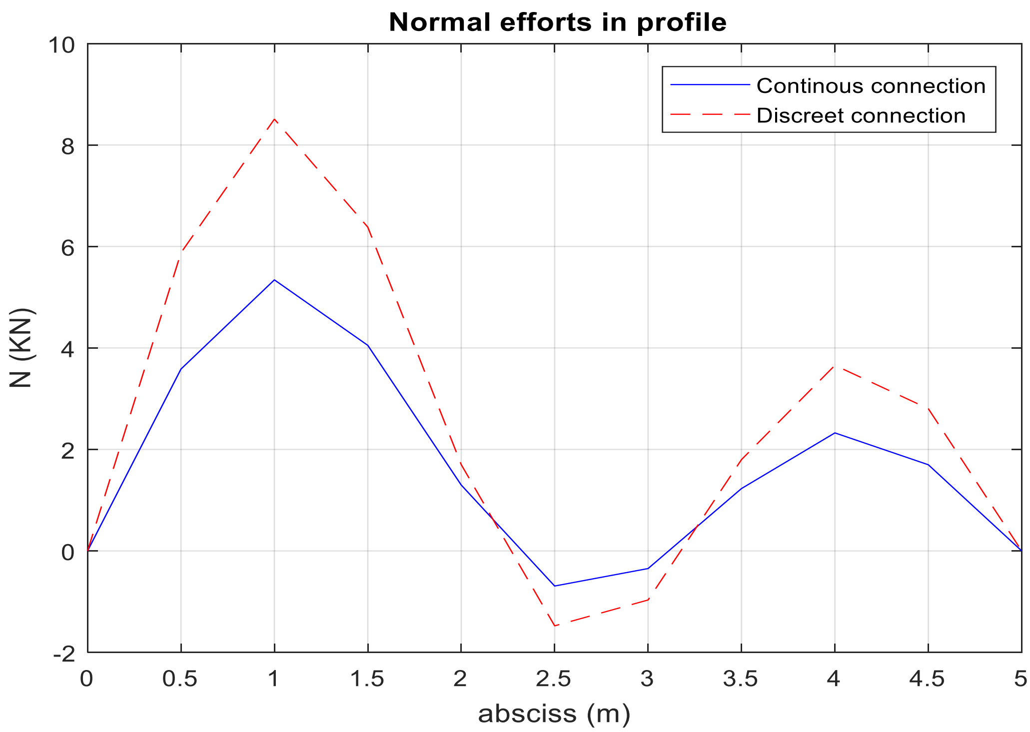


Figure 5.8 Beam B1: Distribution of the normal force in the steel section along the beam length

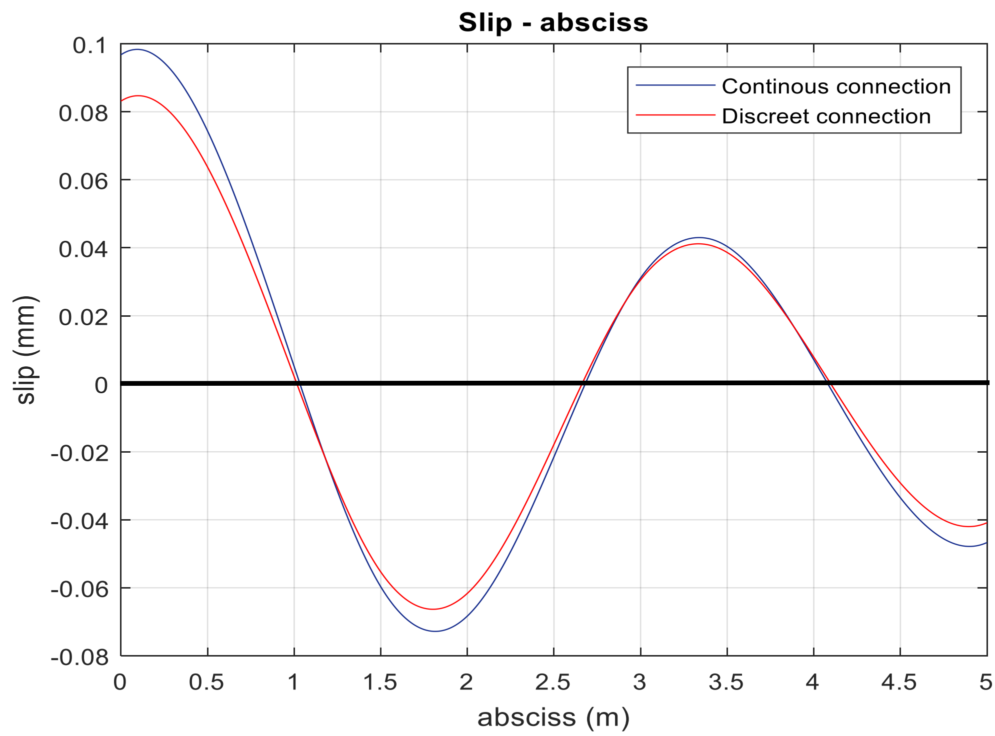


Figure 5.9 Beam B1: Distribution of the slip along the beam length

I can be seen from figure 5.9 that that maximum slip occurs on the supports, which means that the maximum slip follows the maximus shear forces.

5.2.2 Comparison for different shear bond stiffness:

In particular, the comparison can be practically carried out in terms of the longitudinal slip between steel-concrete and the deflection evaluated using the two presented models. Fig. 5.10 and 5.11 show the distribution of the longitudinal slip and deflection respectively obtained with the proposed models for different shear bond stiffness (*ksc*).

As expected, the deflection and slip predicted by the proposed models are highly affected by different shear bond stiffness (*ksc*). In particular, the red line refers to the case of practically full interaction (rigid connection with *ksc* =10000 *MPa*): The other curves, related to cases of lower shear stiffness, monotonically reduce to the case of (practically) no interaction (loose connection with *ksc* = 1 *MPa*).

➢ **In Continuous connection:**

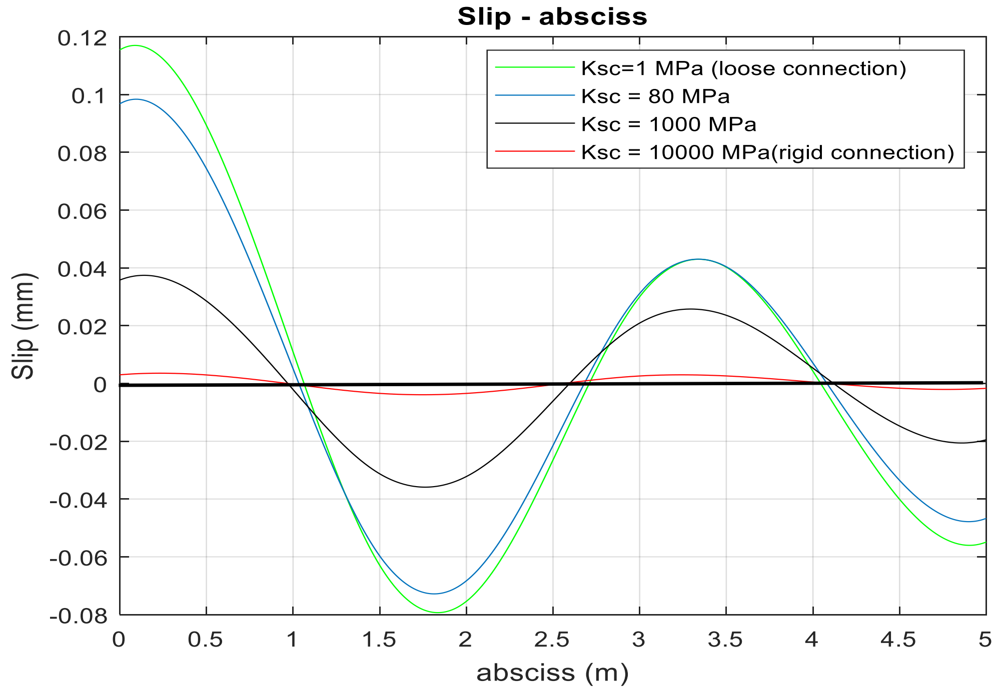


Figure 5.10 Beam slip versus the span for different shear bond stiffness

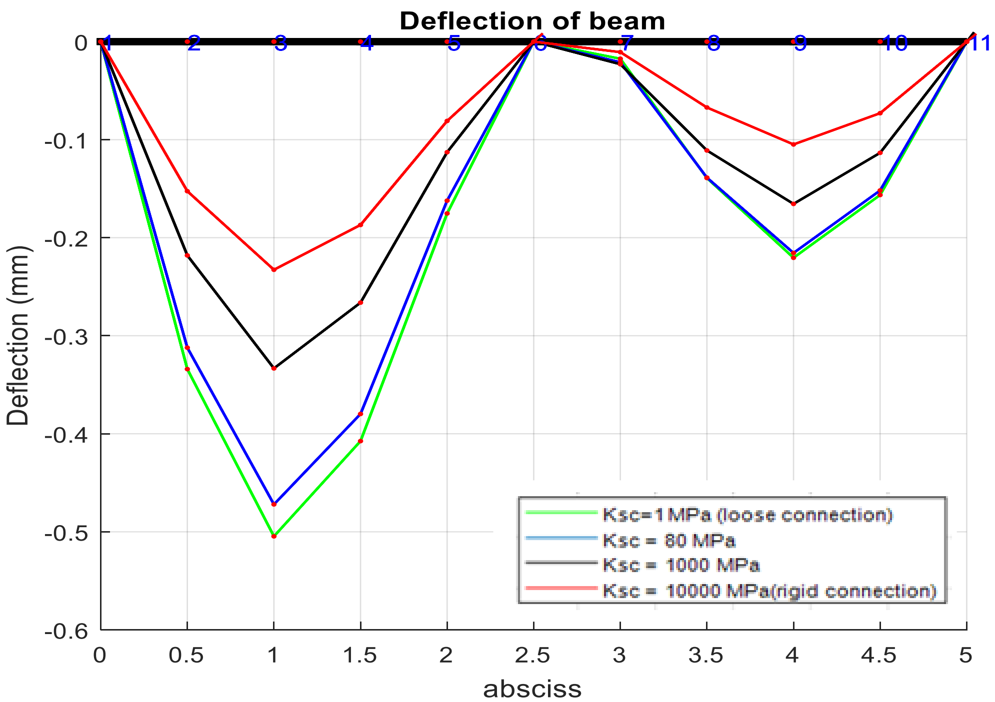


Figure 5.11 Deflection at load point versus the span for different shear bond stiffness

It can be seen that partial interaction results in a reduction of mixed beam stiffness.

