MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESEARCH
ECHAHID CHEIKH LARBI TEBESSI UNIVERSITY - TEBESSA
MINING INSTITUTE
DEPARTMENT OF MINES AND GEOTECHNOLOGIE
وزارة التعليم العالي والبحث العلمي
جامعة الشهيد الشيخ العربي التبسي – تبسة-
معهد المناجم
قسم المناجم والجيوتكنولوجيا

Lecture notes

Advanced soil mechanics

محاضرات في ميكانيك التربة المتقدمة

للسنة الاولى ماستر جيوتقنية

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COURSE AIM

The aim of this course is to give knowledge and understanding of advanced soil mechanics to the class of ***Master 1 Geotechnics in the mining institute of Tebessa***. This course includes several topics e.g., stress-strain behaviour of soils, steady state flow and flow nets, settlement analysis, earth presures and slope stability.

TEXT BOOK and REFERENCES

Course work:

1. Das, Braja M. "Advanced Soil Mechanics Third edition." (2018).
2. Budhu, M. (2011). Soil Mechanics & Foundations, 3rd Edition, John Wiley & Sons, Inc.
3. Craig, R.F. (1987), *Soil Mechanics*, 4th edition, T.J. Press (Padstow) Ltd, Great Britain
4. Das, B. M. and Sobhan, K. (2010). Principles of Geotechnical Engineering, 8th Edition, Cengage Learning
5. Coduto, D. P. (2001). Foundation Design: Principles and Practices. Prentice Hall. Pearson Education International.
6. Das, Braja M. (2000), *Fundamentals of Geotechnical Engineering*, Brooks/Cole, Pacific Grove, CA, USA
7. Das, Braja M. (1983), *Advanced Soil Mechanics*, International edition, McGraw-Hill, London, UK

Laboratory:

1. Das, B. M. (2012). Soil Mechanics Laboratory manual" 8th Edition, Oxford University Press.

Filière : Génie minier **Option : Géotechnique** **Niveau : Master 1**

Semestre : 1
UE : UEF 1.1
Matière : Mécanique des sols avancée

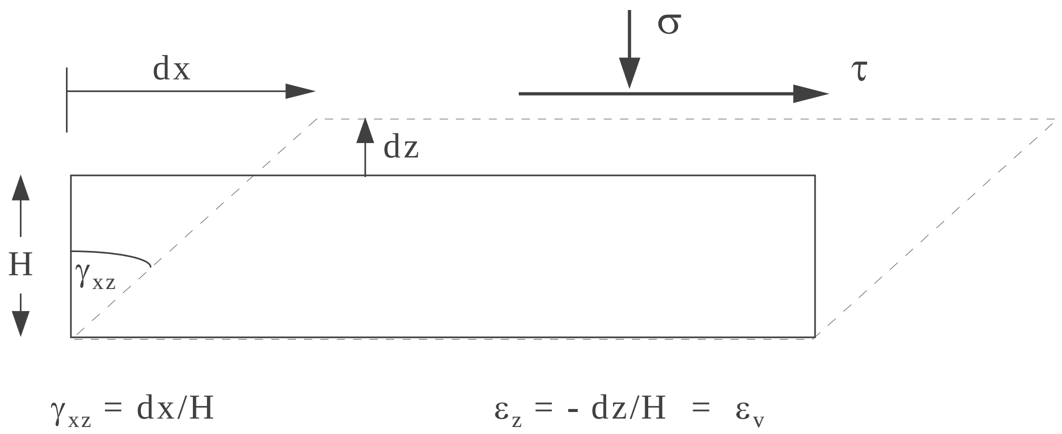
Objectifs de l'enseignement : *La maîtrise de l'état de contraintes dans les sols, leur résistance au cisaillement, le calcul aux états limites et apporter les éléments indispensables à l'étude du comportement des sols saturés et non saturés.*
Connaissances préalables recommandées : *Eléments de mécanique des sols.*
Contenu de la matière :
Chapitre 1. *Introduction et rappel de la théorie des contraintes*
Notions de résistance dans les sols. Contraintes en un point, cercle de Mohr, critère de rupture, contraintes totale et effective.
Chapitre 2. *Evaluation de la résistance au cisaillement*
Facteurs d'influence ; état de contrainte dans les essais de cisaillement direct et triaxial ; état drainé et non drainé.
Chapitre 3. *Résistance au cisaillement des sols pulvérulents*
Courbes contraintes-déformations, dilatance et contractance ; courbe enveloppe de résistance ; sources de résistance ; pression et indice des vides critiques.
Chapitre 4. *Résistance au cisaillement des sols cohérents*
Contrainte effective et pression interstitielle ; courbe enveloppe de résistance et cohésion non drainée ; cheminement des contraintes ; anisotropie ; résistance des argiles raides ; effet du fluage et du taux de déformation ; sol non saturé ; choix des conditions d'essai pour la mesure de la résistance.
Chapitre 5. *Résistance au cisaillement non drainé des matériaux granulaires*
Réponse des pressions interstitielles ; notion de cavitation ; notion d'état critique ; liquéfaction statique.
Chapitre 6. *Analyse de la stabilité des talus*
Principes d'analyse ; développement historique des méthodes d'analyse ; détermination des surfaces de rupture critique ; méthode d'analyse en contrainte effective ; méthodes d'analyse à surface générale ; utilisation d'abaques.
Prise en compte des pressions d'eau dans les analyses ; mesures et évaluation des pressions interstitielles ; utilisation de l'enveloppe critique de contrainte ; stabilité des remblais sur fondation molle ; choix des paramètres de résistance et du type d'analyse.
Chapitre 7. *Principe des calculs aux états limites*
Combinaison d'actions et sollicitations : combinaison d'actions ; Etats limites ultimes ; Etats limites de services.
Mode d'évaluation : *Contrôle Continu 50%, examen 50%*
Références :
[1] Vincent Robitaille, 1997, Mécanique des sols. Théorie et pratique, MODULO.
[2] Cordary, 1995, Mécanique des sols, Tec et Doc.
[3] Exercices de mécanique des sols, 1995, François Schlosser, Presses de l'école nationale des Ponts et Chaussées (ENPC).
[4] Un polycopié sera distribué en classes

CONTENTS

Chapter 1. STRESS-STRAIN BEHAVIOUR OF SOILS

1.1 The behaviour of sands

In practice sands are usually sheared under drained conditions because their relatively high permeability ensures that excess pore pressures are not generated. This behaviour can be investigated in a variety of laboratory apparatus. We will consider the behaviour in simple shear tests. The simple shear test is similar to the shear box test but it has the advantage that the strain and stress states are more uniform enabling us to investigate the stress-strain behaviour. The name simple shear refers to the plane strain mode of deformation shown below:



For this deformation there are only two non-zero strain components, these are the shear strain, $\gamma_{xz} = dx/H$, and the normal strain $\varepsilon_z = dz/H$. The volume strain, $\varepsilon_v = \varepsilon_z$.

For sands the two most important parameters governing their behaviour are the Relative Density, I_d, and the effective stress level, σ′. The Relative density is defined by

$$I_d = \frac{e_{\max} - e}{e_{\max} - e_{\min}}$$

where e_{max} and e_{min} are the maximum and minimum void ratios that can be measured in standard tests in the laboratory, and e is the current void ratio. This expression can be re-written in terms of dry density as

$$\gamma_d = \frac{G_s\ \gamma_w}{1 + e}$$

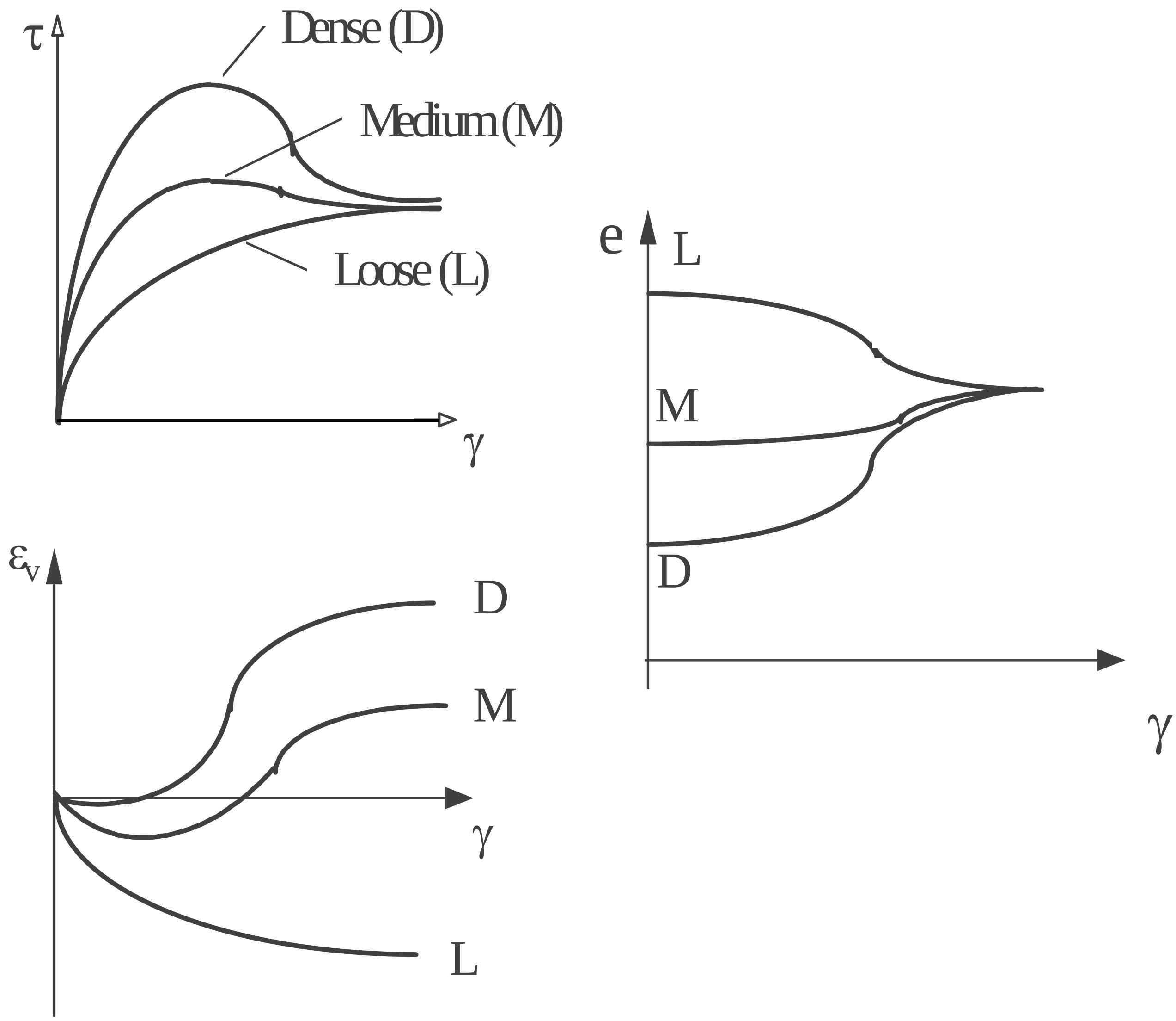
and hence

$$I_d = \frac{\dfrac{1}{\gamma_{dmin}} - \dfrac{1}{\gamma_d}}{\dfrac{1}{\gamma_{dmin}} - \dfrac{1}{\gamma_{dmax}}}$$

Sand is generally referred to as dense if $I_d > 0.6$ and loose if $I_d < 0.3$.

1.1.1 Influence of Relative Density

The influence of relative density on the behaviour can be seen in the plots below for tests all performed at the **same normal stress**.

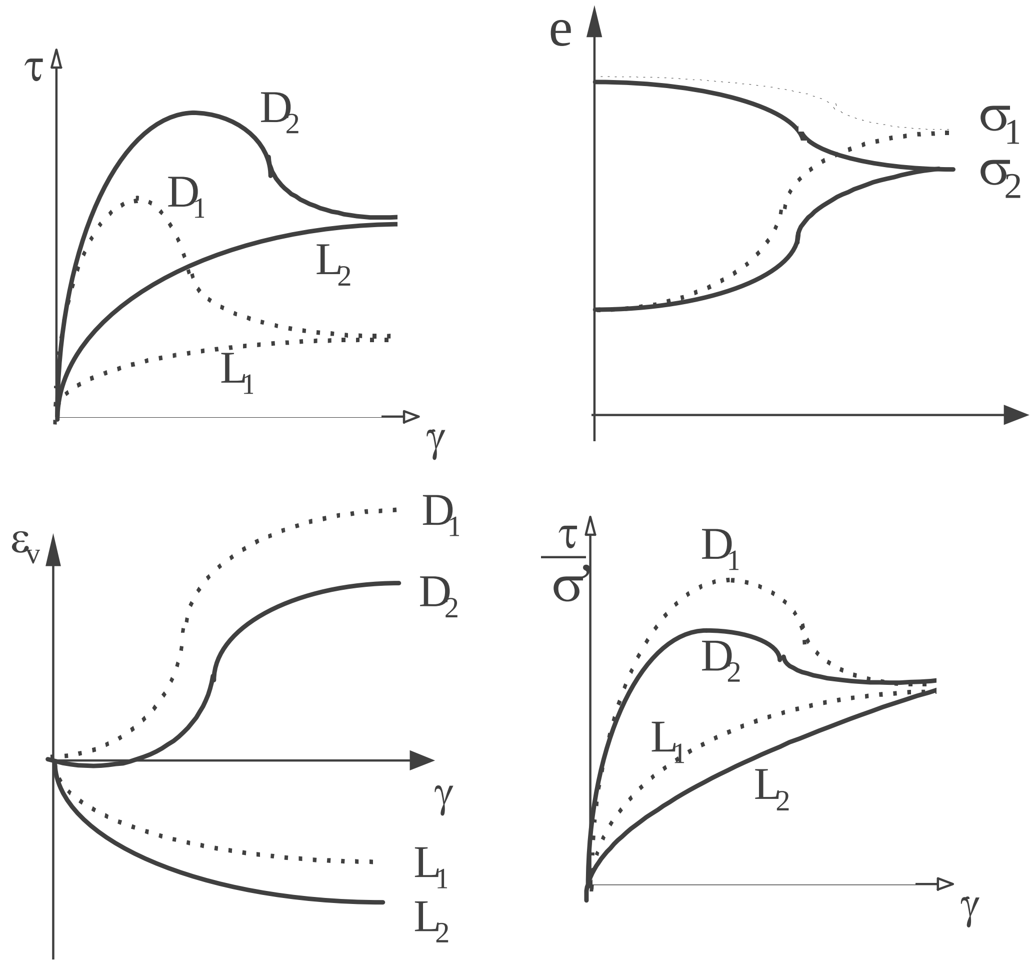


The following observations can be made:

- All samples approach the same ultimate conditions of shear stress and void ratio, irrespective of the initial density
- Initially dense samples attain higher peak angles of friction ($\phi' = \tan^{-1}(\tau/\sigma')$)
- Initially dense soils expand (dilate) when sheared, and initially loose soils compress

1.1.2 Influence of Effective Stress Level

The influence of stress level can be seen in the plots below where the two dense samples have the same initial void ratio, e_1 and similarly the loose samples both have the same initial void ratio e_2.



The following observations can be made:

- The ultimate values of shear stress and void ratio, depend on the stress level, but the ultimate angle of friction ($\phi'_{ult} = \tan^{-1} (\tau/\sigma')_{ult}$) is independent of both density and stress level

- Initially dense samples attain higher peak angles of friction ($\phi' = \tan^{-1} (\tau/\sigma')$), but the peak friction angle reduces as the stress level increases.

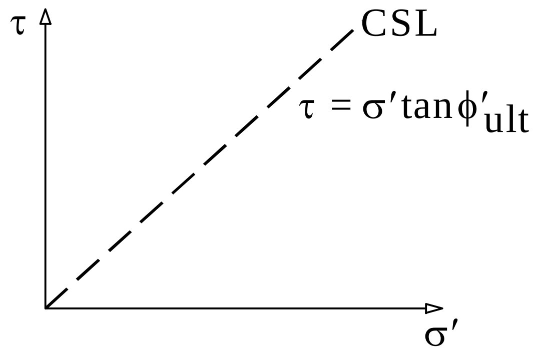
- Initially dense soils expand (dilate) when sheared, and initially loose soils compress. Increasing stress level causes less dilation (greater compression).