We can use the deferential equation of deflection to evaluate the ratio of the mixed beam stiffness with full connection and without loss connection.

$$y'' = \frac{M}{EI} \quad \Rightarrow \quad y = \iint \frac{M}{EI} + c_1 x + c_2$$

By knowing that M and E are the same for both cases, and for a given (x), we have:

$$\frac{y_f}{y_l} = \frac{I_l}{I_f} \quad \Rightarrow \quad I_f = \frac{y_l}{y_f} I_l$$

For $x = 1m$, we have for the discrete connection model:

$$I_f = \frac{0.5}{0.24} I_l \quad \Rightarrow \quad I_f = 0.21 \times I_l$$

- **In discreet connection:**

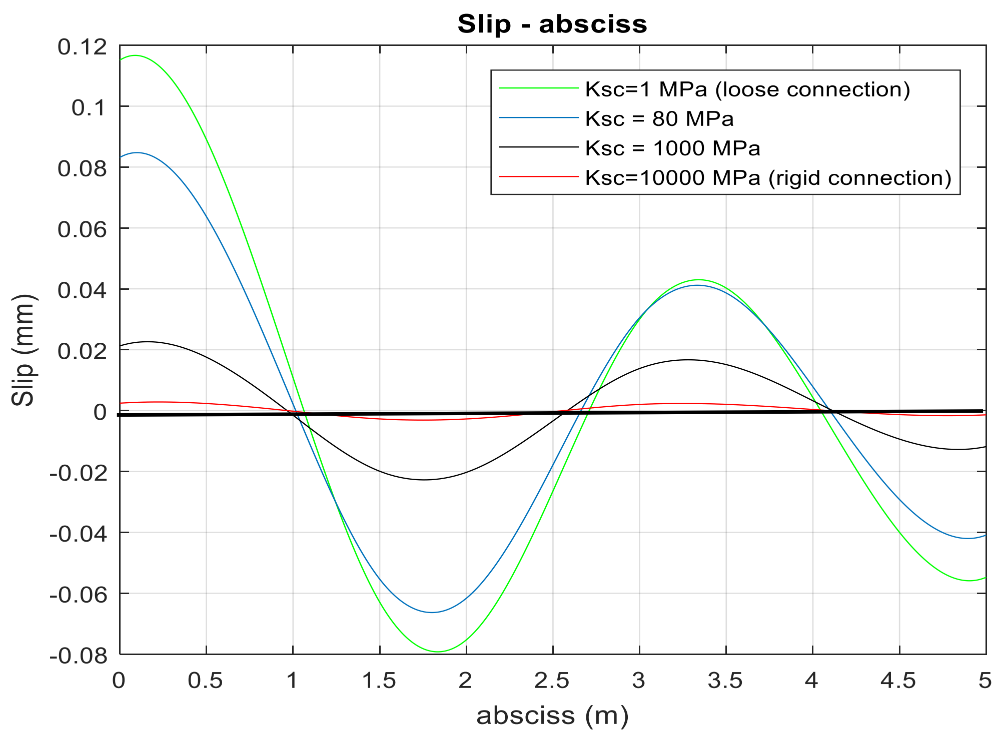


Figure 5.12 Beam slip versus the span for different shear bond stiffness

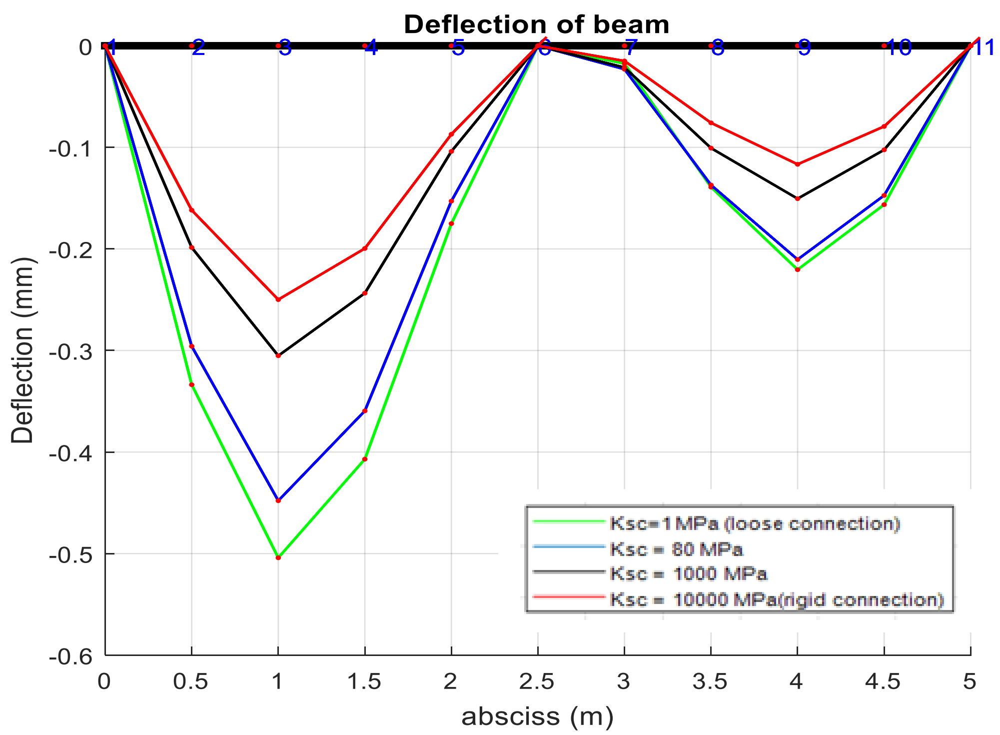


Figure 5.13 Deflection at load point versus the span for different shear bond stiffness

It can also be observed that the discrete and continuous modes give the same results in two cases: the case of full connection and the case of loose connection.

5.3 The clamped with two simply supported composite beam (B2):

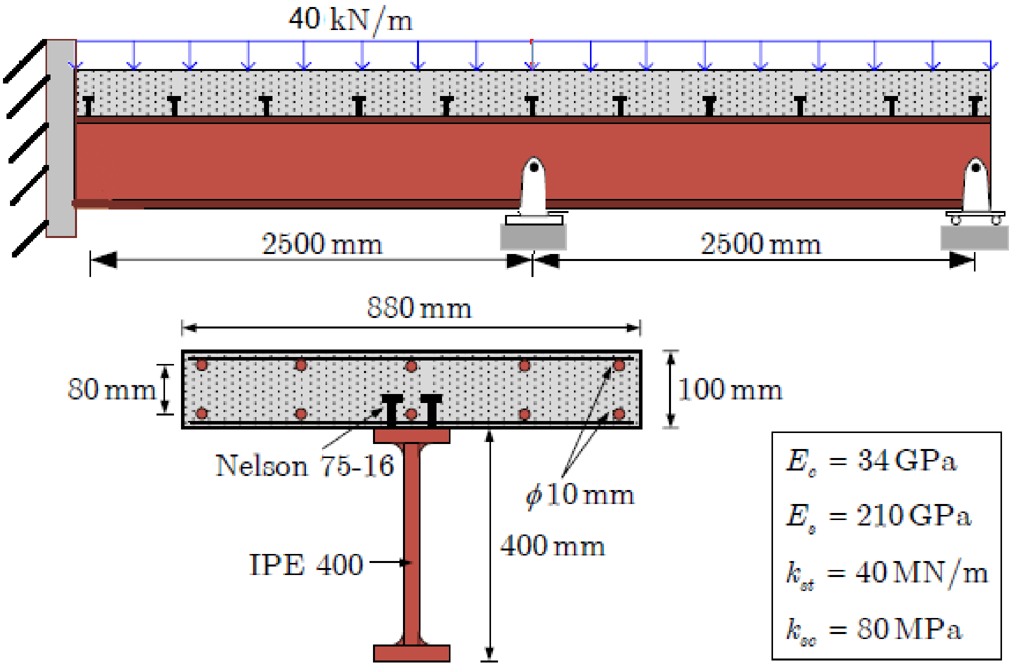


Figure 5.14 Description of the reference beam B2

We analyze in this example a mixed beam with two equal spans resting on two simple supports and one fixed end as presented in figure 5.14. It has a total length of 5 *m*. The beam consists of an IPE 400 connected to a concrete slab whose section is 880 × 100 mm^2, reinforced by two layers of 5 HA10 reinforcement. The span is subjected to a uniformly distributed load of 40 KN/m.

The connection is made by 11 pairs of Nelson studs, h = 75 *mm*, ϕ = 16 *mm*, evenly spaced along the length of the beam. In other words, the longitudinal spacing of the studs, (*a*), is equal to 0.5 *m*. The stiffness of each stud will be taken equal to 20 *MN*/*m*. The stiffness "k_{st}" of the connection used in the simulation with the discrete connection model, is therefore equal to 2×20 =

40 *MN*/*m*. The equivalent linear stiffness of connection "k_{sc}", used in the simulation with the continuous connection model is then:

$$k_{sc} = \frac{k_{st}}{a} = \frac{40}{0.5} = 80\ MPa$$

The kinematic boundary conditions are: $v\ (x = 0) = 0$, $v\ (x = 2.5\ m) = 0$, $v\ (x = 5m) = 0$, u_c (x = 0) = 0, $u_s\ (x = 0\ m) = 0$, $u_s\ (x = 2.5m) = 0$ and theta $(x = 0) = 0$.

Ec= 34 GPa; Es= 210 GPa; Esr= 210 GPa

5.3.1 Comparison of connection types:

The beam is simulated with 10 elements of mixed beam with discrete connection at first, and then with 10 elements of mixed beam with continuous connection. By observing the results provided by two connection models, we find while in the first span the distribution of deflection and slip are very close for both models (see Figure 5.15), the max deflection and slip are slightly deferent in the second span. The slip distribution is given in Figure 5.18 for the two models.

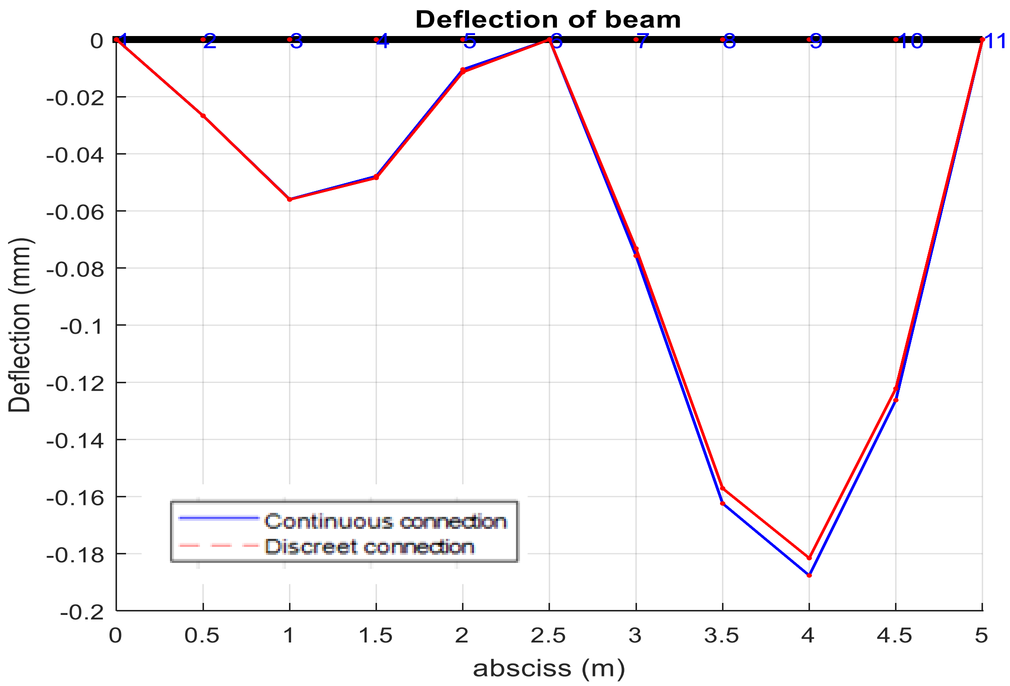


Figure 5.15 Beam B2: Distribution of the deflection along the beam length

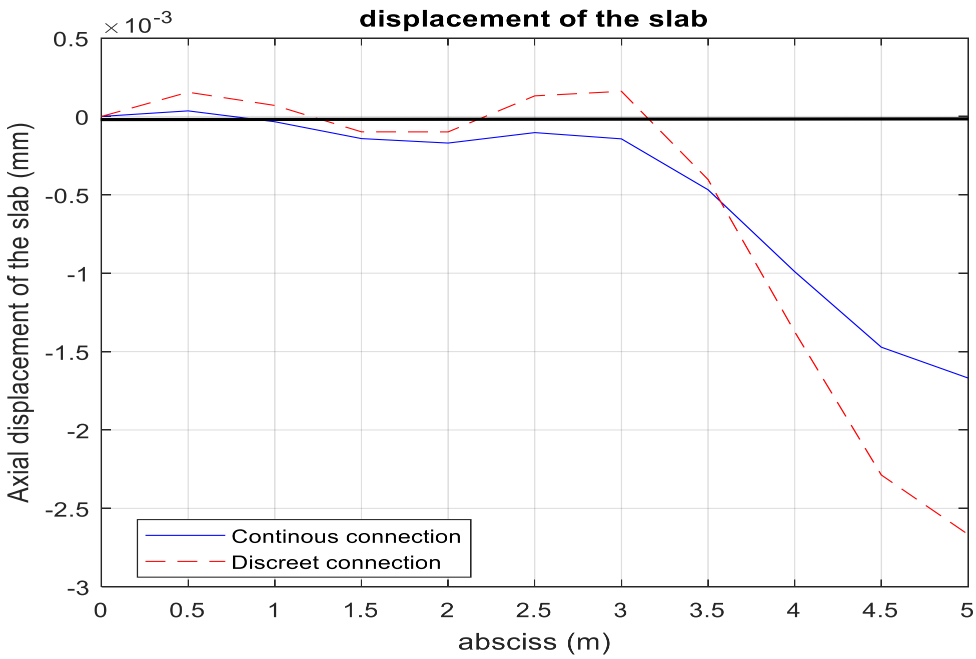


Figure 5.16 Beam B2: Distribution of the axial displacement in the slab along the beam length

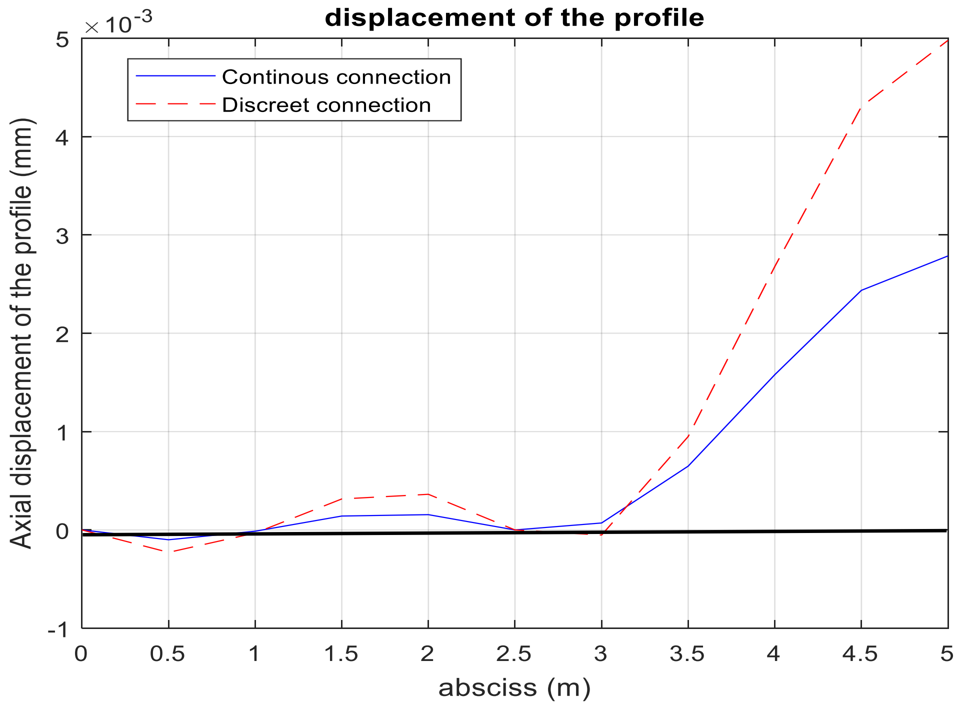


Figure 5.17 Beam B2: Distribution of the axial displacement in the steel section along the beam length