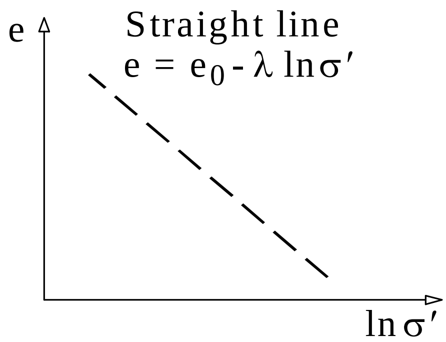
1.1.3 Ultimate or Critical States

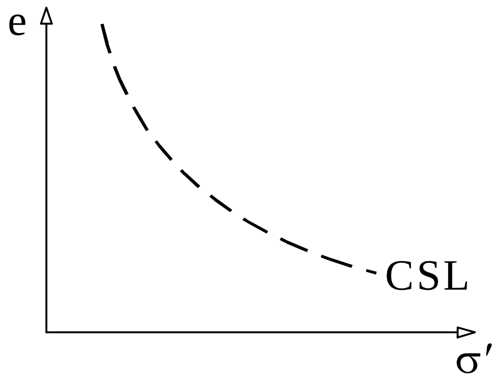
All soil when sheared will eventually attain a unique stress ratio given by $\tau/\sigma' = \tan \phi'_{ult}$, and reach a critical void ratio which is uniquely related to the normal stress. This ultimate state is referred to as a Critical State, defined by

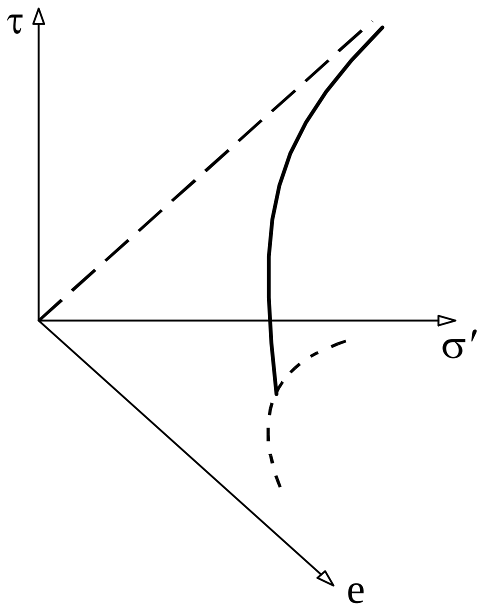
$$\frac{d\tau}{d\gamma} = \frac{d\sigma'}{d\gamma} = \frac{d\,\varepsilon_v}{d\gamma} = 0$$

The locus of these critical states defines a line known as the Critical State Line (CSL). This may be represented by







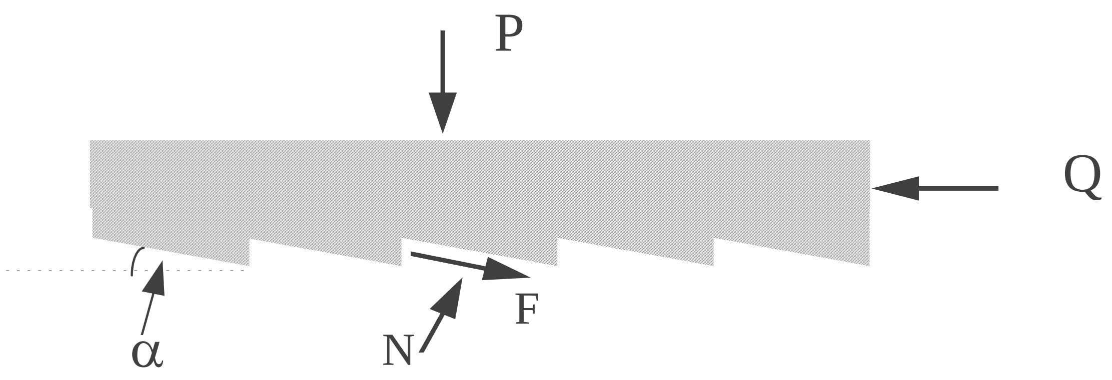


At critical states soil behaves as a purely frictional material

$\phi' = \phi'_{ult} = \phi'_{cs}$ = constant = F (mineralogy, grading, angularity)

1.1.4 Stress-Dilatancy Relation

During a simple shear test on dense sand the top platen is forced up against the applied normal stress. Work must be done against this external force in addition to the work done in overcoming friction between the particles. Thus the frictional resistance of the soil may appear to be greater than ϕ'_{ult}. Another way to demonstrate this is to consider a "saw-tooth" analogy.



$$Q = F\cos\alpha + N\sin\alpha$$

$$P = -F\sin\alpha + N\cos\alpha$$

$$\frac{Q}{P} = \frac{F\cos\alpha + N\sin\alpha}{-F\sin\alpha + N\cos\alpha}$$

$$\frac{Q}{P} = \frac{\left(F/N\right) + \tan\alpha}{1 - \left(F/N\right)\tan\alpha}$$

$$\text{Now} \quad \frac{Q}{P} = \tan\phi' \quad \text{and} \quad \frac{F}{N} = \tan\phi'_{ult}$$

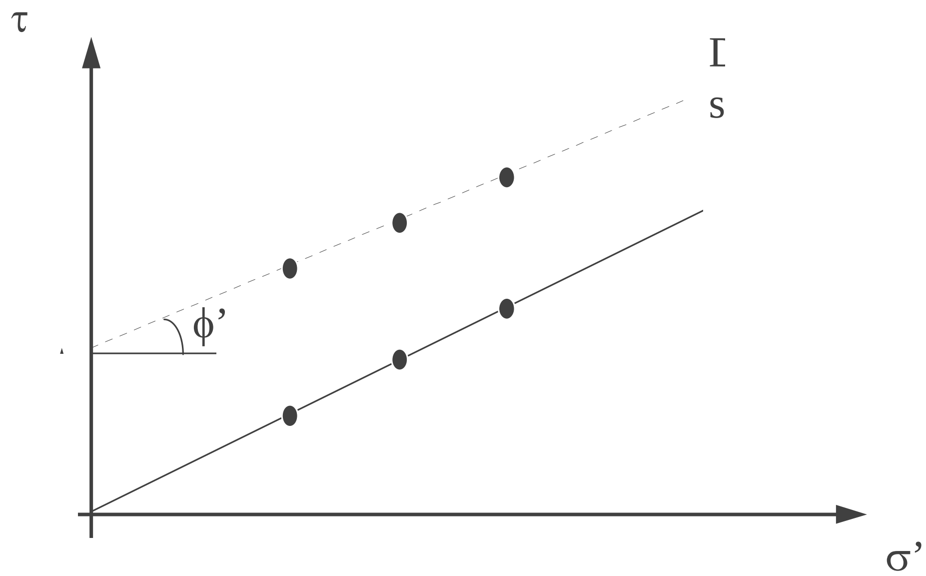
$$\tan\phi' = \frac{\tan\phi'_{ult} + \tan\alpha}{1 - \tan\phi'_{ult}\tan\alpha}$$

$$\boxed{\phi' = \phi'_{ult} + \alpha}$$

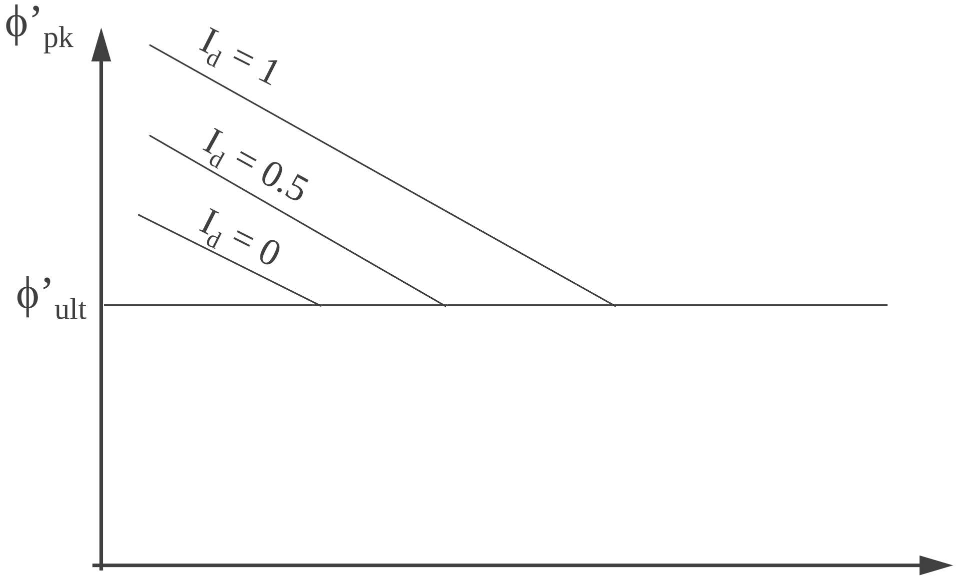
$$\tan\alpha = \frac{dy}{dx} \approx \frac{d\varepsilon_v}{d\gamma}$$

1.1.5 Peak Conditions

The failure conditions are normally expressed by a Mohr-Coulomb criterion using parameters c´, ϕ´. This is the approach that we will be following in estimating the stability of soil constructions.



However, this approach obscures the fact that c´ is only an apparent cohesion. An alternative method of presenting the results is to determine the maximum friction angle ϕ'_{pk} which in shear box type tests is simply given by $\tan^{-1}(\tau/\sigma')$. The relation between ϕ'_{pk} and effective stress is then as shown below.

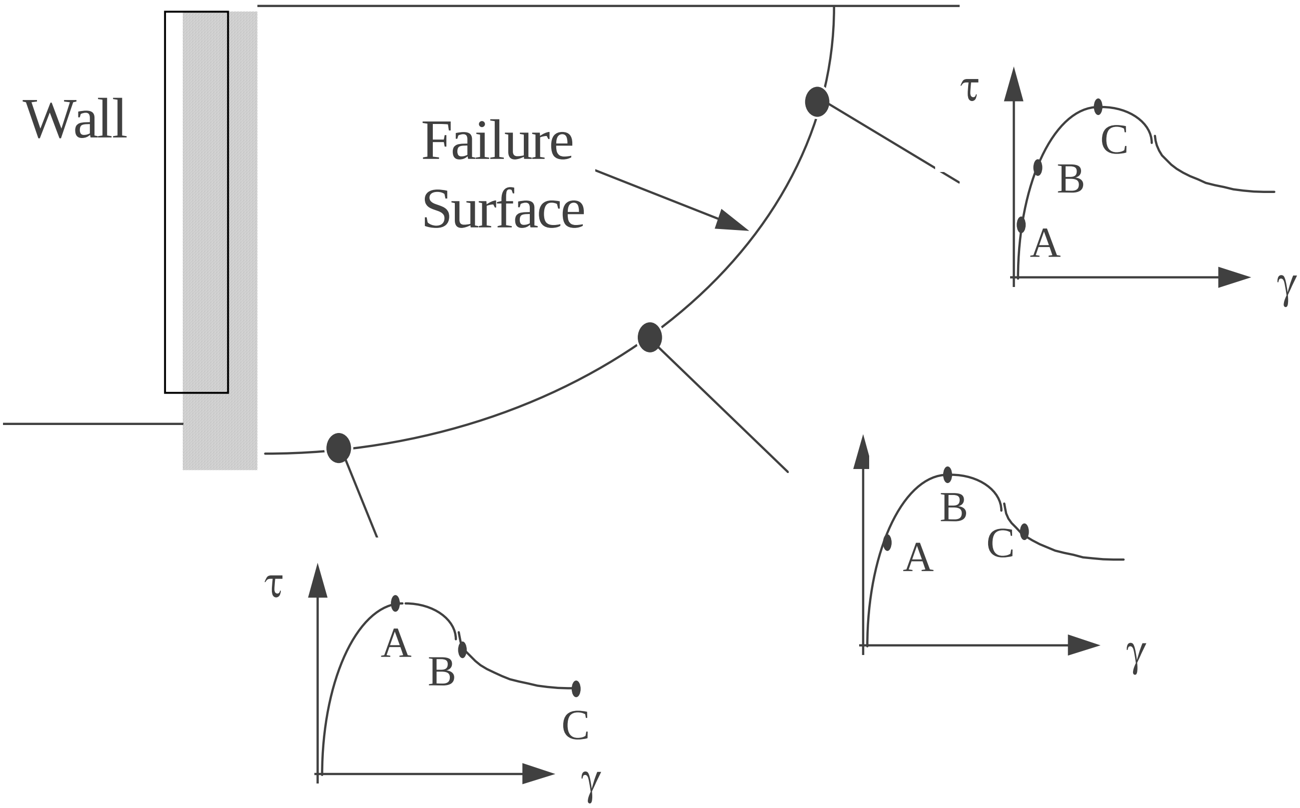


The position of the lines in this plot is a function of the mineralogy and angularity of the soil.

Note that even loose sand can have $\phi'_{pk} > \phi'_{ult}$ if the stress is low enough. This means that loose sands may expand when sheared.

1.1.6 Implications for stability analysis

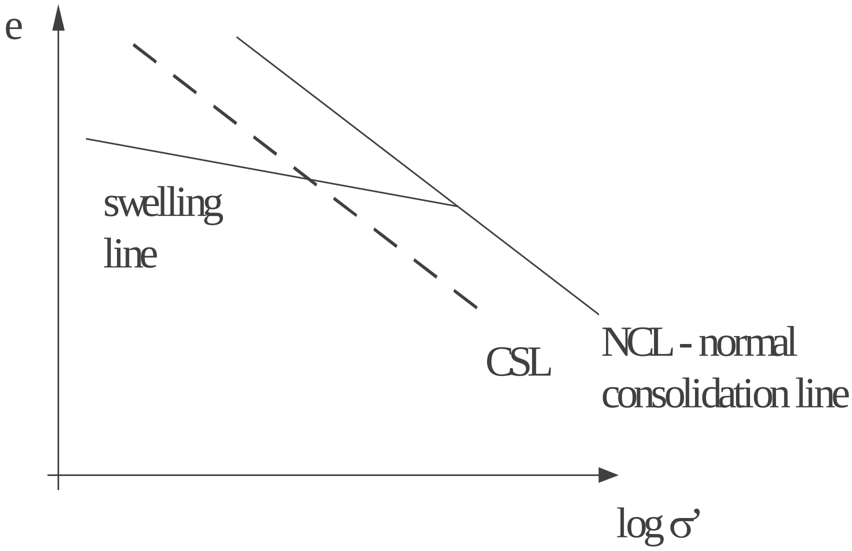
If you choose to use ϕ'_{pk} (or c′, ϕ′ with $c' \neq 0$) in stability calculations then you are saying that everywhere on the critical failure surface the soil will be dilating at failure. In most practical cases this is unlikely to be realistic. For instance consider the case of a retaining wall.



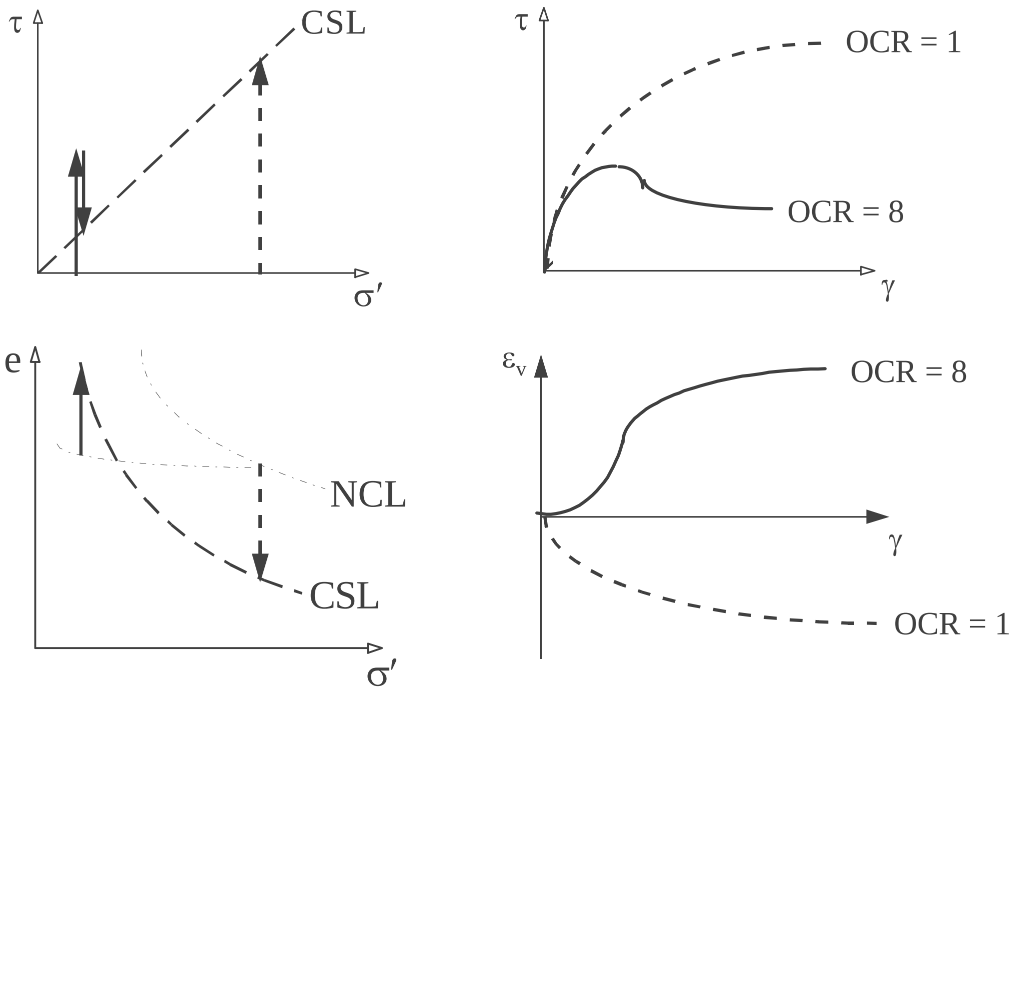
It is conservative to use c´ = 0 and ϕ´ = ϕ'_{ult} for stability analyses.

1.2 Behaviour of clays

The behaviour of clays is essentially identical to that of sands. The data however is usually presented in terms of the soils stress history (OCR) rather than relative density.
To predict the behaviour of soil we need to combine the CSL with our previous knowledge concerning the consolidation behaviour. Experience has shown that the CSL is parallel to the normal consolidation line and lies below it in a void ratio, effective stress plot.

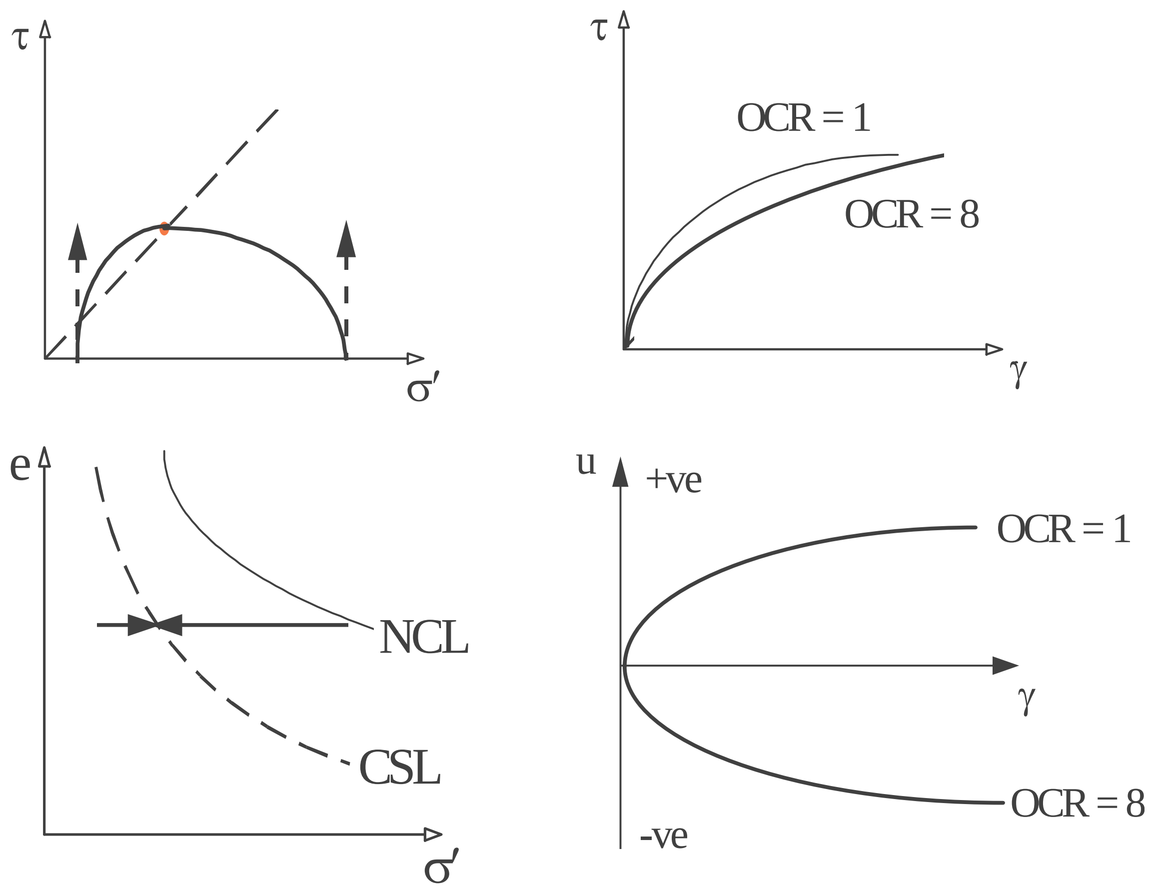


We find that normally consolidated clays behave similarly to loose sands and heavily over-consolidated clays behave similarly to dense sands. As the OCR increases there is a gradual trend between these extremes. The response in drained simple shear tests with σ´ constant is as follows



1.*2.1 Undrained response*

In an undrained test volume change is prevented and therefore the void ratio must remain constant. Because the soil always heads towards a critical state when sheared it is possible to show the path that will be followed in an e, σ′ plot. This is shown below for normally consolidated (OCR=1) and heavily over-consolidated (OCR>8) samples having the same initial void ratio. Once the final states in this plot are known, so too are the final states in the τ, σ′ plot. Also if the final total stresses are known then the excess pore pressures can be determined.



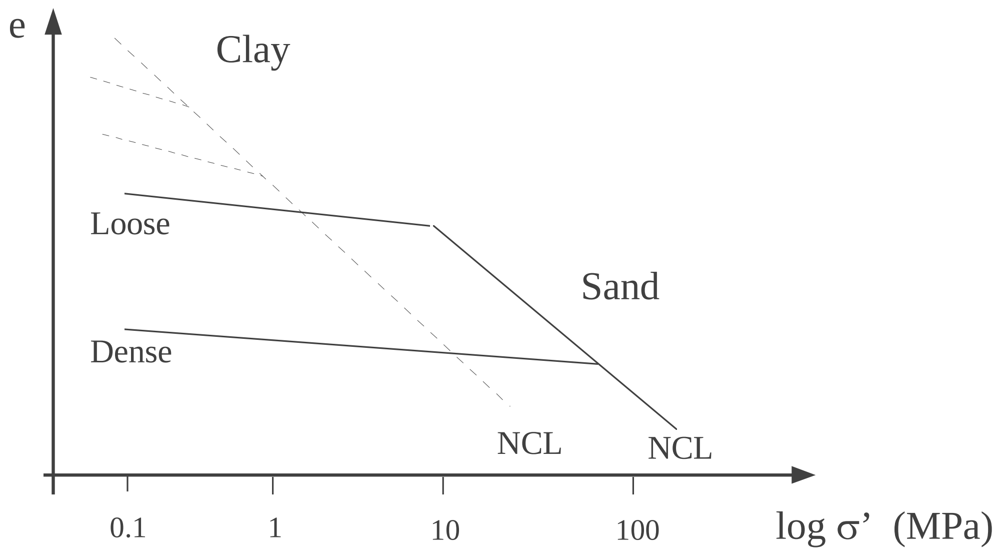
- Knowledge of the Critical State Line enables an explanation for the existence of apparent cohesion (undrained strength) in frictional materials

- It is also clear that if the moisture content changes then so will the undrained strength, because failure will occur at a different point on the CSL

2.3 Differences between sand and clay

When considering the behaviour of sands and clays we generally use different parameters. For sands stress level and relative density are considered to be the important parameters, whereas for clays the parameters are stress level and stress history (OCR).

However, the broad patterns of behaviour observed for sands and clays are very similar. To understand why different "engineering" parameters are used it is useful to consider the positions of the consolidation and CSL lines in the void ratio, effective stress plot.



TUTORIAL SHEET

1. A saturated sample of clay 50 mm diameter by 100 mm long was extracted from the ground. The sample was installed in a triaxial apparatus without allowing drainage and a cell pressure of 350 kPa applied. A back pressure of 200 kPa was set and the drainage taps opened. After leaving it for some time the sample reached equilibrium with no net flow of water into or out of the sample. The drainage taps were then closed and the sample was sheared undrained to failure. The following data were recorded:

ΔF (N)	0	49	74	112	150	181
Δh (mm)	0	1	2	5	10	20
Δu (kPa)	0	17	27	47	75	100

where ΔF, Δh, Δu are respectively the changes in deviator load, axial displacement, and pore pressure.

(a) Calculate the deviator stress and axial strain

(b) Plot deviator stress versus axial strain and pore pressure versus axial strain (Note that this is the conventional way of presenting triaxial test results)

(c) Draw the total and effective stress Mohr circles at failure

(d) Determine the undrained strength s_u and friction angle ϕ' assuming that $c' = 0$.

2. A specimen of clay has been compressed to a state where $\tau = 0$, $\sigma' = 150$ kPa, $u = 0$, and the void ratio, $e = 1.2$. Determine the ultimate undrained strength, s_u and the excess pore pressure at ultimate conditions (a) if the total stress remains constant, (b) if the total stress changes are such that $\Delta\tau = \Delta\sigma$.

The Critical State Line for this clay is given by $\tau = \sigma' \tan\phi'$, $e = \Gamma - \lambda \ln \sigma'$, and $\phi' = 21^o$, $\Gamma = 2.0$, $\lambda = 0.20$.

3. Two identical soil samples have been one-dimensionally consolidated in a simple shear apparatus under an effective stress $\sigma' = 300$ kPa, with a void ratio $e = 0.50$. The two samples were then subjected to standard tests to failure (keeping the total stress constant), one drained and the other undrained, and the following information was recorded.

Drained			Undrained		
τ (kPa)	γ_{xy} (%)	ε_v (%)	τ (kPa)	γ_{xy} (%)	Δu (kPa)
0	0	0	0	0	0
120	1	0.25	86	1	29
250	2.5	-0.5	150	2.5	20
225	5	-1.25	205	5	0
210	10	-1.5	225	10	-35
202	20	-1.6	240	20	-56

Estimate the critical state parameters ϕ', Γ, λ.

What can you deduce about the initial state of the soil, and suggest giving your reasons what type of soil the samples were composed of.

4. Critical thonking

A lightly over-consolidated sample is tested undrained in a simple shear test, and at failure the excess pore pressure is zero.

Sketch the shear stress, τ, shear strain, γ, response and the volume strain, ε_v, shear strain response you would expect an identical sample to follow in a drained test. Explain your answer.

Assume that the total normal stress remains constant in both drained and undrained tests.

Chapter 2. SOIL STRENGTH

Soils are essentially frictional materials. They are comprised of individual particles that can slide and roll relative to one another. In the discipline of soil mechanics it is generally assumed that the particles are not cemented.

One consequence of the frictional nature is that the strength depends on the effective stresses in the soil. As the effective stresses increase with depth, so in general will the strength.

The strength will also depend on whether the soil deformation occurs under fully drained conditions, constant volume (undrained) conditions, or with some intermediate state of drainage. In each case different excess pore pressures will occur resulting in different effective stresses, and hence different strengths. In assessing the stability of soil constructions analyses are usually performed to check the short term (undrained) and long term (fully drained) conditions.

2.1 Mohr-Coulomb failure criterion

The limiting shear stress that may be applied to any plane in the soil mass is found to be given by an equation of the form

$$\tau = c + \sigma_n \tan \phi$$

where c = cohesion (apparent)
ϕ = friction angle

This is known as the Mohr-Coulomb failure criterion

The parameters c and ϕ are not generally soil constants. The Mohr-Coulomb criterion is an empirical criterion, and the failure locus is only locally linear. Extrapolation outside the range of normal stresses for which it has been determined is likely to be unreliable. The parameters depend on:

- the initial state of the soil
  Overconsolidation ratio (OCR) for clays
  Relative density (I_d) for sands

- the type of test
  Drained - slow fully drained, no excess pore water pressures
  Undrained - no drainage, excess pore water pressures develop

- the use of total or effective stresses

  In terms of effective stress the failure criterion is written

$$\tau = c' + \sigma'_n \tan \phi'$$

c' and ϕ' are referred to as the effective (drained) strength parameters.
Soil behaviour is controlled by effective stresses, and the effective strength parameters are the fundamental strength parameters. But they are not necessarily soil constants. They are

fundamental in the sense that if soil is at failure the state will always be described by an effective stress failure criterion. The parameters can be determined from any test provided that the pore pressures are known.
In terms of total stress the failure criterion is written

$$\tau = c_u + \sigma_n \tan \phi_u = s_u$$

c_u, ϕ_u are referred to as the undrained (total) strength parameters. These parameters can only be determined from undrained tests.

The undrained strength parameters are not soil constants, they depend strongly on the moisture content of the soil. The total stress criterion has limited applicability as it is only valid if soil deformation occurs without drainage.

The undrained strengths measured in the laboratory are only relevant in practice to clayey (low permeability) soils that initially deform without drainage, and that have the same moisture content in-situ.

2.2 Strength Tests

The engineering strength of soil materials is often determined from tests in either the shear box apparatus or the triaxial apparatus.

2.2.1 The Shear Box Test

The soil is sheared along a predetermined plane by placing it in a box and then moving the top half of the box relative to the bottom half. The box may be square or circular in plan and of any size, however, the most common shear boxes are square, 60 mm x 60 mm, and test specimens are typically 20 mm thick. Larger boxes of 300 mm x 300 mm are used to test specimens with larger particle sizes. The shear box is constructed in two separate halves (which may be held together by locating screws so that the box can be filled with the soil to be tested).

A load normal to the plane of shearing may be applied to a soil specimen through the lid of the box. Provision is made for porous plates to be placed above and below the soil specimen. These enable drainage to occur which is necessary if a specimen is to be consolidated under a normal load, and if a specimen is to be tested in a fully drained state. The soil specimen may be submerged, by filling the containing vessel with water, to prevent the specimens from drying out. Undrained tests may be carried out, but in this case solid spacer blocks rather than the porous disks must be used.
Notation

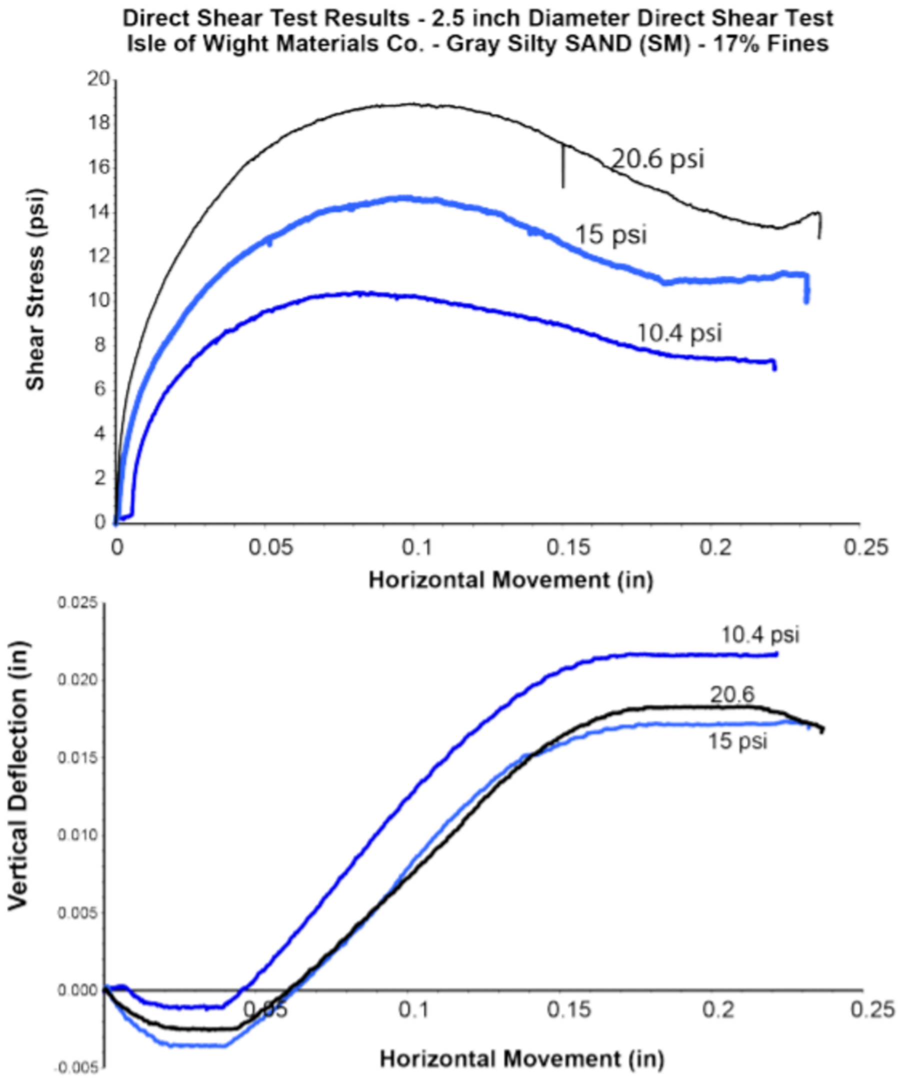
N	=	*Normal Force*		
F	=	*Tangential (Shear) Force*		
σ_n	=	*N/A*	=	*Normal Stress*
τ	=	*F/A*	=	*Shear Stress*
A	=	*Cross-sectional area of shear plane*		
dx	=	*Horizontal displacement*		
dy	=	*Vertical displacement*		

Usually only relatively slow *drained* tests are performed in shear box apparatus. For clays the rate of shearing must be chosen to prevent excess pore pressures building up. For freely draining sands and gravels tests can be performed quickly. Tests on sands and gravels are usually performed dry as it is found that water does not significantly affect the (drained) strength.

Provided there are no excess pore pressures the pore pressure in the soil will be approximately zero and the total and effective stresses will be identical. That is, $\sigma_n = \sigma'_n$

The failure stresses thus define an effective stress failure envelope from which the effective (drained) strength parameters c´, ϕ´ can be determined.

Typical test results



At this stage we are primarily interested in the stresses at failure. It is observed that for a set of initially similar soil samples there is a linear failure criterion that may be expressed as

$$\tau = c' + \sigma'_n \tan \phi'$$

From this the effective (drained) strength parameters c′ and ϕ′ can be determined.

A peak and an ultimate failure locus can be obtained from the results each with different c´ and ϕ´ values. All soils are essentially frictional materials and continued shearing results in them approaching a purely frictional state where $c' \approx 0$. Normally consolidated clays (OCR =1) and loose sands do not usually show peak strengths and have $c' = 0$, whereas, overconsolidated clays and dense sands have $c' > 0$. Note that dense sands (OC clays) do not possess any true cohesion (bonds), and the apparent cohesion results from the tendency of soil to expand when sheared.

As a soil test the shear box is far from ideal. Disadvantages of the test include:

- Non-uniform deformations and stresses. The stresses determined may not be those acting on the shear plane, and no stress-strain curve can be obtained.
- There are no facilities for measuring pore pressures in the shear box and so it is not possible to determine effective stresses from undrained tests.
- The shear box apparatus cannot give reliable undrained strengths because it is impossible to prevent localised drainage away from the shear plane.

However, it has many apparent advantages:
- It is easy to test sands and gravels
- Large deformations can be achieved by reversing the shear box. This involves pushing half of the box backwards and forwards several times, and is useful in finding the residual strength of a soil.
- Large samples may be tested in large shear boxes. Small samples may give misleading results due to imperfections (fractures and fissures) or the lack of them.
- Samples may be sheared along predetermined planes. This is useful when the shear strengths along fissures or other selected planes are required.

In practice the shear box is used to get quick and crude estimates of the failure parameters. It is sometimes used to obtain undrained strengths but this use should be discouraged.