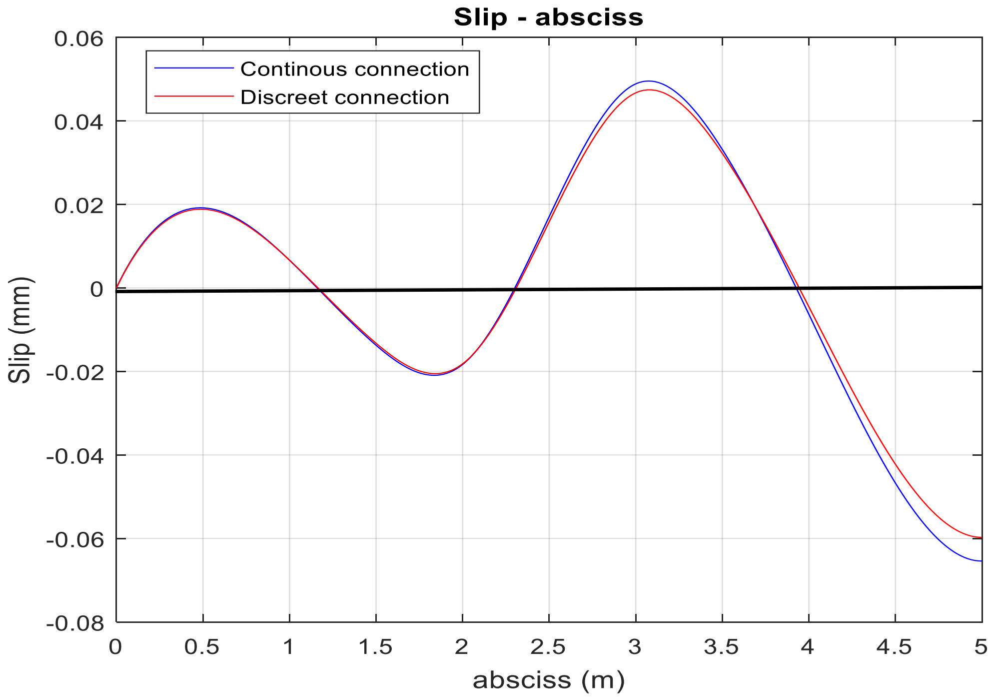


Figure 5.18 Beam B2: Distribution of the slip along the beam length

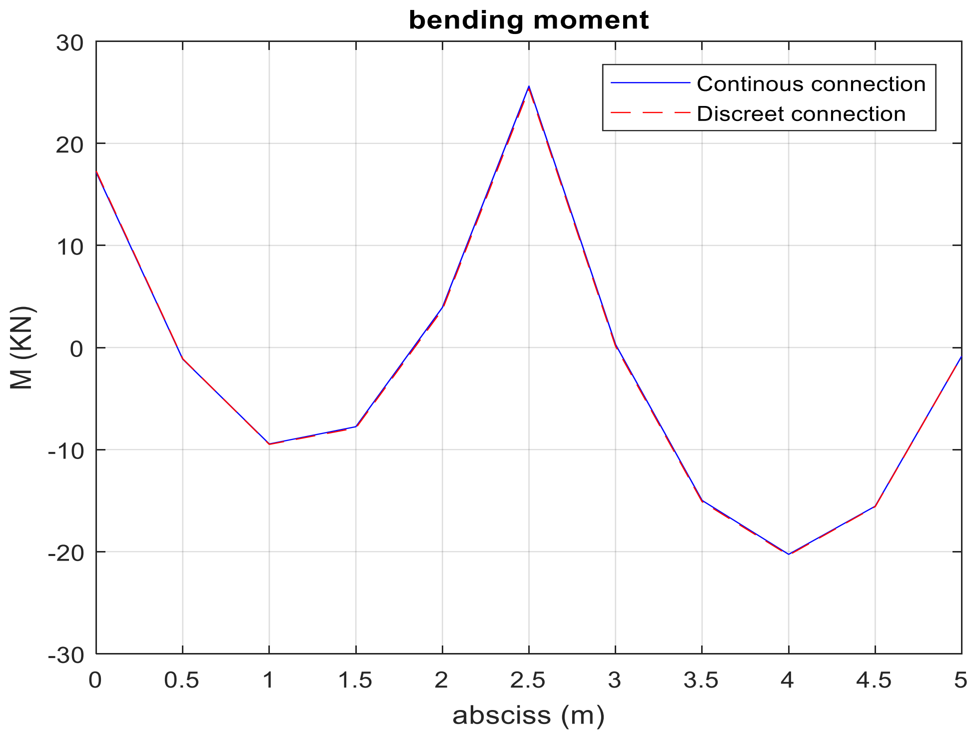


Figure 5.19 Beam B2: Distribution of the total bending moment along the beam length

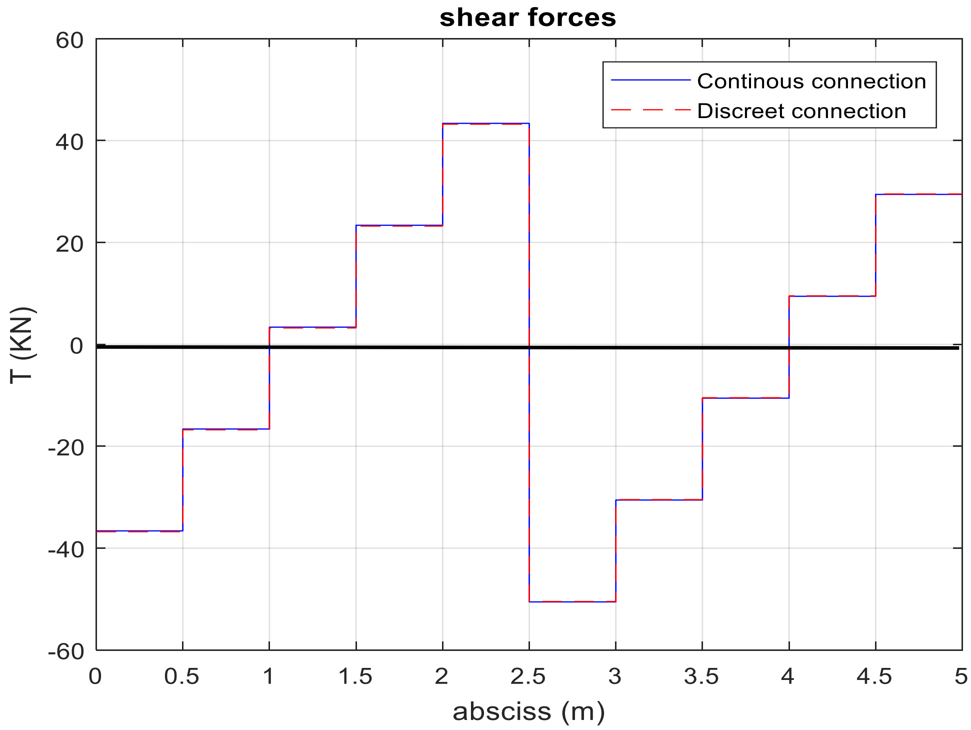


Figure 5.20 Beam B2: Distribution of the total shear force along the beam length

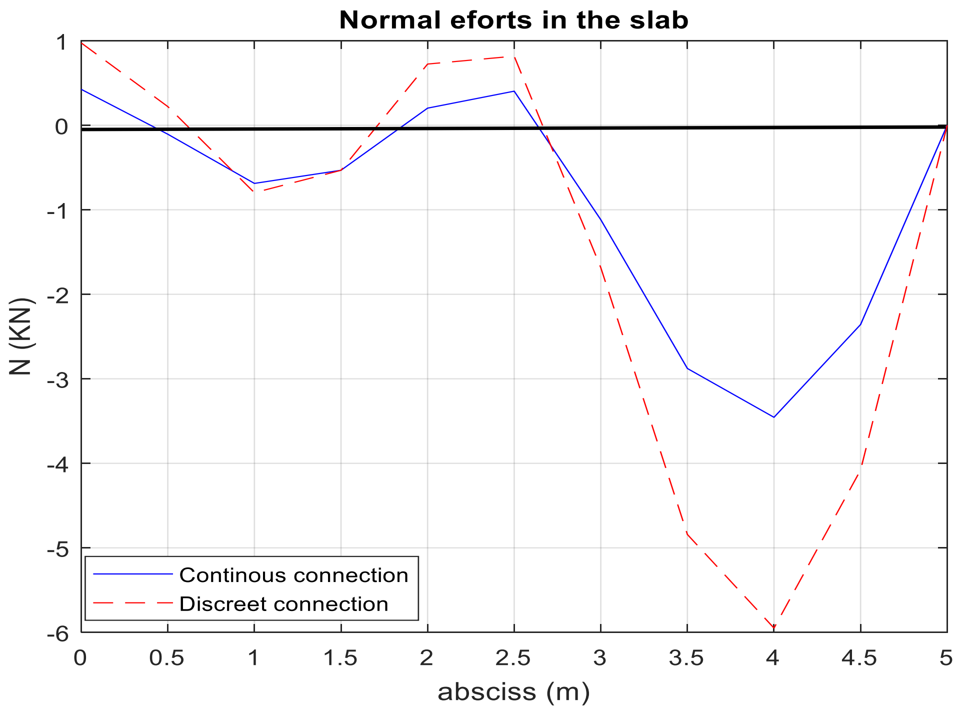


Figure 5.21 Beam B2: Distribution of the normal force in the slab along the beam length

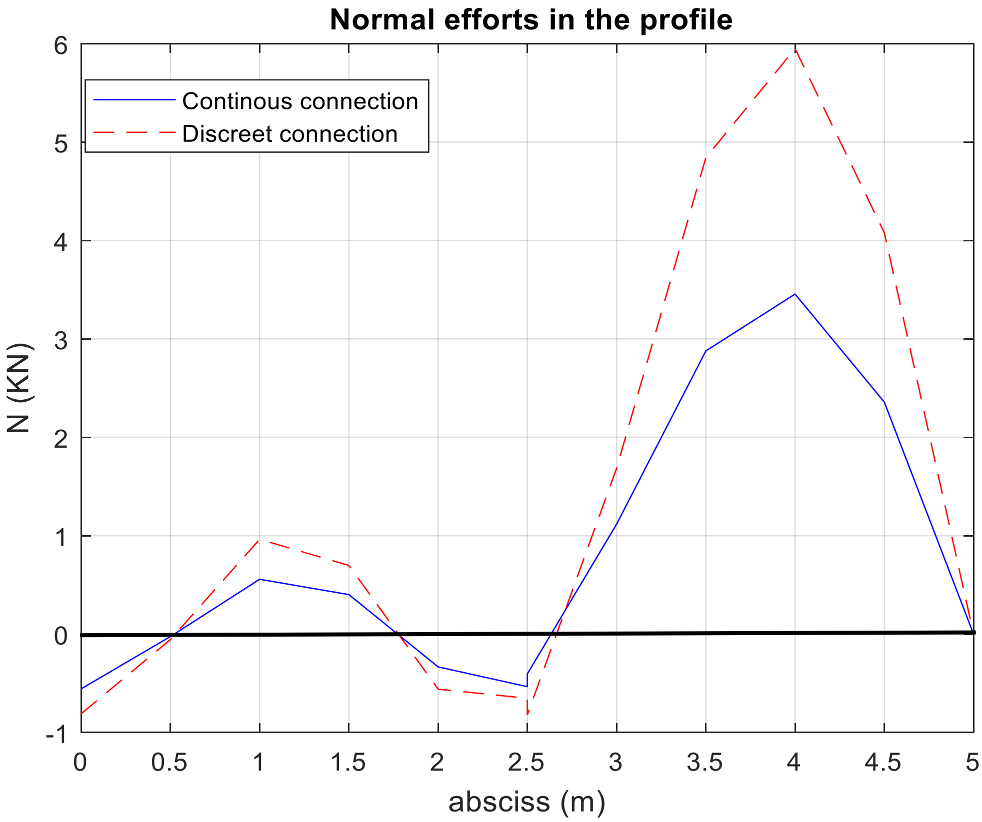


Figure 5.22 Beam B2: Distribution of the normal force in the steel section along the beam length