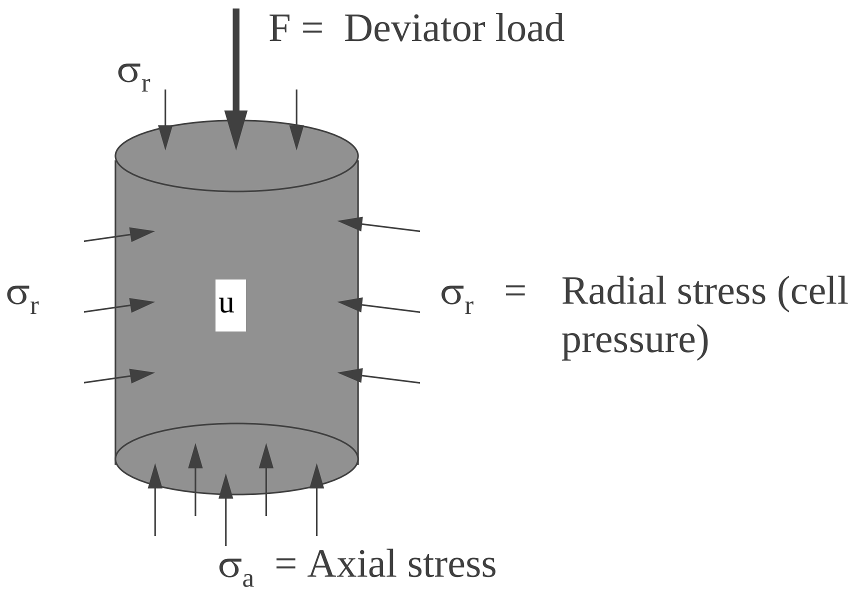
2.2.2 The Triaxial Test

The triaxial test is carried out in a cell and is so named because three principal stresses are applied to the soil sample. Two of the principal stresses are applied to the sample by a water pressure inside the confining cell and are equal. The third principal stress is applied by a loading ram through the top of the cell and therefore may be different to the other two principal stresses. A diagram of a typical triaxial cell is shown below.

A cylindrical soil specimen as shown is placed inside a latex rubber sheath which is sealed to a top cap and bottom pedestal by rubber O-rings. For drained tests, or undrained tests with pore pressure measurement, porous disks are placed at the bottom, and sometimes at the top of the specimen. For tests where consolidation of the specimen is to be carried out, filter paper drains may be provided around the outside of the specimen in order to speed up the consolidation process.

Pore pressure generated inside the specimen during testing may be measured by means of pressure transducers. These transducers must operate with a very small volume change, since fluid flowing out of the specimen would cause the pore water pressure that was being measured to drop.

2.2.2.1 Stresses



From vertical equilibrium we have $\sigma_a = \sigma_r + \frac{F}{A}$

The term F/A is known as the deviator stress, and is usually given the symbol q.

Hence we can write $q = \sigma_a - \sigma_r = \sigma_1 - \sigma_3$ (The axial and radial stresses are principal stresses)

If q = 0 increasing cell pressure will result in:

- volumetric compression if the soil is free to drain. The effective stresses will increase and so will the strength
- increasing pore water pressure if soil volume is constant (that is, undrained). As the effective stresses cannot change it follows that $\Delta u = \Delta\sigma_r$

Increasing q is required to cause failure

2.2.2.2 Strains

From the measurements of change in height, dh, and change in volume dV we can determine

Axial strain $\varepsilon_a = -dh/h_0$
Volume strain $\varepsilon_v = -dV/V_0$

where h_0 is the initial height, and V_0 the initial volume. The conventional small strain assumption is generally used.

It is assumed that the sample deforms as a right circular cylinder. The cross-sectional area, A, can then be determined from

$$A = A_o \left(\frac{1 + \frac{dV}{V_0}}{1 + \frac{dh}{h_0}} \right) = A_o \left(\frac{1 - \varepsilon_v}{1 - \varepsilon_a} \right)$$

It is important to make allowance for the changing area when calculating the deviator stress,

$$q = \sigma_1 - \sigma_3 = F/A$$

2.2.2.3 Test procedure

There are many test variations. Those used most in practice are

UU (unconsolidated undrained) test.

Cell pressure applied without allowing drainage. Then keeping cell pressure constant increase deviator load to failure without drainage.

CIU (isotropically consolidated undrained) test.

Drainage allowed during cell pressure application. Then without allowing further drainage increase q keeping σ_r constant as for UU test.

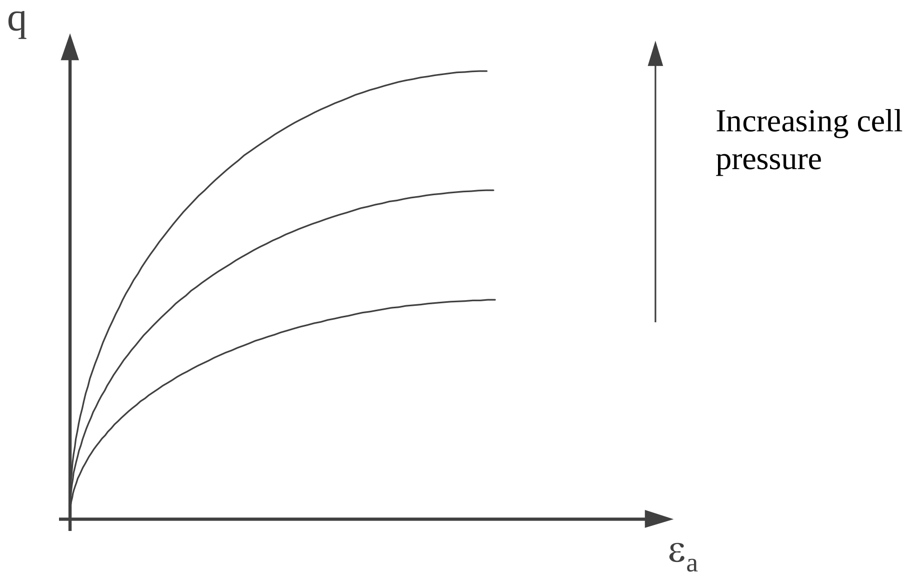
CID (isotropically consolidated drained) test

Similar to CIU except that as deviator stress is increased drainage is permitted. The rate of loading must be slow enough to ensure no excess pore pressures develop.

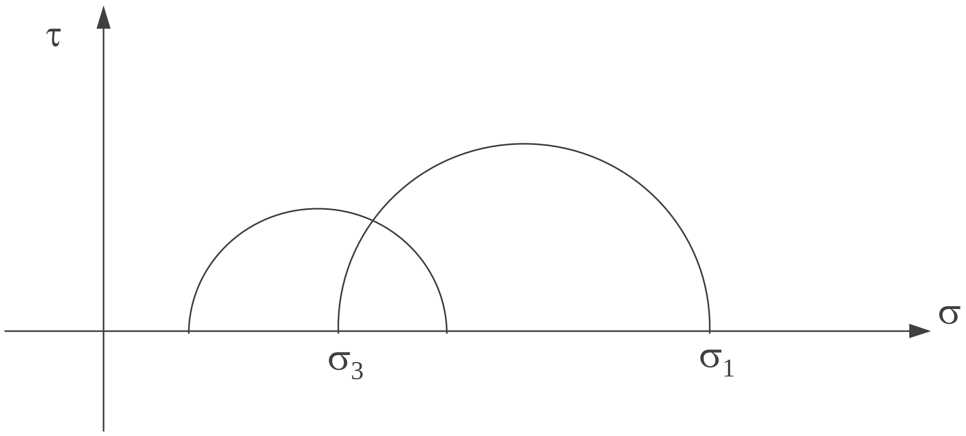
As a test for investigating the behaviour of soils the triaxial test has many advantages over the shear box test:

- Specimens are subjected to uniform stresses and strains
- The complete stress-strain behaviour can be investigated
- Drained and undrained tests can be performed
- Pore water pressures can be measured in undrained tests
- Different combinations of confining and axial stress can be applied

Typical results from a series of drained tests consolidated to different cell pressures would be as follows.



The triaxial test gives the strength in terms of the principal stresses, whereas the shear box gives the stresses on the failure plane directly. To relate the strengths from the two tests we need to use some results from the Mohr circle transformation of stress.



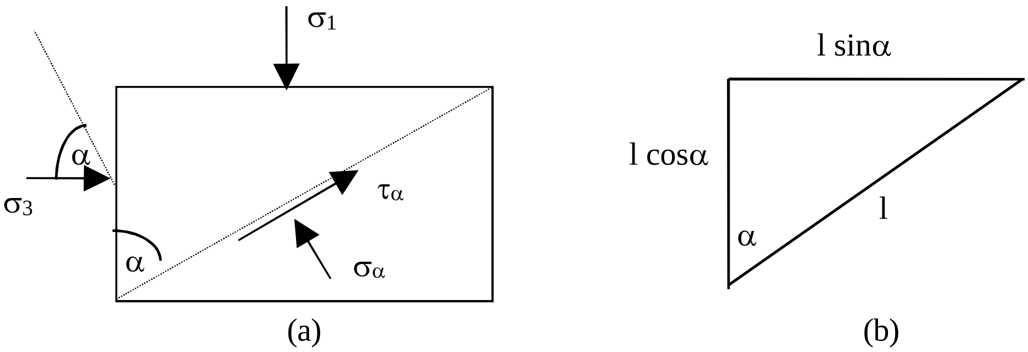
2.3 Mohr Circles

The Mohr circle construction enables the stresses acting in different directions at a point on a plane to be determined, provided that the stress acting normal to the plane is a principal stress. The Mohr circle construction is very useful in Soil Mechanics as many practical situations can be approximated as plane strain problems.

The sign convention is different to that used in Structural Analysis because for Soils it is conventional to take the compressive stresses as positive.

Sign convention: Compressive normal stresses are positive
Anti-clockwise shear stresses are positive (from inside soil element)
Angles measured clockwise positive

Let us consider the stresses acting on different planes for an element of soil



(a) shows the stresses on a plane at angle α to the minor principal stress, and (b) shows the relevant lengths.

Now resolving forces gives

$$\sigma_\alpha l = \sigma_1 \sin\alpha \, l \sin\alpha + \sigma_3 \cos\alpha \, l \cos\alpha$$

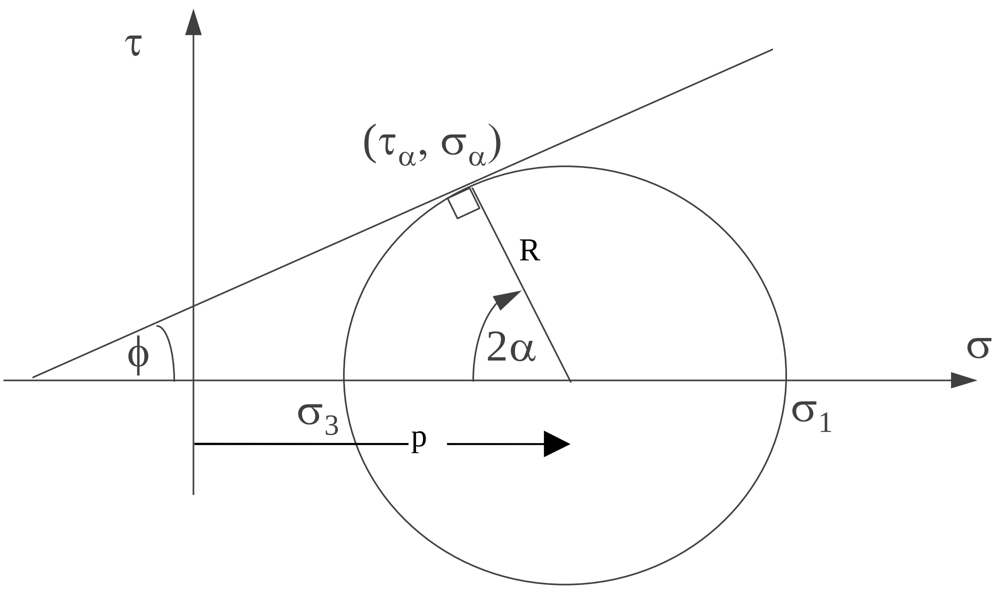
$$\sigma_\alpha = \frac{\sigma_1}{2}(1 - \cos 2\alpha) + \frac{\sigma_3}{2}(1 + \cos 2\alpha)$$

$$\sigma_\alpha = \frac{(\sigma_1 + \sigma_3)}{2} - \frac{(\sigma_1 - \sigma_3)}{2}\cos 2\alpha$$

and similarly

$$\tau_\alpha = \frac{(\sigma_1 - \sigma_3)}{2}\sin 2\alpha$$

which define the Mohr circle relation



From the Mohr Circle we have

$$\sigma_\alpha = p - R\cos 2\alpha$$
$$\tau_\alpha = R\sin 2\alpha$$

where

$$p = \frac{(\sigma_1 + \sigma_3)}{2} = \frac{(\sigma_{xx} + \sigma_{zz})}{2}$$

$$R = \frac{(\sigma_1 - \sigma_3)}{2} = \frac{1}{2}\sqrt{(\sigma_{xx} - \sigma_{zz})^2 + 4\,\tau_{zx}^2}$$

and failure occurs on a plane at an angle α from the plane on which σ_3 acts, and

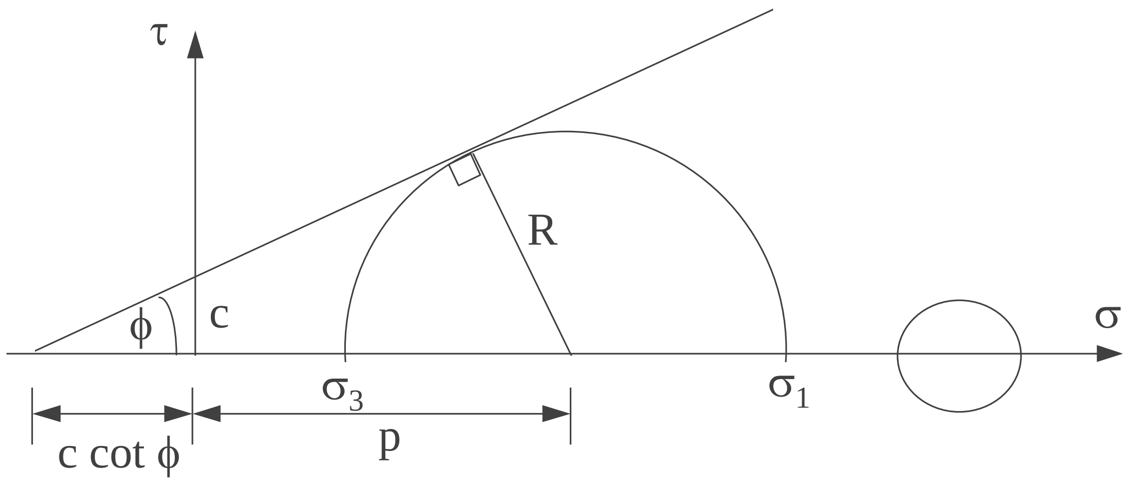
$$\alpha = \left(\frac{\pi}{4} - \frac{\phi}{2}\right)$$

CHAPTER 3: SHEAR STRENGTH OF COHESIVE SOIL

3.1 Mohr-Coulomb Failure Criterion (Principal stresses)

Failure will occur when we can find any direction such that

$$|\tau_\alpha| \geq c + \sigma_\alpha \tan\phi$$



At failure from the geometry of the Mohr Circle

$$R = \sin\phi\,(p + c\cot\phi) = p\sin\phi + c\cos\phi$$

$$\sigma_1 = N_\phi\,\sigma_3 + 2c\sqrt{N_\phi}$$

$$\frac{\sigma_1 + c\cot\phi}{\sigma_3 + c\cot\phi} = \frac{1 + \sin\phi}{1 - \sin\phi} = \tan^2\left[\frac{\pi}{4} + \frac{\phi}{2}\right] = N_\phi$$

3.2. Mohr-Coulomb Failure Criterion for Saturated Soil

As mentioned above it is the effective strength parameters c', ϕ' that are the fundamental soil strength parameters. To use these parameters the Mohr-Coulomb criterion must be expressed in terms of effective stresses, that is

$$\tau = c' + \sigma'_n \tan\phi'$$

$$\sigma'_1 = N_\phi\,\sigma'_3 + 2c'\sqrt{N_\phi}$$

with $$N_\phi = \frac{1 + \sin\phi'}{1 - \sin\phi'}$$

and the effective stresses are given by

$$\sigma'_n = \sigma_n - u$$
$$\sigma'_1 = \sigma_1 - u$$
$$\sigma'_3 = \sigma_3 - u$$

Note that the difference between the total and effective stresses is simply the pore pressure u. Thus the total and effective stress Mohr circles have the same diameter and are displaced along the σ axis by the value of the pore pressure.

3.3 Interpretation of Laboratory Data

It is helpful to distinguish between drained and undrained loading.

3.3.1 Drained loading

In drained laboratory tests the loading rate is sufficiently slow so that all excess pore water pressures will have dissipated. From the known pore water pressures the effective stresses can be determined.

The behaviour of drained tests must be interpreted in terms of the effective strength parameters c', ϕ', using the effective stresses. It is possible to construct a series of total stress Mohr Circles but the inferred total strength parameters have no relevance to the soil behaviour.

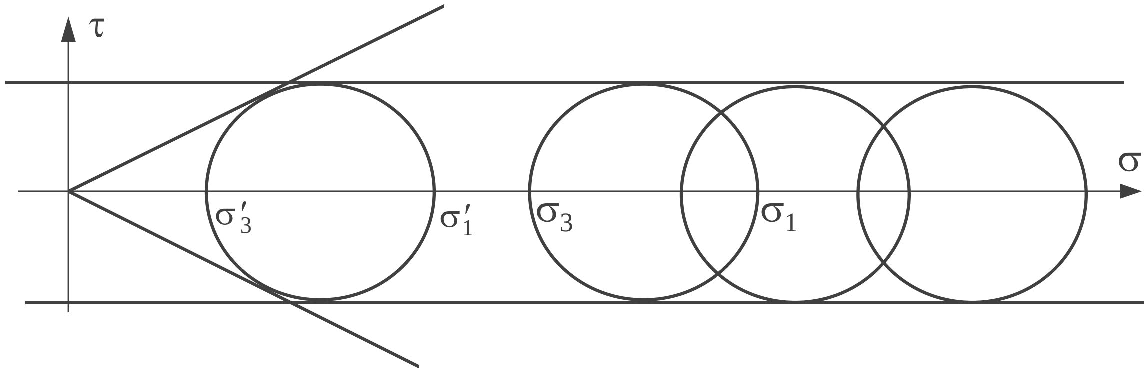
The effective strength parameters are generally used to check the long term (that is when all the excess pore pressures have dissipated) stability of soil constructions. However, for sands and gravels pore pressures dissipate rapidly and for these permeable soils the effective strength parameters can also be used for assessing the short term stability. In principle the effective strength parameters can be used to check the stability at any time for any soil type, but to do this the pore pressures in the ground must be known and in general they are not.

3.*3.2 Undrained loading*

In undrained laboratory tests it is necessary to ensure no drainage from the sample, or moisture redistribution within the sample occurs. In shear box tests this requires fast rates, but because of the more uniform conditions in the triaxial test undrained tests can be performed more slowly simply making sure that no water can drain from the sample.

The behaviour of undrained tests may be interpreted in terms of the effective strength parameters c', ϕ', using the effective stresses. In a triaxial test with pore pressure measurement this is possible. The behaviour may also be interpreted in terms of the total strength parameters c_u, ϕ_u. However, if the total stress parameters are being used they must be determined from Unconsolidated Undrained tests if they are to be relevant to the soil in the ground.

Let us consider the behaviour of three identical saturated soil samples in undrained triaxial tests. No water is allowed to drain and three different confining pressures are applied (Samples are Unconsolidated). The Mohr circles at failure will be as follows



From the total stress Mohr circles we find that $\phi = \phi_u = 0$.

Because all samples are at failure the effective stress failure condition must also be satisfied, and because all the circles have the same radius there must be a single effective stress Mohr circle. The different total stress Mohr circles indicate that the samples must have different pore water pressures.

The explanation for the independence of the undrained strength on the confining stress is that increasing the cell pressure without allowing drainage has the effect of increasing the pore pressure by the same amount ($\Delta u = \Delta\sigma_r$). There is therefore no change in effective stress. As it is the effective stresses that control the soil behaviour the subsequent strength is unaffected. The change in pore pressure during shearing is a function of the initial effective stress and the moisture content. As these are identical for the three samples an identical strength is obtained. As will be shown later the fact that the moisture content remains constant is the most important factor in having a constant strength.

In some series of unconsolidated undrained tests it is found that for different soil samples from a particular site ϕ_u is not zero, or c_u is not constant. If this occurs then either

- the samples are not saturated, or
- the samples have different moisture contents

The undrained strength c_u is not a fundamental soil parameter.

The total stress strength parameters c_u, ϕ_u are often used to assess the short term (undrained) stability of soil constructions. It is important that no drainage should occur otherwise this approach is not valid. Therefore, for sands and gravels which drain rapidly a total stress analysis would not be appropriate.

For soils that do not drain freely this approach is the only simple way of assessing the short term stability, because in general the pore water pressures are unknown.

Note however, that it is possible to measure an undrained strength for any type of soil in the triaxial apparatus.

Example

In an unconsolidated undrained triaxial test the undrained strength is measured as 17.5 kPa. Determine the cell pressure used in the test if the effective strength parameters are $c' = 0$, $\phi' = 26^o$ and the pore pressure at failure is 43 kPa.

Analytical solution

Undrained strength = $17.5 = \dfrac{(\sigma_1 - \sigma_3)}{2} = \dfrac{(\sigma'_1 - \sigma'_3)}{2}$

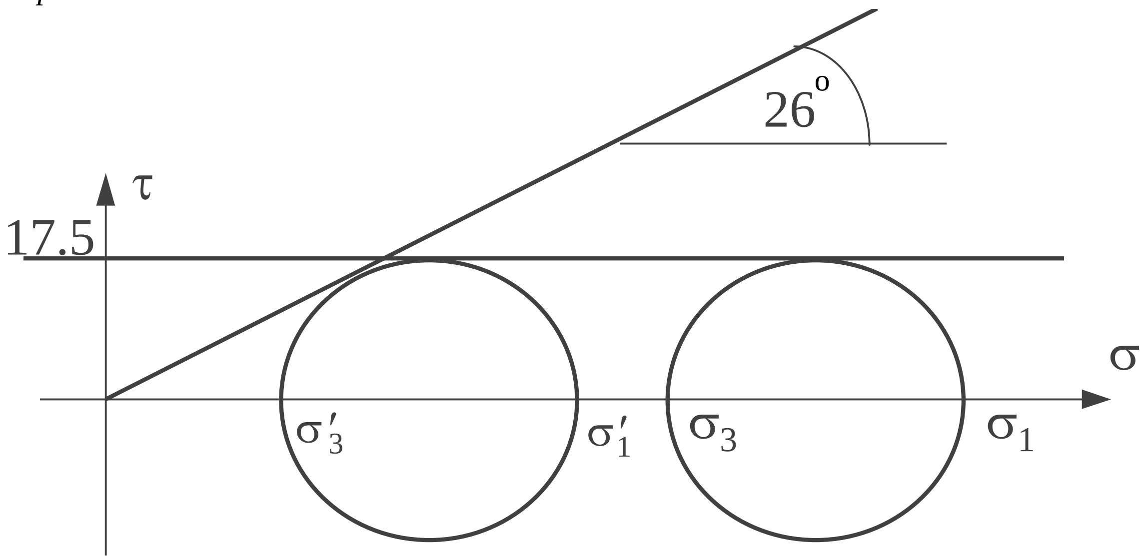
Effective stress failure criterion $\quad \sigma'_1 = N_\phi \sigma'_3 + 2c'\sqrt{N_\phi}$

$c' = 0, \; N_\phi = \dfrac{1 + \sin\phi'}{1 - \sin\phi'} = 2.561$

Hence $\sigma_1' = 57.4$ kPa, $\sigma_3' = 22.4$ kPa

and cell pressure (total stress) = $\sigma_3' + u = 65.4$ kPa

Graphical solution



TUTORIAL

1. Undrained triaxial tests with pore pressure measurement have been performed on three samples of a particular soil, after consolidation to different cell pressures. What information (strength parameters) can be obtained from the results given below?

Cell pressure (kPa)	Failure Deviator Stress (kPa)	Failure Pore Pressure (kPa)
24	31	12
48	76	18
72	104	30

2. Three identical specimens of clay having a small air void content were tested in unconsolidated undrained triaxial tests and the following results obtained.

Axial stress at failure (kPa)	Cell pressure (kPa)
63	12
87	32
118	60

(a) Determine values of c_u and ϕ_u from the results.
(b) What value of undrained strength would you predict for an unconfined (Cell pressure zero) compression test using these values?
(c) What would the pore pressure at failure be in the unconfined test if $c' = 0$, $\phi' = 20°$.
(d) Comment on the significance of the parameters c_u and ϕ_u determined from these tests.

3. A saturated compacted gravel was tested in a large shear box, 300 mm x 300 mm in plan. What properties of the gravel can be deduced from the following results?

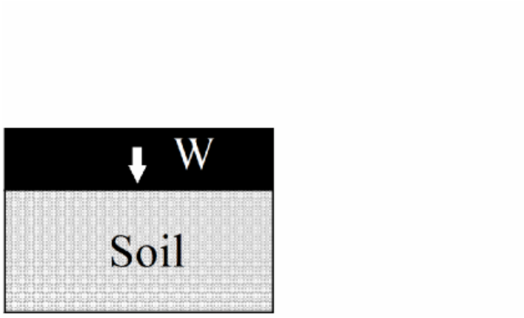
Normal load (N)	Peak Shear Load (N)	Ultimate Shear Load (N)
4500	4500	3520
9200	7890	7190
13800	11200	10780

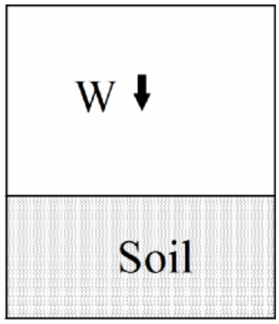
Chapter 4. EFFECTIVE STRESS

4.1 Saturated Soil

A saturated soil is a two-phase material consisting of a soil skeleton and voids, which are saturated with water. It is reasonable to expect that the behavior of an element of such a material will be influenced not only by the forces applied to its surface but also by the water pressure of the fluid in the pores.

Suppose that a soil sample having a uniform cross sectional area A is subjected to an applied load W, as shown in Fig la, then it is found that the soil will deform. If however, the sample is loaded by increasing the height of water in the containing vessel, as shown in Fig lb, then no deformation occurs.





Fig(1a) Soil loaded by an applied weight W **Fig(1b) Soil loaded by water weighing W**

In examining the reasons for this observed behavior, it is helpful to use the following quantities:

$$\sigma_v = Vertical \quad Stress \quad = \frac{Vertical \quad Force}{Cross \quad Sectional \quad Area} \qquad (1)$$

and to define an additional quantity the vertical effective stress, by the relation

$$\sigma_v' = \sigma_v - u_w \qquad (2)$$

Let us examine the changes the vertical stress, pore water pressure and vertical effective stress for the two load cases considered above.

	Ds_v	Du_w	Ds_v'
Case (a)		0	
Case (b)			0

These experiments indicate that if there is no change in effective stress there is no change in deformation, or alternatively that deformation only occurs when there is a change in effective stress.

Another situation in which effective stresses are important is the case of two rough blocks sliding over one another, with water pressure in between them as shown in Fig 2.

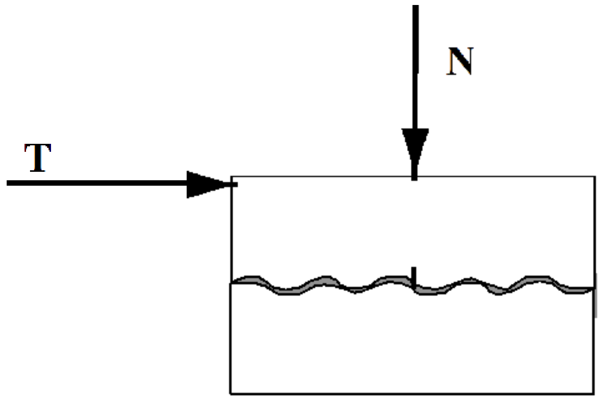


Fig 2 Two pieces of Rock in contact

The effective normal thrust transmitted through the points of contact will be

$$N' = N - U \qquad (3a)$$

where U is the force provided by the water pressure

The frictional force will then be given by *T=μ N'* where m is the coefficient of friction. For soils and rocks the actual contact area is very small compared to the cross-sectional area so that U/A is approximately equal to u_w the pore water pressure. Hence dividing through by the cross sectional area A this becomes:

$$\tau = \mu \sigma'_v \qquad (3b)$$

where t is the average shear stress and s¢$_v$ is the vertical effective stress.

Of course it is not possible to draw a general conclusion from a few simple experiments, but there is now a large body of experimental evidence to suggest that both deformation and strength of soils depend upon the effective stress. This was originally suggested by Terzaghi in the 1920's, and equation 2 and similar relations are referred to as the Principle of Effective Stress.

4.2 Calculation of Effective Stress

It is clear from the definition of effective stress that in order to calculate its value it is necessary to know both the total stress and the pore water pressure. The values of these quantities are not always easy to calculate but there are certain simple situations in which the calculation is quite straightforward. The most important is when the ground surface is flat as is often the case with sedimentary (soil) deposits.

4.2.1 Calculation of Vertical (Total) Stress

Consider the horizontally "layered" soil deposit shown schematically in Fig.3,

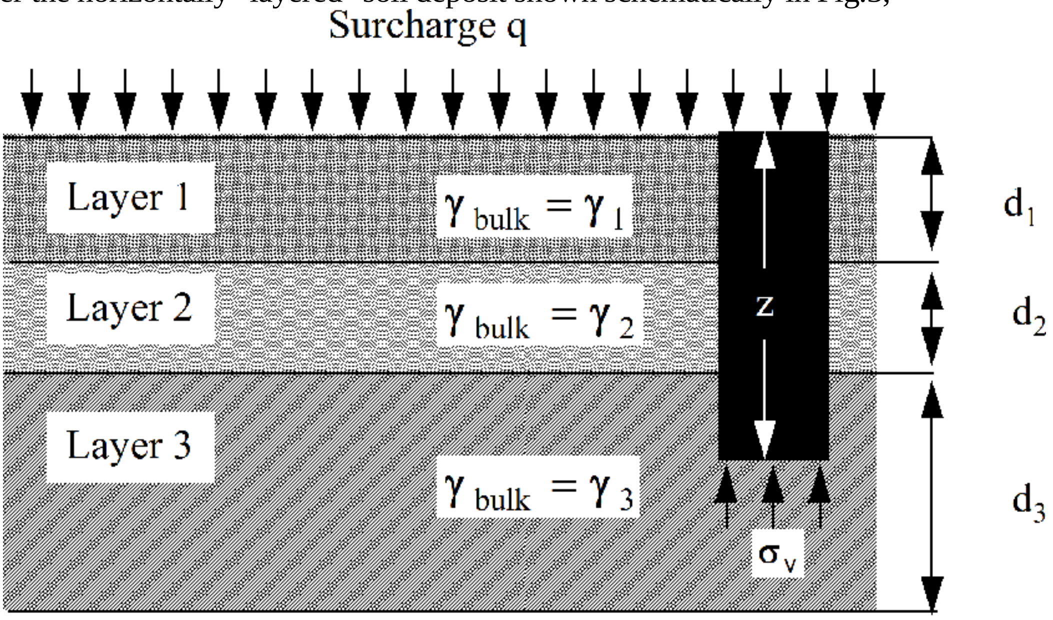


Fig 3 Soil Profile

If we consider the equilibrium of a column of soil of cross sectional area A it is found that

$$\begin{aligned} \text{Force on base} &= \text{Force on Top} + \text{Weight of Soil} \\ A\sigma_v &= Aq + A\gamma_1 d_1 + A\gamma_2 d_2 + A\gamma_3(z - d_1 - d_2) \\ \sigma_v &= q + \gamma_1 d_1 + \gamma_2 d_2 + \gamma_3(z - d_1 - d_2) \end{aligned} \quad (4)$$

- Calculation of Pore Water Pressure

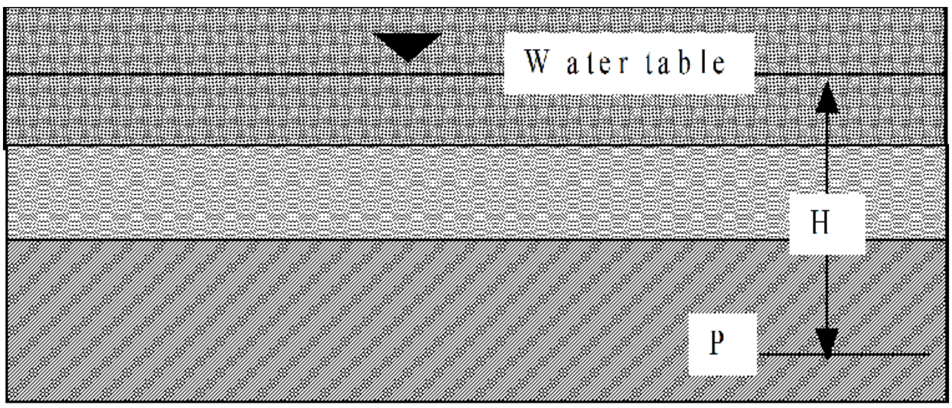


Fig 4 Soil with a static water table

Suppose the soil deposit shown in Fig. 4 has a static water table as indicated. The water table is the water level in a borehole, and at the water table $u_w = 0$. The water pressure at a point P is given by

$$u_w(P) = \gamma_w H \quad (5)$$

Example

A uniform layer of sand 10 m deep overlays bedrock. The water table is located 2 m below the surface of the sand which is found to have a voids ratio e = 0.7. Assuming that the soil particles have a specific gravity Gs = 2.7 calculate the effective stress at a depth 5 m below the surface.