5.4 Hyper-static beam with two unequal spans (B3):

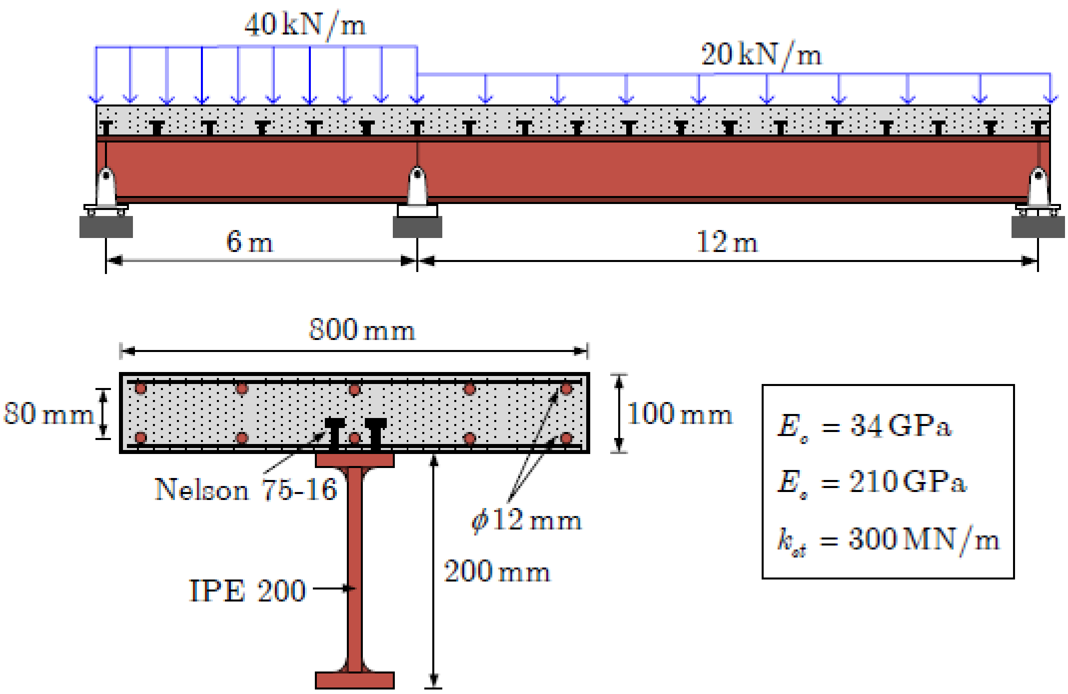


Figure 5.23 Description of the reference beam B3

We analyze in this example a mixed beam with two unequal spans resting on three simple supports. It has a total length of 18 *m* with the first span of 6 *m* and the second span of 12 *m*. The beam consists of an IPE 200 steel section connected to a concrete slab whose section is 800×100 mm^2, reinforced by two layers of 5 HA12 reinforcement. The first span is subjected to a uniform distributed load of 40 *KN/m* while the second is subjected to a uniform distributed load of 20 *KN/m* as shown in Figure 5.20. The connection is made by 19 pairs of Nelson studs, $h = 75$ *mm*, $\phi = 16$ *mm*, regularly spaced along the length of the beam. In other words, the longitudinal spacing of the studs (a), is equal to 1 *m*. The rigidity of each stud will be taken equal to 150 *MN/m*.

The stiffness "k_{st}" of the connection used in the simulation with the discrete connection model, is therefore equal to 2×150 = 300 MN/m. The equivalent linear stiffness of connection "k_{sc}", used in the simulation with the continuous connection model is then:

$$k_{sc} = \frac{k_{st}}{a} = \frac{300}{1} = 300\ MPa$$

The kinematic boundary conditions are: v (x = 0) = 0, v (x = 6m) = 0, v (x = 12m) = 0 and u_s (x = 6m) = 0.

5.4.1 Comparison of connection types:

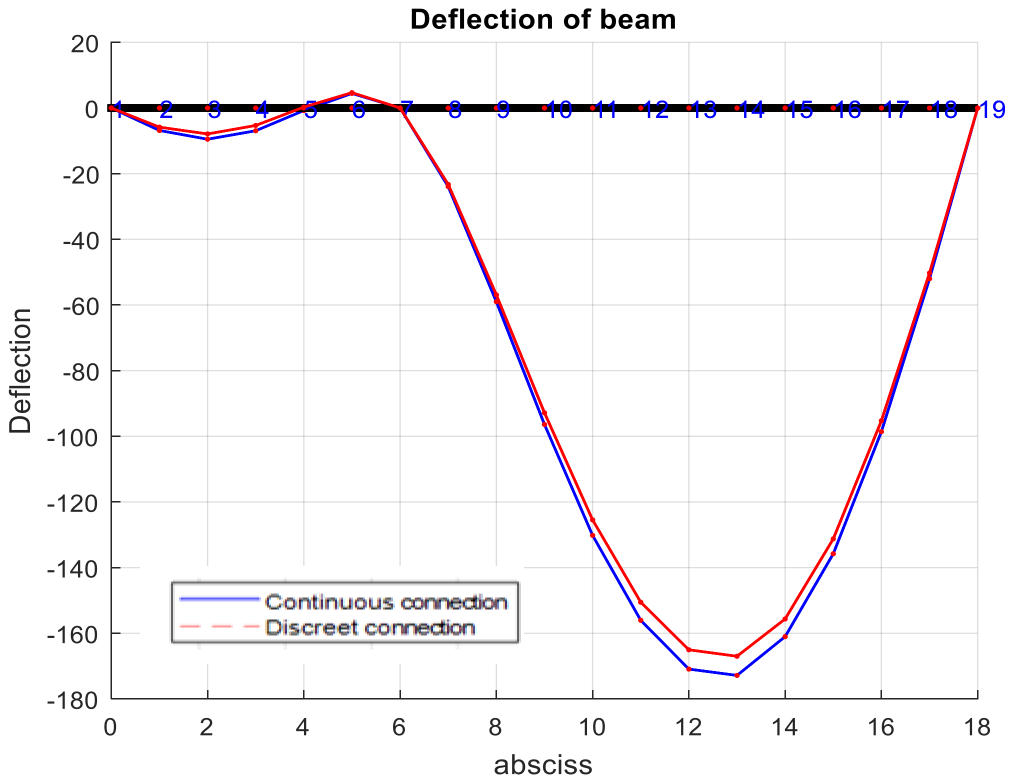


Figure 5.24 Beam B3: Distribution of the deflection along the beam length

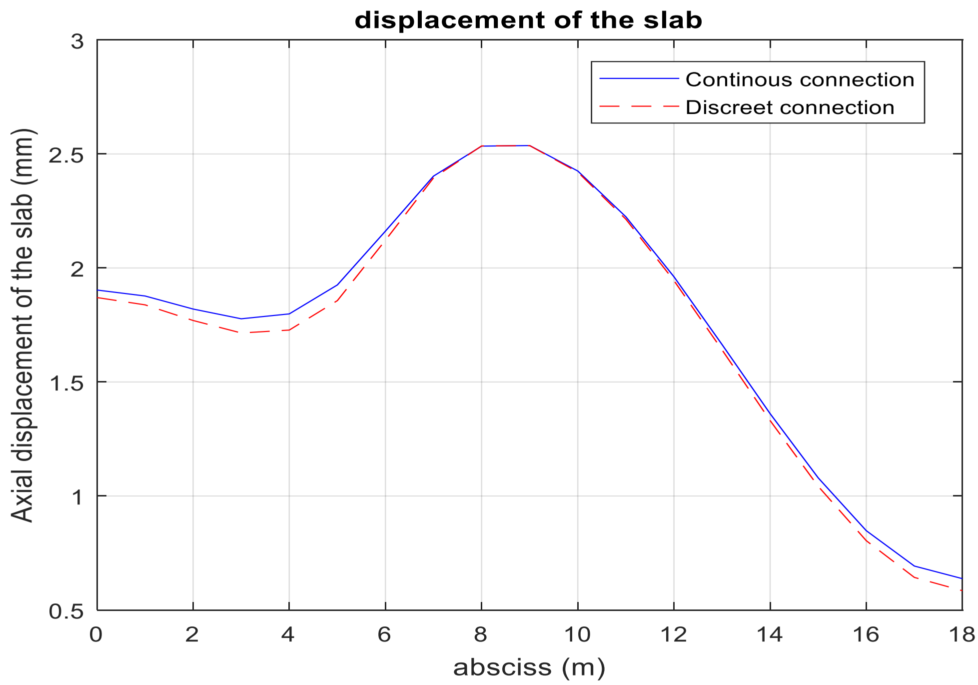


Figure 5.25 Beam B3: Distribution of the axial displacement in the slab along the beam length