Step one: Draw ground profile showing soil stratigraphy and water table

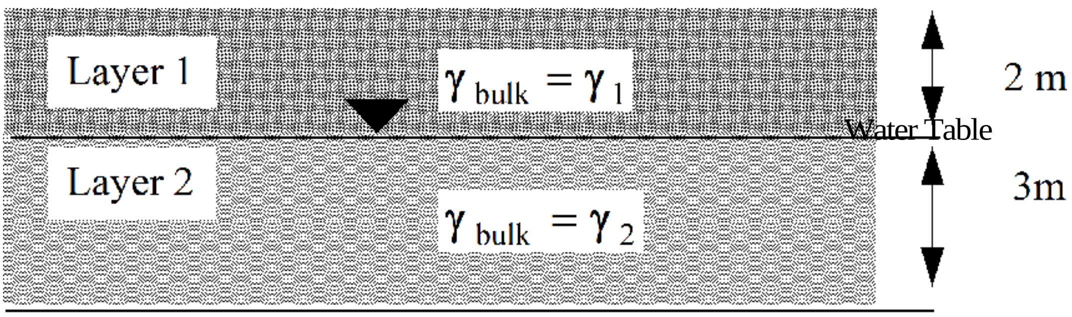


Fig 5 Soil Stratigraphy

Step two: Calculation of Dry and Saturated Unit Weights

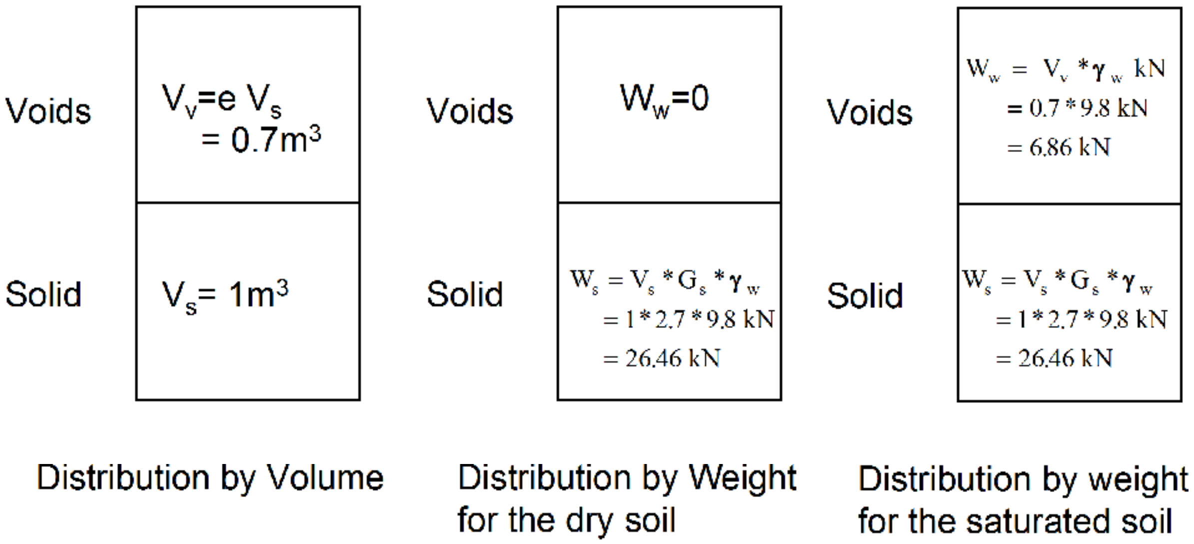


Fig 6 Calculation of dry and saturated unit weight

$$\gamma_{dry} = \frac{26.46 \text{ kN}}{1.70 \text{ m}^3} = 15.56 \text{ kN/m}^3$$

$$\gamma_{sat} = \frac{(26.46 + 6.86) \text{ kN}}{1.70 \text{ m}^3} = 19.60 \text{ kN/m}^3$$

Step three: Calculation of (Total) Vertical Stress

$$\sigma_v = 15.56 \times 2 + 19.60 \times 3 = 89.92 \ kPa \ (kN/m^2) \quad \textbf{(7)}$$

Step four: Calculation of Pore Water Pressure

$$u_w = 3 \times 9.8 = 29.40 \ kPa \quad \textbf{(8)}$$

Step five: Calculation of Effective Vertical Stress

$$\sigma'_v = \sigma_v - u_w = 89.92 - 29.40 = 60.52 \ kPa \quad (9)$$

Note that in practice if the void ratio is known the unit weights are **not** normally calculated from first principles considering the volume fractions of the different phases. This is often the case for saturated soils because the void ratio can be simply determined from

$$e = mG_s$$

The unit weights are calculated directly from the formulae given in the data sheets, that is

$$\gamma_{dry} = \frac{G_s \gamma_w}{1 + e}$$

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1 + e}$$

Effective Stress under general conditions

In general the state of stress in a soil cannot be described by a single quantity, the vertical stress. To fully describe the state of stress the nine stress components (6 of which are independent), as illustrated in Fig. 7 need to be determined. Note that in soil mechanics a compression positive sign convention is used.

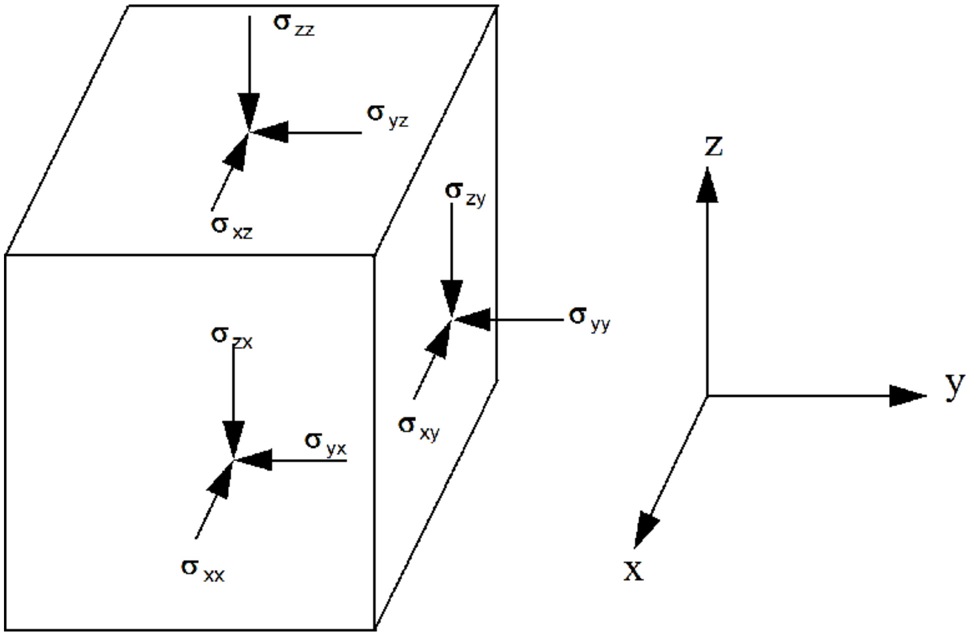


Figure 7 Definition of Stress Components

The effective stress state is then defined by the relations

$$
\begin{aligned}
\sigma'_{xx} &= \sigma_{xx} - u_w \quad ; \quad \sigma'_{yz} = \sigma_{yz} \\
\sigma'_{yy} &= \sigma_{yy} - u_w \quad ; \quad \sigma'_{zx} = \sigma_{zx} \\
\sigma'_{zz} &= \sigma_{zz} - u_w \quad ; \quad \sigma'_{xy} = \sigma_{xy}
\end{aligned}
\qquad (10)
$$

Example – Effects of groundwater level changes

Initially a 50 m thick deposit of a clayey soil has a groundwater level 1 m below the surface. Due to groundwater extraction from an underlying aquifer the regional groundwater level is lowered by 2 m. By considering the changes in effective stress at a depth, z, in the clay investigate what will happen to the ground surface.

Due to decreasing demands for water the groundwater rises (possible reasons include de-industrialisation and greenhouse effects) back to the initial level. What problems may arise?

Assume

- g_{bulk} is constant with depth
- g_{bulk} is the same above and below the water table (clays may remain saturated for many metres above the groundwater table due to capillary suctions)

The vertical total and effective stresses at depth z are given in the Table below.

	Initial GWL	Lowered GWL
σ_v	$\gamma_{bulk} \cdot z$	$\gamma_{bulk} \cdot z$
μ	$\gamma_w .(z -1)$	$\gamma_w .(z -1)$
σ'_v	$z.(\gamma_{bulk} - \gamma_w) - \gamma_w$	$z.(\gamma_{bulk} - \gamma_w) + 3. \gamma_w$

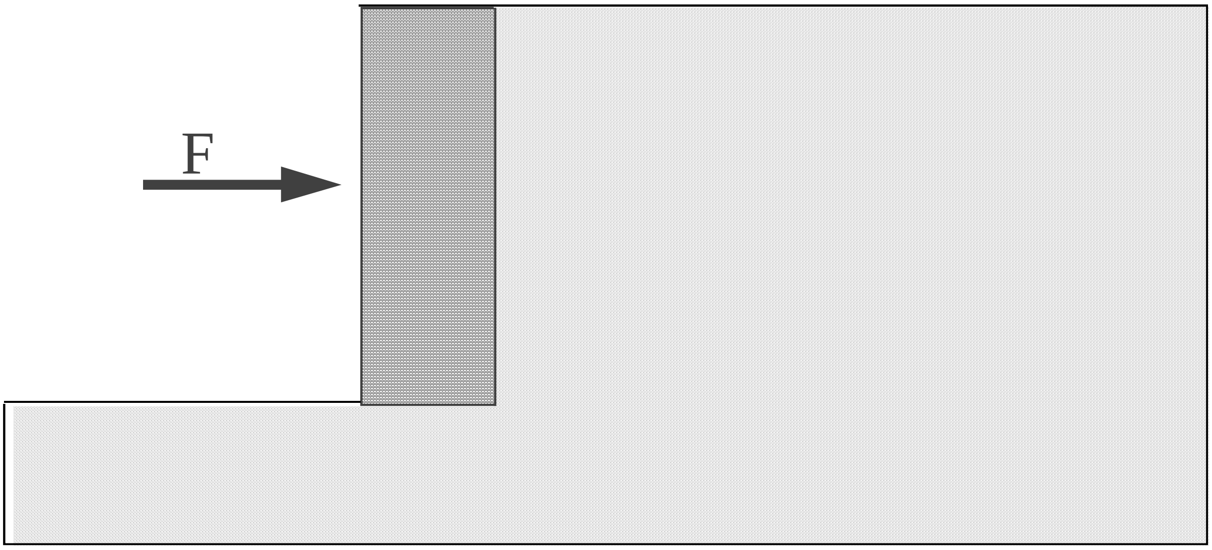
At all depths the effective stress increases and as a result the soil compresses. The cumulative effect throughout the clay layer can produce a significant settlement of the soil surface.

When the groundwater rises the effective stress will return to its initial value, and the soil will swell and the ground surface heave (up). However, due to the inelastic nature of soil, the ground surface will not in general return to its initial position. This may result in:

- surface flooding
- flooding of basements built when GWL was lowered
- uplift of buildings
- failure of retaining structures
- failure due to reductions in bearing capacity

Chapter 5 : EARTH PRESSURES (Rankine's Method)

5.1 Modes of failure



Some force is required to support the soil. This force may be provided by

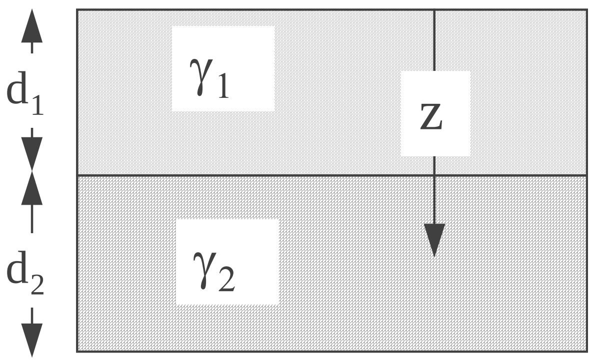
- friction at the base (gravity retaining walls)
- founding the wall into the ground (sheet retaining walls)
- anchors and struts
- external loads

If the force is too small the soil behind the wall will reach a state of failure with the wall moving away from the soil (active failure). If the force is too large the soil will reach another state of failure with the wall moving into the soil (passive failure).

Rankine's theory allows the limiting pressures on retaining walls to be determined.

5.2 Rankine's theory

In Rankine's method it is assumed that the wall is frictionless. The normal stress acting on the wall will therefore be a principal stress. If the wall is vertical and the soil surface horizontal, the vertical and horizontal stresses throughout the retained soil mass will be principal stresses. In this situation the vertical stress at any depth can be simply determined, as follows:



$$\sigma_v = \gamma_1 d_1 + \gamma_2 (z - d_1)$$

The horizontal stress can then be calculated from the Mohr-Coulomb failure criterion. If short term stability is being considered this can be achieved using undrained (total stress) parameters while if long term stability is being considered drained (effective stress) parameters must be used.

From Mohr-Coulomb failure criterion we can write for soil at failure

$$\sigma_1 = N_\phi \ \sigma_3 + 2c\sqrt{N_\phi}$$

The implications of this expression are most easily investigated by considering the response of soil adjacent to a frictionless retaining wall. Then we can identify two limiting conditions:

5.2.1 Active failure

There is insufficient force to support the soil. Assuming that the vertical stress is given simply by the weight of the overlying soil and does not change during deformation, the minimum horizontal stress may be determined from

$$\sigma_{hmin} = \frac{\sigma_v - 2c\sqrt{N_\phi}}{N_\phi}$$

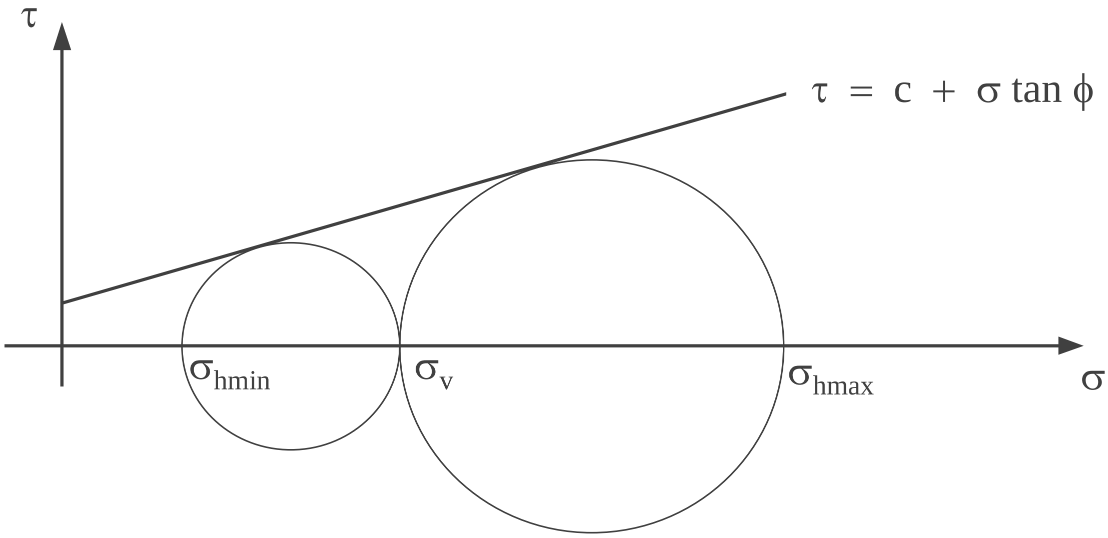
5.2.2 Passive failure

The force on the wall is greater than the resistance provided by the soil. The horizontal stress reaches a maximum value given by

$$\sigma_{hmax} = N_\phi \ \sigma_v + 2c\sqrt{N_\phi}$$

In the Rankine method a stress state is found that is in equilibrium with the applied loads and has the soil at failure. In plasticity theory this approach is referred to as a lower bound method, a method which can be shown to produce safe, conservative solutions.

The relation between active and passive states can be seen by considering the Mohr circles as shown below.



For a given σ_v it is impossible for the horizontal stresses to drop below σ_{hmin} or rise above σ_{hmax}.

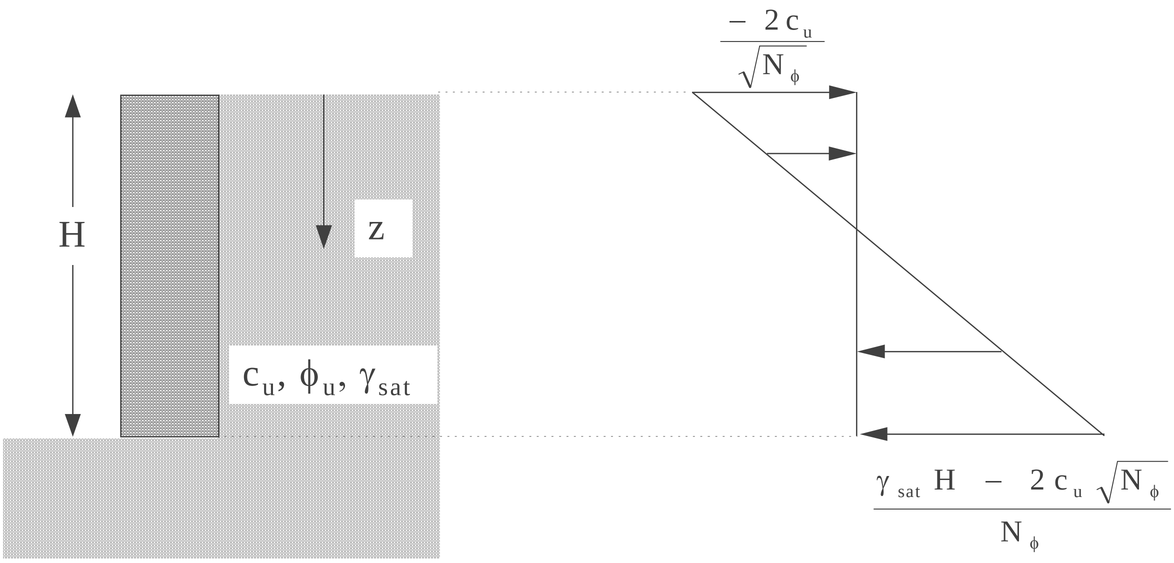
5.3 Total Stress Analysis

Only appropriate if the soil remains undrained. In practice this implies that total stress analysis can only be used to investigate the stability of clayey soils with low permeabilities.

Use undrained parameters c_u, ϕ_u and total stresses σ_1, σ_3, σ_v, σ_h with

$$N_\phi = \frac{1 + \sin \phi_u}{1 - \sin \phi_u} \qquad c = c_u$$

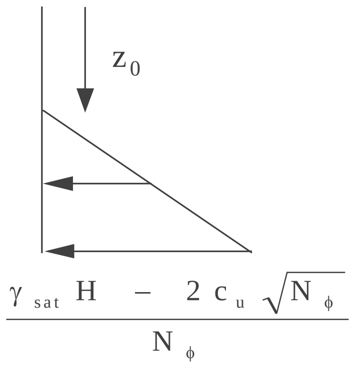
Consider the undrained active failure of a wall in a saturated clayey soil



- $\phi_u \neq 0$ This implies that the undrained strength s_u increases with depth. It does not imply that the soil is unsaturated; if this were true an undrained analysis would be inappropriate.