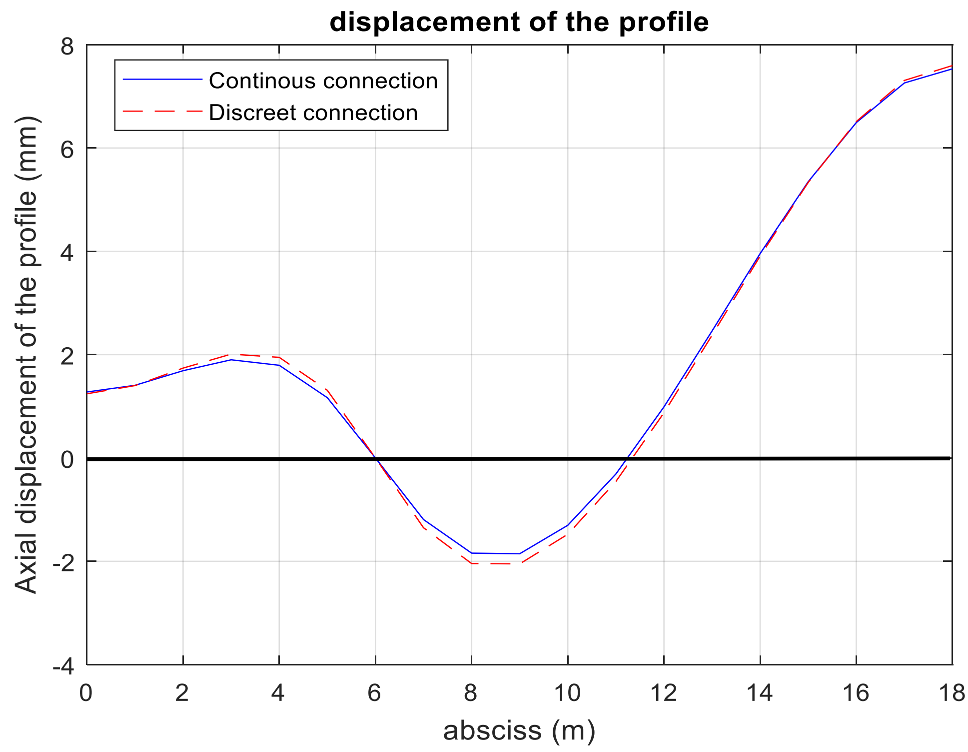


Figure 5.26 Beam B3: Distribution of the axial displacement in the steel section along the beam length

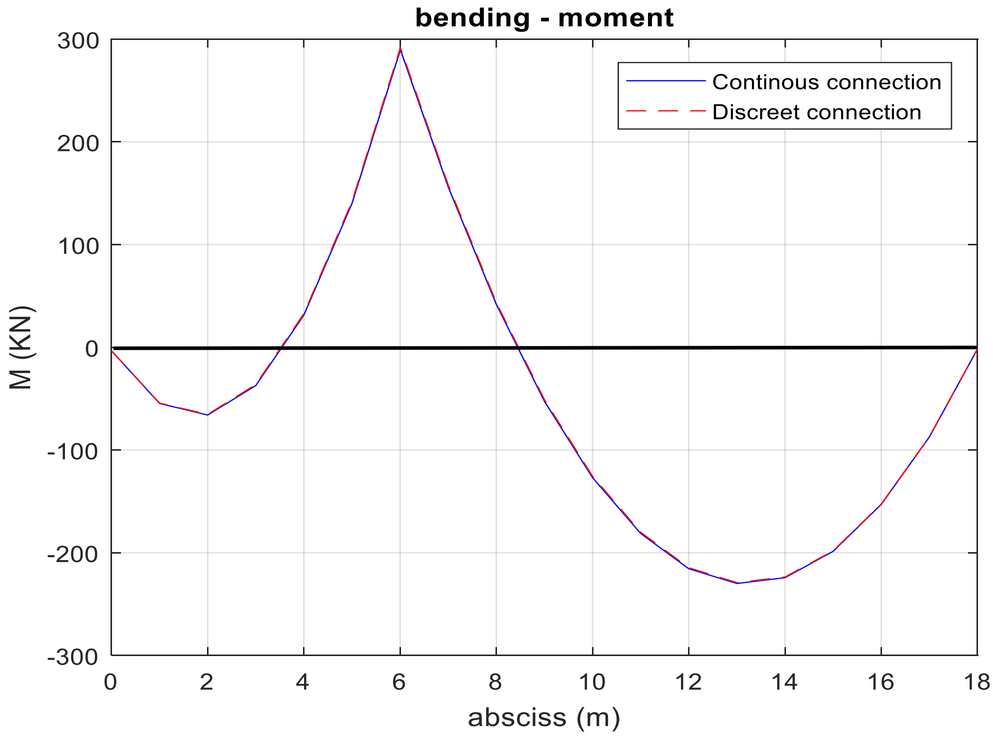


Figure 5.27 Beam B3: Distribution of the total bending moment along the beam length

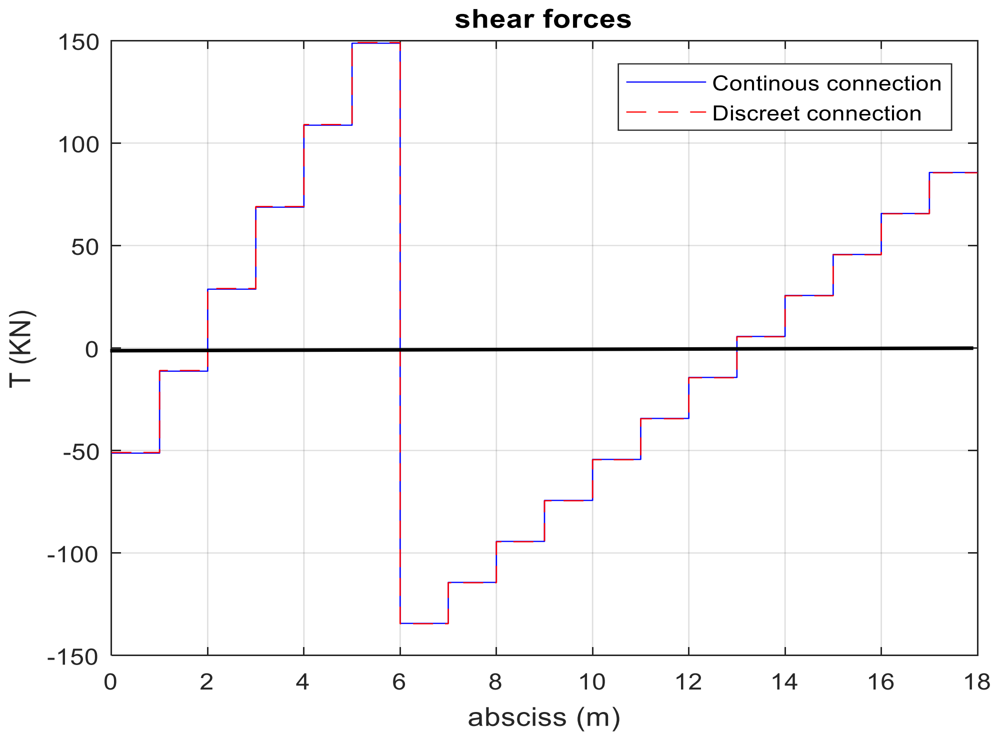


Figure 5.28 Beam B3: Distribution of the total shear force along the beam length

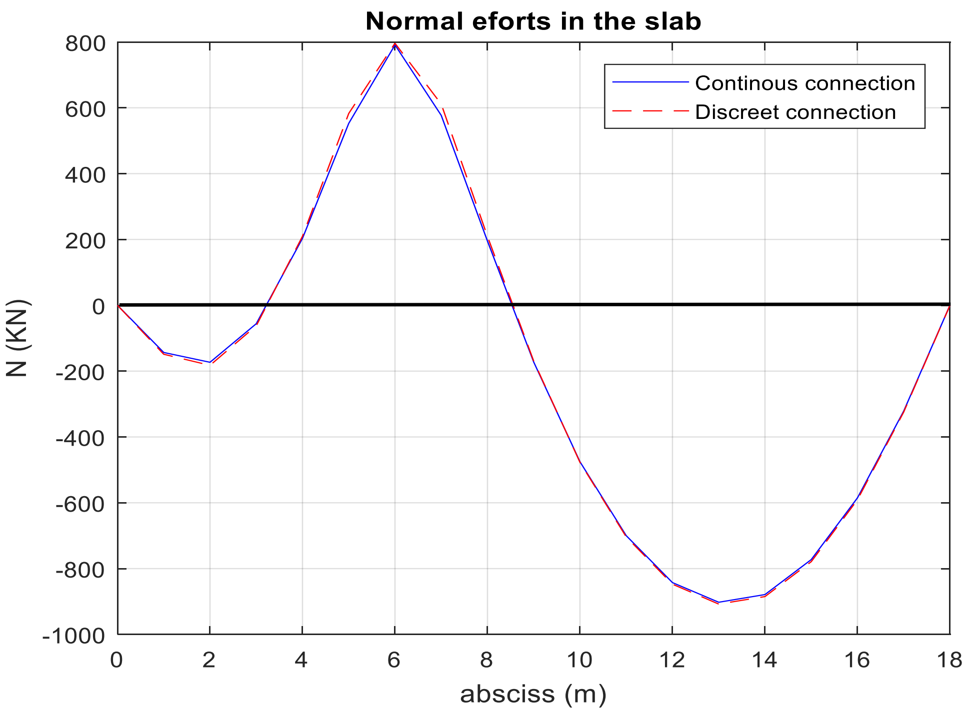


Figure 5.29 Beam B3: Distribution of the normal force in the slab along the beam length