- Values of c_u, ϕ_u can be measured for sands from undrained triaxial tests. However, these are almost never relevant because of drainage

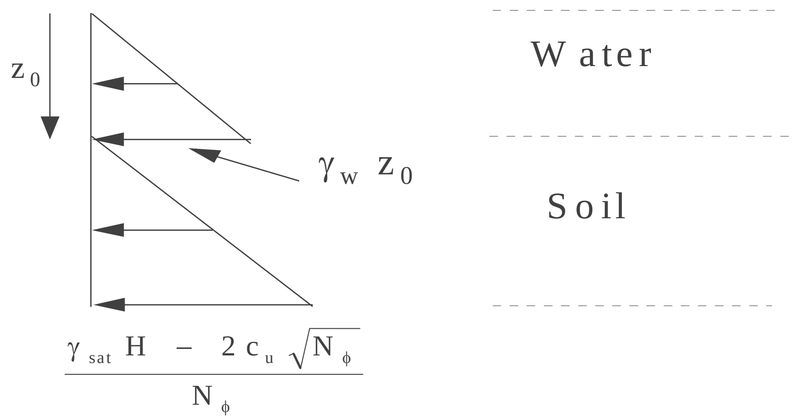
- Tension cracks. The analysis indicates negative, tensile, stresses at the surface. However, soil particles cannot provide tension. The negative stresses have to come from suctions in the pore water. It is difficult to rely on the tensile forces and they are usually ignored. The tensile stresses reduce the force required for stability of the wall. Ignoring the tensile stresses therefore gives a more conservative solution. The pressure distribution on the wall becomes



where the depth of the tension region z_o is given by

$$z_0 = \frac{2 c_u \sqrt{N_\phi}}{\gamma_{sat}}$$

If water is available it can fill up the tension crack and provide additional pressures on the wall. In this situation the pressure diagram becomes



- The position of the water table is only important in as far as it affects the total stresses.

5.4 Effective stress analysis

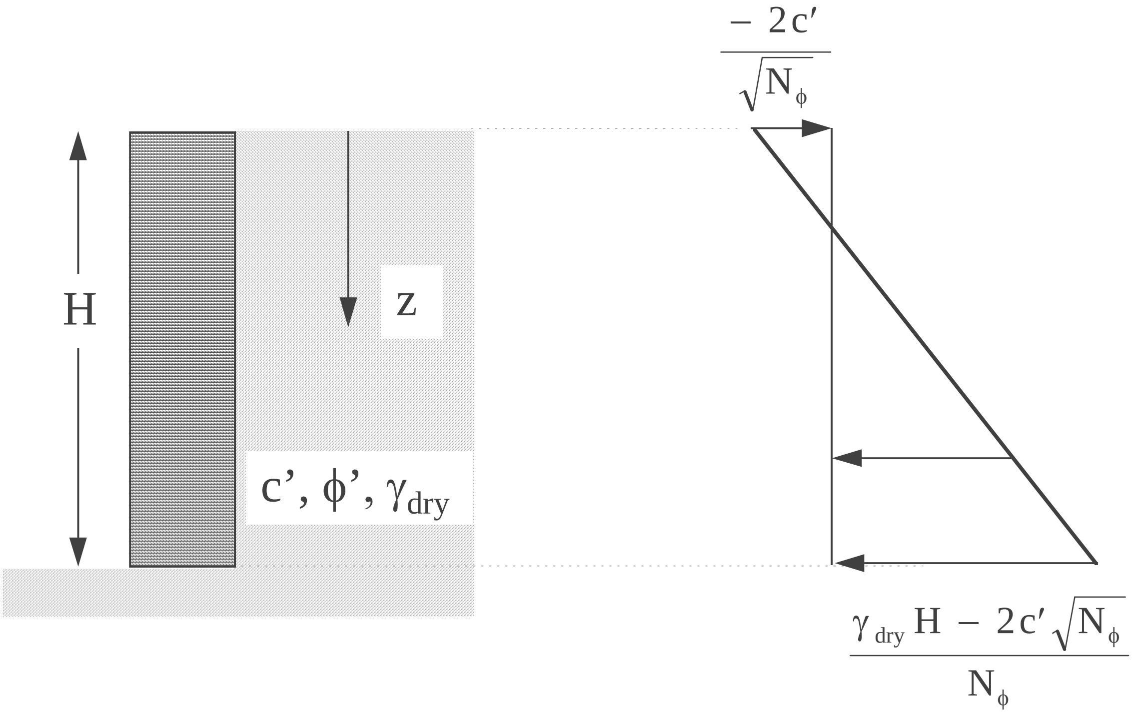
This is always appropriate, irrespective of the drainage conditions. But to perform an effective stress analysis the pore water pressures in the soil must be known, and unfortunately they are often unknown. In the long term a steady state will be reached where the pore pressures can be detemined either from knowing the position of the static water table or from a flow net.

Use effective soil parameters c', ϕ' and effective stresses σ_1', σ_3', σ_v', σ_h' with

$$N_\phi = \frac{1 + \sin\phi'}{1 - \sin\phi'} \qquad c = c'$$

and $\sigma' = \sigma - u$

Consider the active failure of a wall in a dry sandy soil



- c', ϕ' are peak strength values. It is generally more appropriate and safer to use the ultimate or critical state parameters, $c' = 0$, $\phi' = \phi_{ult}'$

- The critical state parameters require a larger active force to be provided to maintain the wall stability, thus providing a safe, conservative, estimate.

- For passive failure the critical state parameters give a smaller force on the wall than the peak strength parameters. Again this gives a safe, conservative, estimate. In dry sand the limiting passive pressure is given by

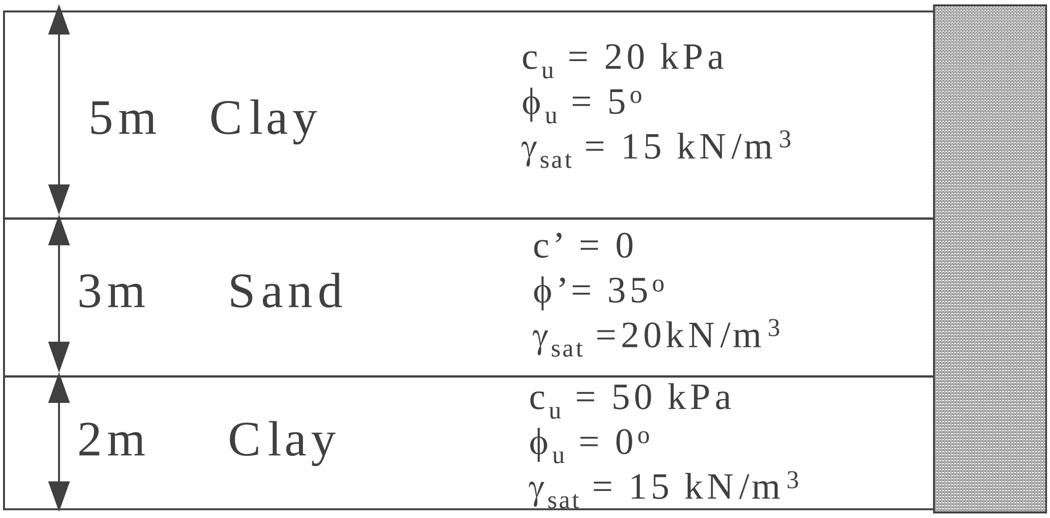
$$\sigma_h' = \gamma_{dry} z N_\phi + 2c'\sqrt{N_\phi}$$

- It is important to remember to use effective stresses, $\sigma_v' = \sigma_v - u$ when calculating the horizontal effective stresses σ_h'. Then to calculate the total horizontal stress on the wall the pore water pressure must be added to obtain $\sigma_h = \sigma_h' + u$

- If the water level is not the same on each side of the wall, water will flow. The pore water pressures must then be determined from a flow net before calculating σ_v'.

Example 1

A 10 m high retaining wall retains 5 m of clay which overlays 3 m of sand which overlays 2 m of clay. The water table is at the surface of the retained soil. Calculate the limiting active pressure immediately after construction.



Layer 1: A clay layer so will be undrained in the short term. Will require a total stress (undrained) analysis

$$c = c_u = 20\,\text{kPa} \qquad N_\phi = \frac{1 + \sin\phi_u}{1 - \sin\phi_u} = 1.19$$

Active failure thus $\sigma_1 = \sigma_v$ and $\sigma_3 = \sigma_h$

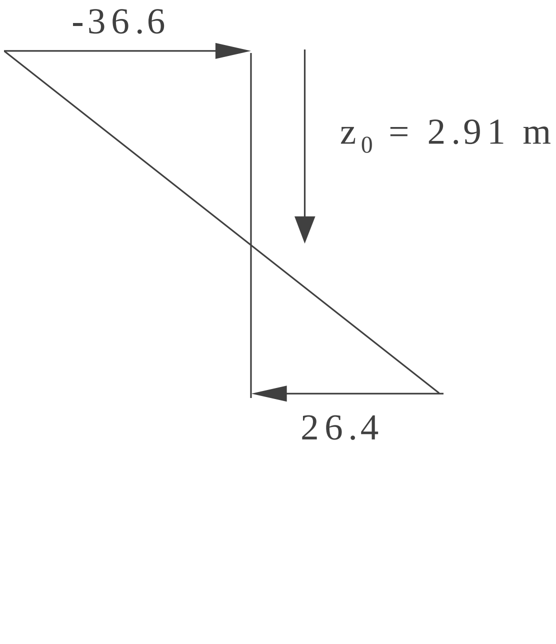
From the Mohr-Coulomb failure criterion

$$\sigma_h = \frac{\sigma_v - 2c_u\sqrt{N_\phi}}{N_\phi} = \frac{\sigma_v - 43.6}{1.19}$$

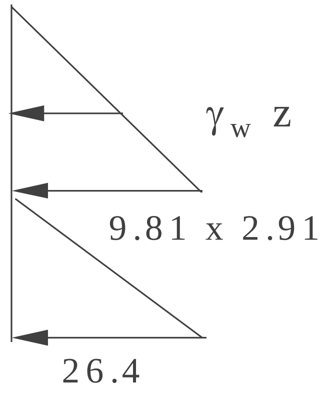
At the surface z = 0, σ_v = 0, σ_h = - 36.6 kPa

At base of layer z = 5 m, σ_v = 5x15, σ_h = 26.4 kPa

This gives the following pressure distribution on the wall



The analysis predicts tensile stresses between the soil and the wall. These are not likely, and a tension crack may develop. Because the water table is at the surface the crack will fill with water, and a more pessimistic pressure distribution will be



Layer 2: Sand so excess pore pressures will dissipate rapidly. Therefore total stress analysis cannot be used. For sand in short term assume fully drained. Must use effective stress analysis.

$$c = c' = 0 \qquad N_\phi = \frac{1 + \sin\phi'}{1 - \sin\phi'} = 3.69$$

Active failure so $\sigma'_1 = \sigma'_v$ and $\sigma'_3 = \sigma'_h$ and from the Mohr-Coulomb criterion

$$\sigma'_h = \frac{\sigma'_v - 2c'\sqrt{N_\phi}}{N_\phi} = \frac{\sigma'_v}{3.69}$$

z	σ_v	u	$\sigma'_v = \sigma_v - u$	$\sigma'_h = \sigma'_v/3.69$	u	$\sigma_h = \sigma'_h + u$
5	75	49	26	7	49	56
8	135	78.4	56.6	15.3	78.4	93.7

Note that most of horizontal pressure is due to water

Layer 3: Clay, therefore total stress (undrained) analysis for short term

$$c = c_u = 50\,\text{kPa} \qquad N_\phi = \frac{1 + \sin\phi_u}{1 - \sin\phi_u} = 1$$

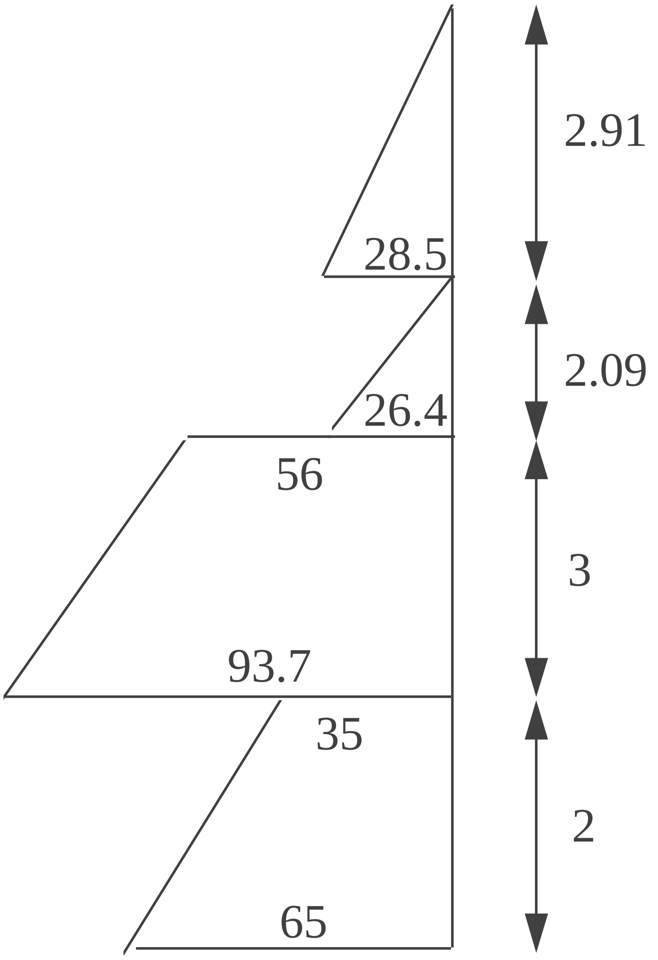
When $\phi_u = 0$ the Mohr-Coulomb criterion reduces to

$$\sigma_1 = \sigma_3 + 2\,c_u$$

$$\sigma_h = \sigma_v - 2\,c_u$$

z	σ_v	σ_h
8	135	35
10	165	65

The final pressure diagram is then



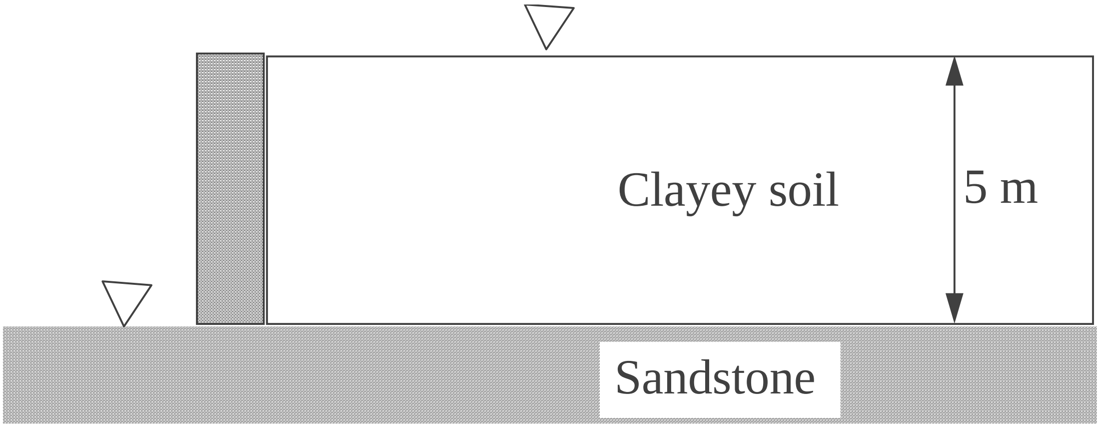
The force required to prevent active failure can be determined from the pressure diagram

```
F = 0.5x28.5x2.91
     +  0.5x26.4x2.09
     +  56x3 + 0.5x(93.7-56)x3
     +  35x2 + 0.5x(65-35)x2
  = 393.7 kN/m
```

Example 2

A 5m high retaining wall retains a clayey soil, which overlies a highly permeable sandstone. If the water level remains at the surface of the clay in the retained soil, and is level with the top of the sandstone determine the minimum force required to maintain the stability of the wall for short and long term. The soil parameters are:

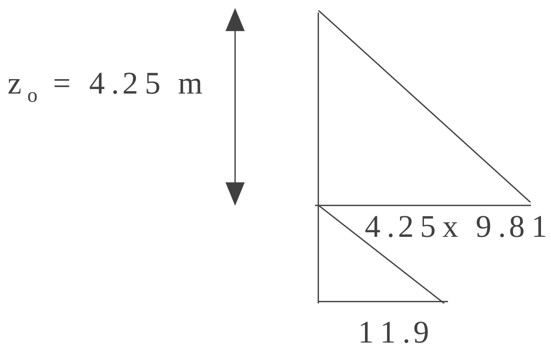
$c_u = 37$ kPa, $\phi_u = 5^o$ $c' = 0$, $\phi_{ult}' = 25^o$, $\gamma_{sat} = 19$ kN/m^3



Short term undrained - total stress analysis
Minimum force for stability - active failure