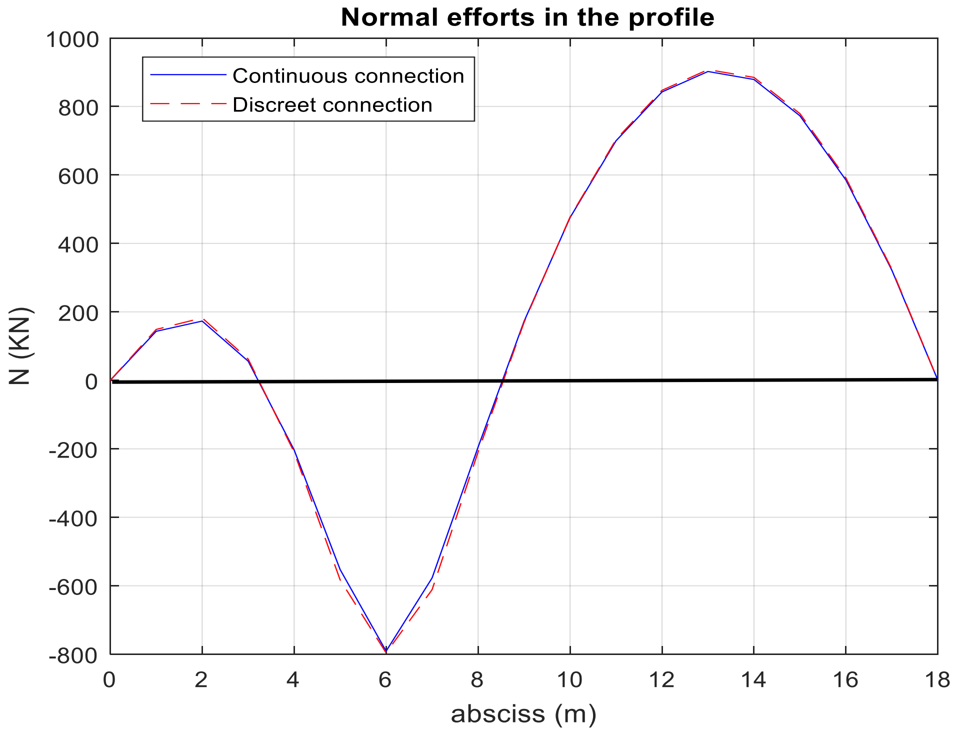


Figure 5.30 Beam B3: Distribution of the normal force in the steel section along the beam length

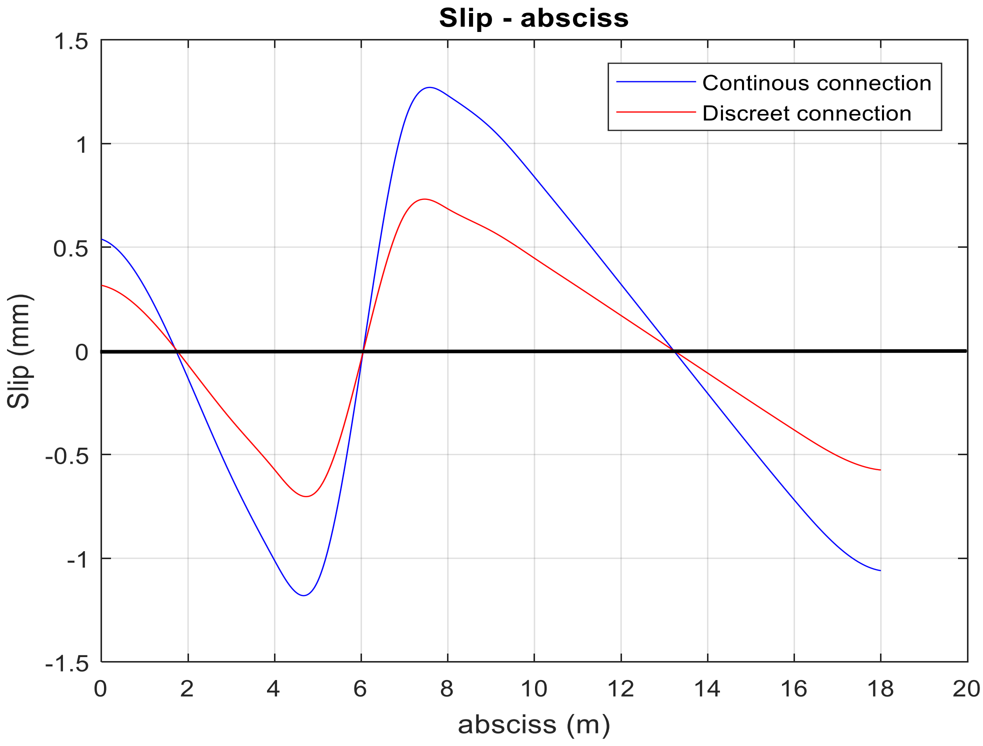


Figure 5.31 Beam B3: Distribution of the slip along the beam.

5.5 Frame structure:

In this example, the analysis of a mixed frame with clamped supports has been carried out. It has a Hight of 3 m and a Length of 5 m. The columns of the frame consists of an IPE 200 steel section while the beam of the frame consists of an IPE 200 steel section connected to a concrete slab whose section is 800×100 mm2, reinforced by two layers of 5 HA12 reinforcement. The span is subjected to a uniform distributed load of 80 KN/m. The connection is made by 10 pairs of Nelson studs, h = 75 mm, ϕ = 16 mm, regularly spaced along the length of the beam. In other words, the longitudinal spacing of the studs (a), is equal to 0.5 m. The rigidity of each stud will be taken equal to 150 MN/m.

The stiffness "kst" of the connection used in the simulation with the discrete connection model, is therefore equal to 2×150 = 300 MN/m. The equivalent linear stiffness of connection "ksc", used in the simulation with the continuous connection model is then 40 MPa.

The column is discretized with 6 elements as shown in figure 5.28.

Ec= 34 GPa; Es= 210 GPa; Esr= 210 GPa; Ecolumn =210 Gpa

5.5.1 Comparison of connection types:

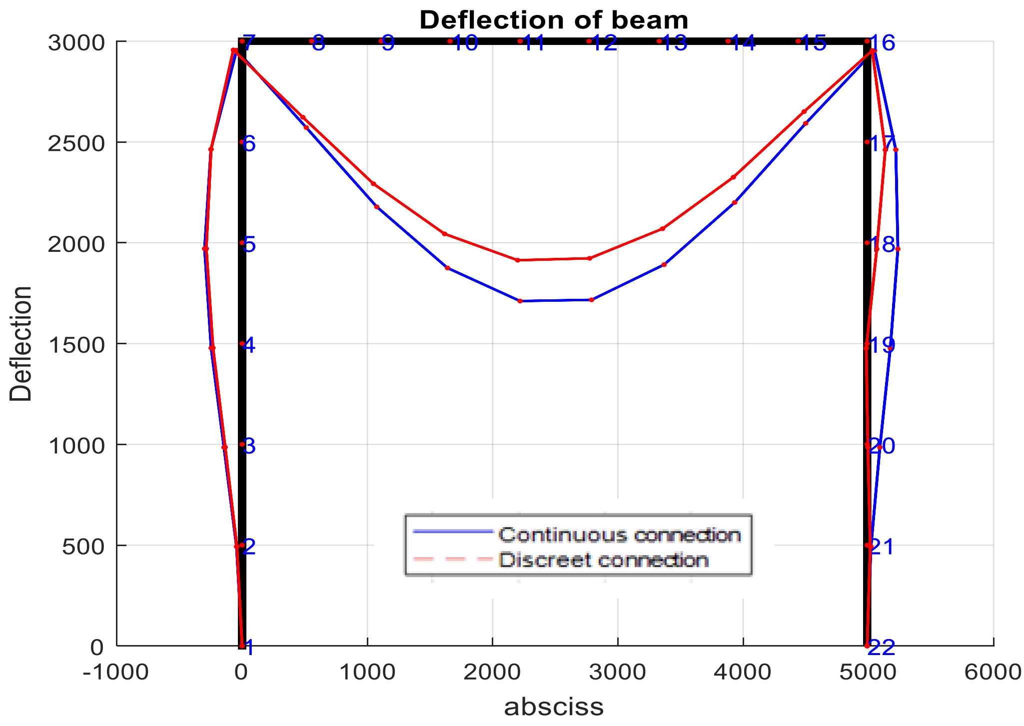


Figure 5.32 Frame: Distribution of the deflection in the frame.

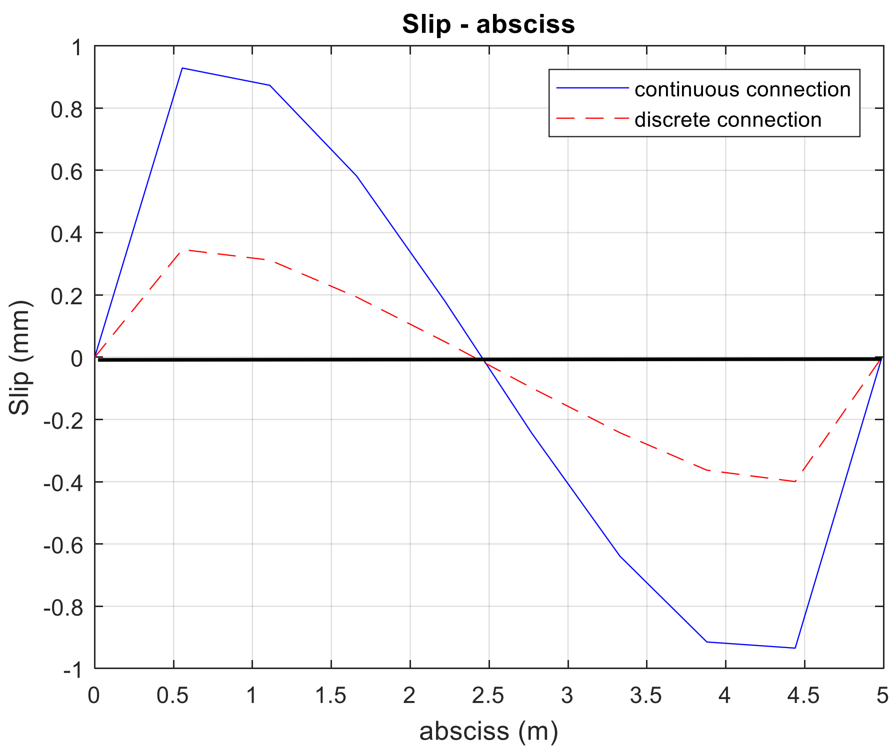


Figure 5.33 Frame: Distribution of the slip along in the frame

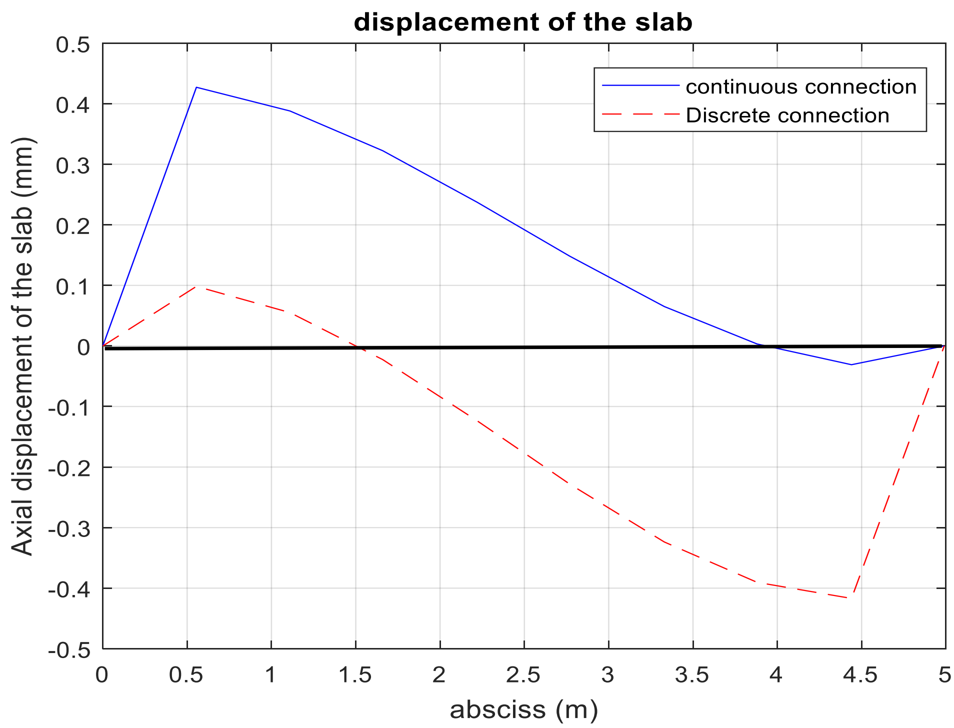


Figure 5.34 Frame: Distribution of the axial displacement in the slab in the frame

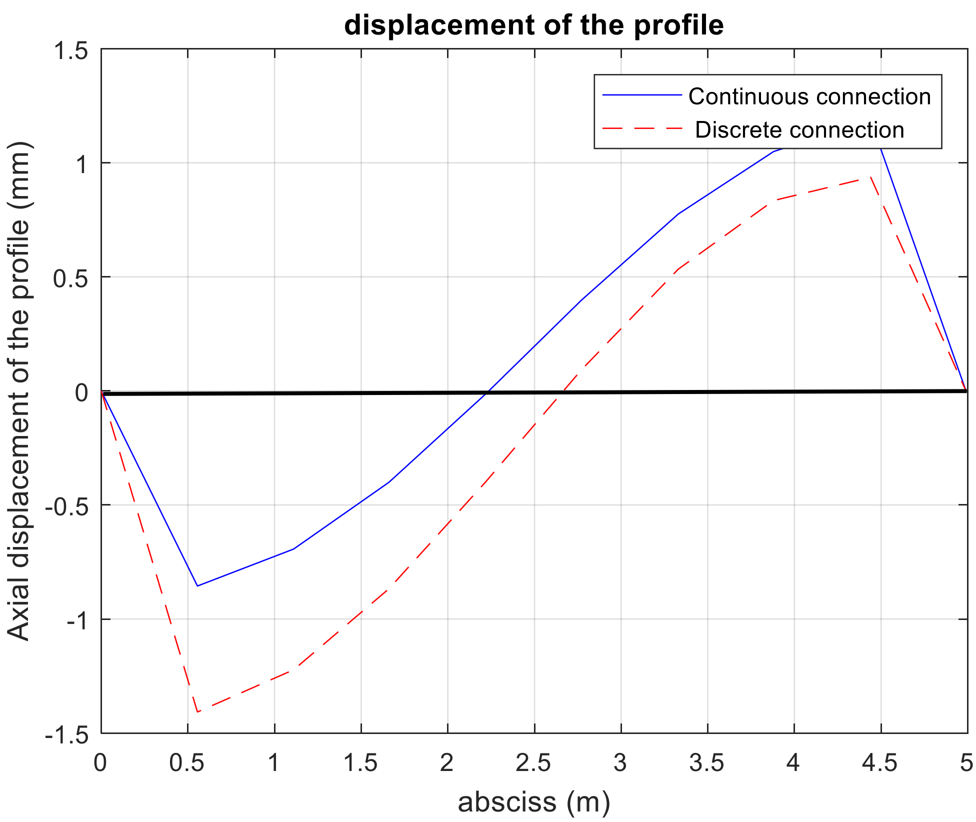


Figure 5.35 Frame: Distribution of the axial displacement in the steel section in the frame

Conclusion:

In this paper, Bernoulli kinematics for thin beam elements are used to model a steel-concrete mixed beam for linear elastic analysis of the global response of mixed beam structures. The partial shear interaction caused by the longitudinal interfacial slip due to the deformability of shear connectors is considered and to models considering continuous and discrete connection are presented.

Several numerical examples of mixed beams and frames are solved by the developed Matlab program. The obtained results are utilised to assess the performance of the presented continuous and discrete models. The numerical results shows that the mixed finite element beam model can efficiently predict the global response of mixed beams.

Finally, the influence of partial interaction on the overall behaviour has been also investigated. A parametric analysis, based on various values of the shear bond stiffness, has been performed.

REFERENCES

1. SAMHAL. E. April (2005), Composite Construction, SSEDTA European Steel Computer Aided Learning, pp.1-3

2. MALEKI. SH, MEHRDAD MAHOUTIAN.M. (2009), Experimental and analytical study on channel shear connectors in Fiber-reinforced concrete, University of Technology, Azadi Ave., Tehran, Iran, pp 1787.

3. SHARIATI. A, SULONG.H, SHARIATI M, January (2012). Various types of shear connectors in composite structures: A review, Article *in* International journal of physical sciences, pp. 186

4. CHIOREAN. C, BURU.S, (2016). Practical nonlinear inelastic analysis method of composite steel-concrete beams with partial composite action, University of Cluj-Napoca, Romania, pp. 74

5. NARDIN.S, EL DEBS.A, (2008). Study of partially encased composite beams with innovative position of stud bolts, Department of Structural Engineering, University of Sao Paulo at Sao Carlos, Sao Carlos, Brazil, pp 342-343

6. TARUBRATA R., June (2011). F.E. ANALYSIS OF COMPOSITE BEAMS WITH PARTIAL INTERACTIONS USING HIGHER ORDER SHEAR DEFORMATION THEORY, Department of civil Engineering Indian Institute Of Technology Roorkee,(INDIA), PP.12

7. BARAN E, TOPKAYA C. (2012), An experimental study on channel type shear connectors, Department of Civil Engineering, Atilim University, Turkey and Middle East Technical University, Ankara, 06531, Turkey, pp 108

8. MAZOZ1 A, BENANANE A, OUINAS D. (2016), experimental study and finite element Modelling of push-out tests on a new shear Connector of I-shape, University of Mostaganem and University of M'sila , Algeria, pp.1-2

9. SHARIATI A., RAMLISULONG N. (2012), SUHATRIL M AND SHARIATI M, Department of Civil Engineering, University of Malaya, 50603 Kuala Lumpur, Malaysia, pp 2-9

10. S.SBHAVIKATTI. (2005) Finite Element Analysis, New Age International (P) Ltd., Publishers, Published by New Age International (P) Ltd., Publishers, Ansari Road, Daryaganj, New Delhi, PP.1-2.

11. Bofang Z. (2018), The Finite Element Method Fundamentals and Applications in Civil, Hydraulic, Mechanical and Aeronautical Engineering, Tsinghua University Press, Beijing, China, PP.04.

12. Daryl L. (2012), A First Course in the Finite Element Method, Fifth Edition, Cengage Learning, Stamford, USA, PP.8-14.

13. TARUBRATA R. June (2011), F.E. ANALYSIS OF COMPOSITE BEAMS WITH PARTIAL INTERACTIONS USING HIGHER ORDER SHEAR DEFORMATION THEORY, Indian Institute Of Technology, (INDIA) PP.22.

14. C.M. Wang, J.N. Reddy. (2000) "SHEAR DEFORMABLE BEAMS AND PLATES". The National University of Singapore, ELSEVIER, PP.11-12.

15. Timoshenko, S.P. (1958), Strength of Materials. Part 1: Elementary Theory and Problems. Van Nostrand Company Inc., New York.

16. O˜nate, E. (2009). Structural Analysis with The Finite Element Method. Vol. 1: Basis and Solids, Springer-CIMNE.

17. Eugenio Oñate, (2013), Structural Analysis with the Finite Element Method Linear Statics Volume 2. School of Civil Engineering Universitat Politècnica de Catalunya (UPC) Barcelona, Spain, PP. 2-4.

18. Hirt M. A. et Bez R. (1994) « Construction Métallique – Vol. 10 : Notions fondamentales et méthodes de dimensionnement ». EPFL, Lausanne.

19. MAZOZ A. (2015), Thèse développement du système de connexion dans les poutres mixtes (acier-béton), Algeria, pp 8-11.

20. Quang-Huy Nguyen. (2009), Thesis Modelling of the nonlinear behavior of composite beams taking into account time effects, Francais, PP 15-26.