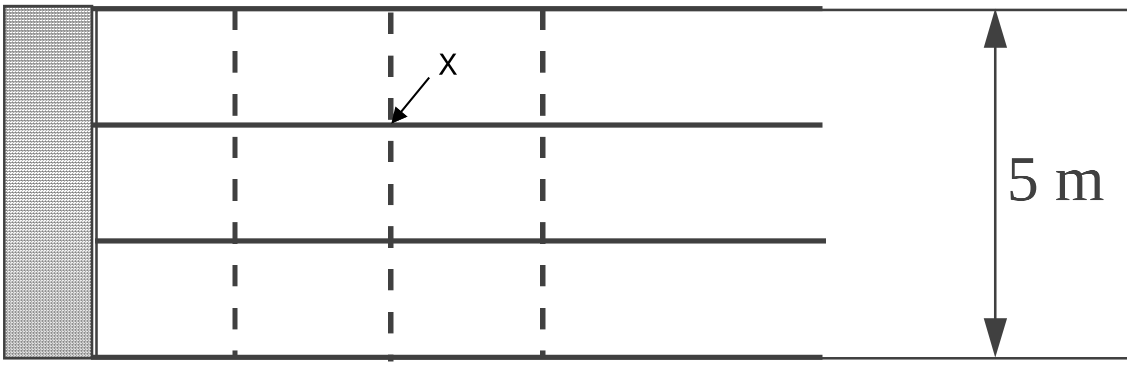
$$\sigma_h = \frac{\sigma_v - 2c_u\sqrt{N_\phi}}{N_\phi} = \frac{\sigma_v}{1.19} - 67.8$$

At the surface σ_h = - 67.8 kPa, and at 5 m σ_h = 11.9 kPa
Allowing for tension crack filling with water, pressures acting on the wall will be



$$F = \frac{1}{2}\times 9.81\times 4.25^2 + \frac{1}{2}\times 11.9\times 0.75 = 93.1\,\text{kN / m}$$

Long term - Effective stress analysis
Pore pressures required - Have to be determined from a flow net



$$u = \gamma_w (h - z)$$

Taking the Datum at the base of the wall

H_o = head at the soil surface = 5 m

At X

$h = h_o - \Delta h = 5 - (5/3)x1 = 10/3$

$z = (2/3)x5 = 10/3$

$u = 0$

Effective stress analysis with $c' = 0, \ \phi' = 25^o$

$$\sigma'_h = \frac{\sigma'_v - 2c'\sqrt{N_\phi}}{N_\phi} = \frac{\sigma'_v}{2.46}$$

Now u = 0, so $\sigma'_v = \sigma_v = \gamma_{sat} \ z$

At wall base $\sigma_h = \sigma'_h = 38.6$ kPa

Hence F $= 0.5 \times 38.6 \times 5 = 96.4$ kN/m

TUTORIAL SHEET

1. A vertical wall 9 m high retains soil level with the top of the wall. If the soil is a saturated clay with $c_u = 20$ kN/m^2, $\phi_u = 0$, $\gamma_{sat} = 19$ kN/m^3, use Rankine's method to calculate the magnitude and line of action of the active earth force on the wall,

   (a) assuming the soil can provide tension

   (b) assuming the soil can provide no tension

   (c) allowing for rain water collecting in the tension cracks.

   in each case sketch the pressure distribution on the wall.

2. The same wall as in Question 1 retains sand for which $\phi' = 30^o$, $c' = 0$, $\gamma_{dry} = 18$ kN/m^3, $\gamma_{sat} = 20$ kN/m^3. Use Rankine's method to obtain the magnitude and line of action of the active earth force on the wall, if the water table lies:

   (a) at the upper soil surface

   (b) below the bottom of the wall

   (c) half-way up the wall

   In each case sketch the pressure distribution on the wall.

3. The figure below shows a 8 m high sea wall at a location where 4 m of sand overlie a deep clay deposit. The water table in the soil is at the same level as the sea level on the other side of the wall. Tests have been performed to determine the relevant soil properties. For the sand $\gamma_{dry} = 17$ kN/m^3, $\gamma_{sat} = 19$ kN/m^3, and a series of shear box tests gave the following results at failure

Shear stress τ (kN/m^2)	Normal stress σ (kN/m^2)
16	20
37	50
72	100

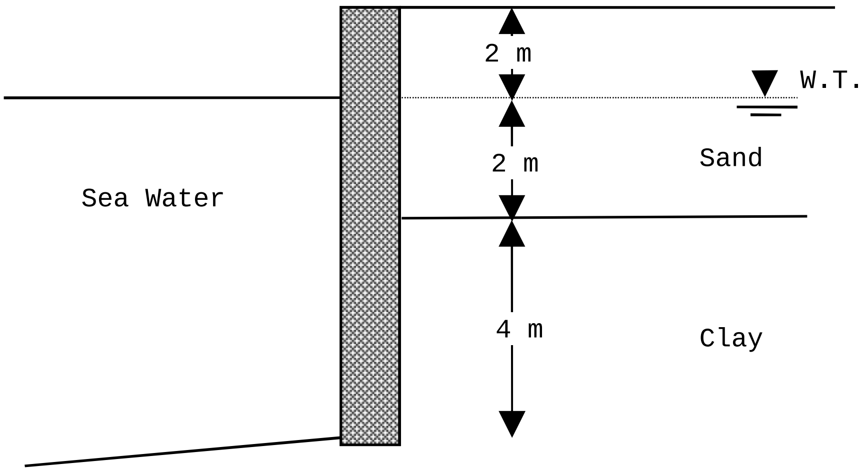
   A series of triaxial tests were performed on samples of the clayey soil ($\gamma_{sat} = 16.5$ kN/m^2). This has included 3 undrained unconsolidated tests in which pore pressures were measured and one consolidated undrained test. The stresses at failure are given below.

Test type	Cell pressure σ_3 (kN/m^2)	Deviator stress $\sigma_1 - \sigma_3$ (kN/m^2)	Pore pressure u (kN/m^2)
Unconsolidated Undrained	0	50	-34.4
Unconsolidated Undrained	50	50	15.6
Unconsolidated Undrained	100	50	65.6
Consolidated Undrained	200	90	127.2

By determining the soil strength parameters calculate, using Rankine's method, the minimum force required to maintain the stability of the wall:

(a) in the short term

(b) in the long term



Chapter 6: RETAINING WALLS

Retaining walls can be broadly split into two categories. Those that rely on their weight for the stability of the wall (Gravity walls), and those that mobilise earth pressures in the ground to provide resistance (Embedded walls). Within each category there are a variety of wall types. The selection of the appropriate wall type depends on many factors that include:

- Soil and groundwater conditions
- Height and ground topography
- Availability of suitable fill material
- Construction constraints (space, access, equipment, specialist techniques available)
- Environment – appearance and impact during construction
- Ground movements and their affects on adjacent structures
- Underground obstructions and services
- Design life and maintenance requirements
- Cost

These notes are primarily concerned with the general design methods used for the two types of wall. Further details of individual wall types and their advantages and disadvantages may be found in many texts on retaining wall and foundation design. For retaining walls used to support excavations particular attention should always be given to the effects of groundwater. Failure to consider this can lead to failure of the soil-wall system by mechanisms not always considered in standard design calculations. For instance, groundwater lowering will lead to settlements which may damage adjacent services and structures, groundwater flow may lead to erosion and piping at the base of excavations, and groundwater pressures may cause heave into an excavation.

6.1 Gravity Walls

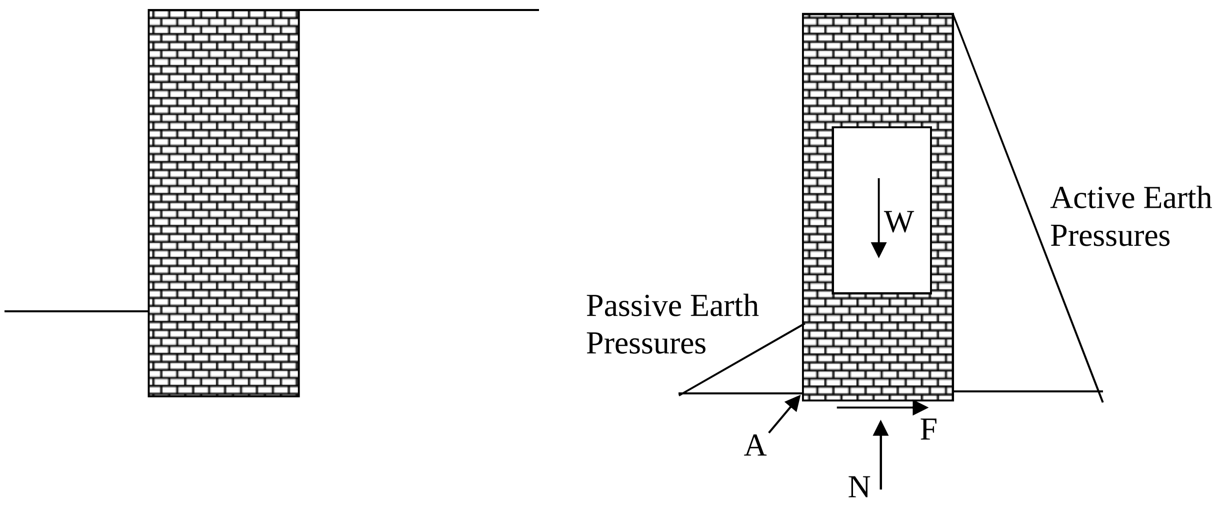
Gravity walls are generally used to retain soil above the existing ground level. The simplest walls rely on the mass of the wall for stability. These include walls made of mass concrete, concrete with masonry facing, unreinforced masonry (bricks and stone), gabions (wire baskets filled with stone), and crib walls (hollow crib formwork filled with soil). These types of wall are common for small retained heights up to 3 m, and are rare for heights greater than 8 m. For walls between 3 and 8 m precast reinforced concrete (cantilever) walls are very common. These walls are usually in the shape of an L or inverted T. Reinforced soil walls are also widely used. These use strips of steel or plastic placed in the soil connected to facing elements that retain the soil. Friction between the reinforcing strips and the soil provides the resistance to hold up the facing elements. To mobilise the soil resistance some movement must occur and reinforced soil walls are therefore more flexible and require relatively large tolerances to ground movement. Soil nailed walls are similar to reinforced soil but are used to support the soil face during excavation.

There are four principal modes of failure that need to be analysed for any gravity wall. These are

- Translation
- Overturning
- Bearing capacity
- Overall failure of the soil and wall

In addition it is generally necessary to check that the wall deformations and the ground movements will not be excessive.

6.1.1 Translation



Translation is the mode of failure where the wall slides because the frictional force, F, is less than the force due to the difference in the active and passive pressures. The active and passive pressures can be determined from either Rankine's method or from a limit equilibrium method. It is found that the factor of safety is very dependent on any passive pressures developed in front of the wall. Because of this it is normal to ignore the upper 0.5 to 1 m of soil contributing to the passive pressures. This reduces the possibility of inadvertent excavation leading to failure.

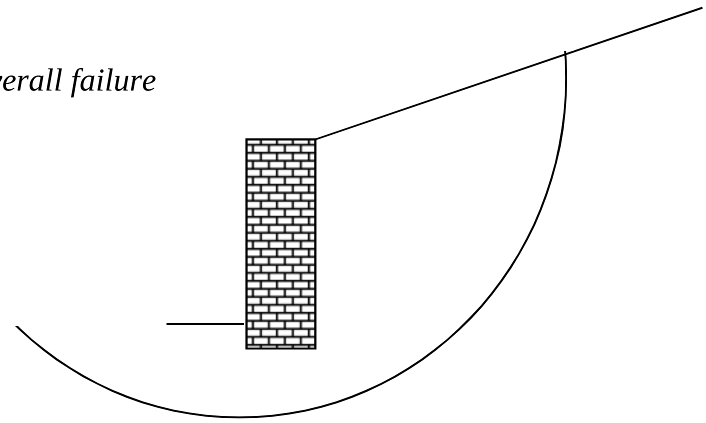
6.1.2 Overturning

If the wall height becomes large then there will be a significant moment due to the active earth pressures. In the limit the wall will topple about the toe, point A in the diagram above. At this limit the overturning moment due to the earth pressures must be balanced by the restoring moment due to the weight of the wall.

6.1.3 Bearing capacity

If the stress due to the weight of the wall is large there is the possibility that the underlying soil will not be able to support it. This is known as a bearing capacity failure. Section 7 of these notes discusses the bearing capacity in more detail. It should be noted that due to the earth pressures acting on the wall there will be a moment (eccentricity of the normal load) and horizontal force acting on the base of the wall. This moment and horizontal load will significantly reduce the bearing capacity (vertical stress) that the soil can support. One method of allowing for these loads is given in the Soil Mechanics Data Sheets (p74, 75). The general bearing capacity formula includes reduction factors that account for the load inclination (horizontal loads) and load eccentricity. The moment is allowed for by using an effective foundation width B' (= $B - 2e$), where e is the eccentricity of the load, in the correction factors for load inclination.

6.1.4 Overall failure



A check is required on the overall stability of the soil and wall combined to check that a failure surface will not occur in the soil. This may be analysed using the methods discussed previously for assessing slope stability. This may include checking a rotational failure mechanism as shown above, and possibly a wedge mechanism if there are weak layers at some depth beneath the wall.

6.2 Embedded retaining walls

Embedded walls are generally used for construction from the ground level down. They can be partly driven and then backfilled, or fully driven or constructed in-situ followed by excavation. There are four main construction methods: walls constructed of sheets of timber, steel or concrete; soldier or king piles with sheeting placed between the piles; bored pile walls; and diaphragm walls. Each wall type may act as a cantilever or be supported by one or more rows of anchors or props. They can be used either as temporary supports during construction, or for permanent structures such as quay or basement walls. The walls range from relatively flexible steel sheet piles to relatively stiff diaphragm walls. These walls are generally more expensive than gravity walls but their cost is balanced by the speed of construction and lack of temporary support. Cantilever walls are only suitable for moderate retained heights, typically less than 5 m, but if a stiff reinforced concrete wall is formed may be suitable to about 10 m. Significant ground movements can occur behind cantilever walls, and they are generally unsuitable if services or foundations of adjacent buildings are close. The use of anchors or props can reduce the required penetration length, the ground deformation and the bending moments in the walls

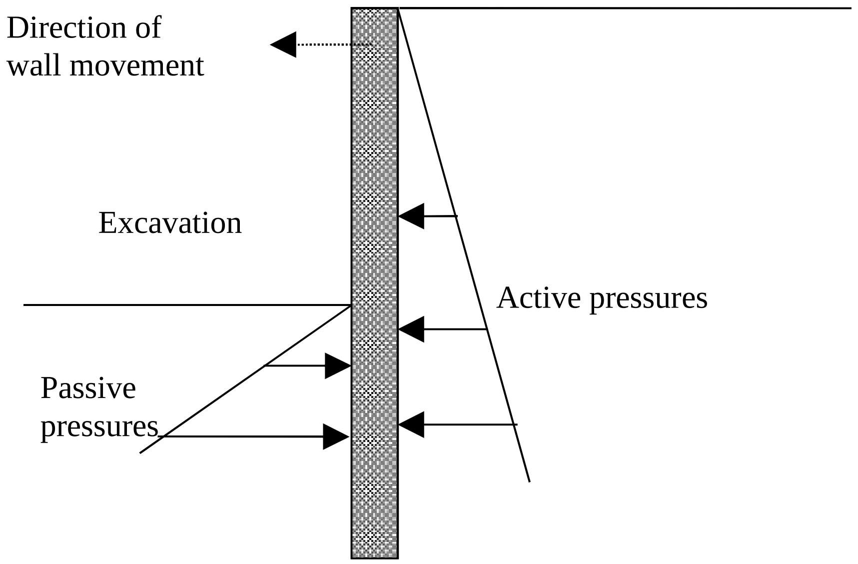
From the design viewpoint we can split these sheet pile walls into three groups

1. Cantilever Walls
2. Walls with a single anchor or prop
3. Walls with multiple props

For any wall type we need to consider:

- The overall stability of the soil/wall system
- The structural strength of the wall
- The possibility of damage to adjacent structures, and services in the ground, due to wall construction

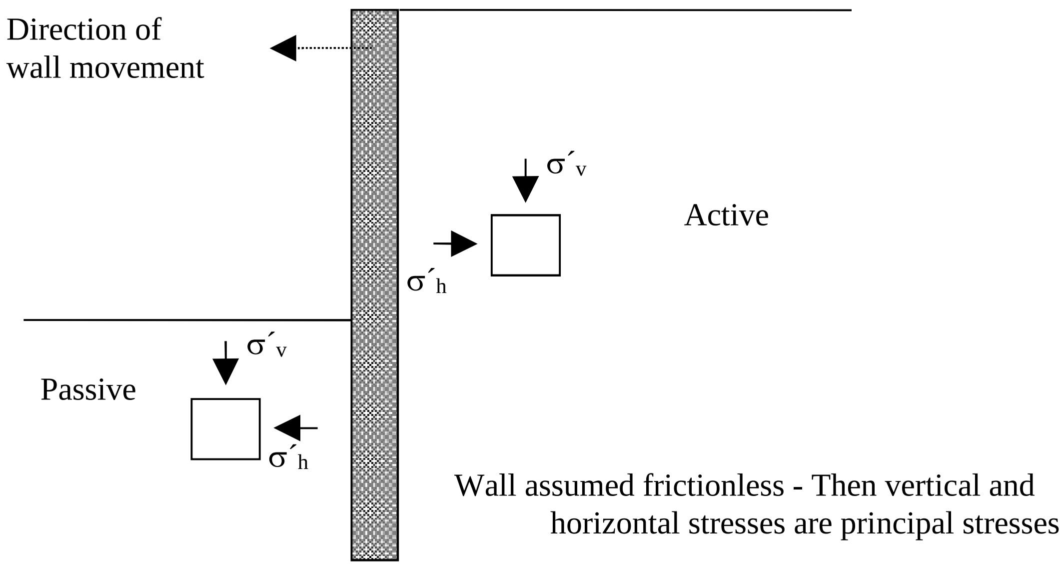
6.2.1 Cantilever walls

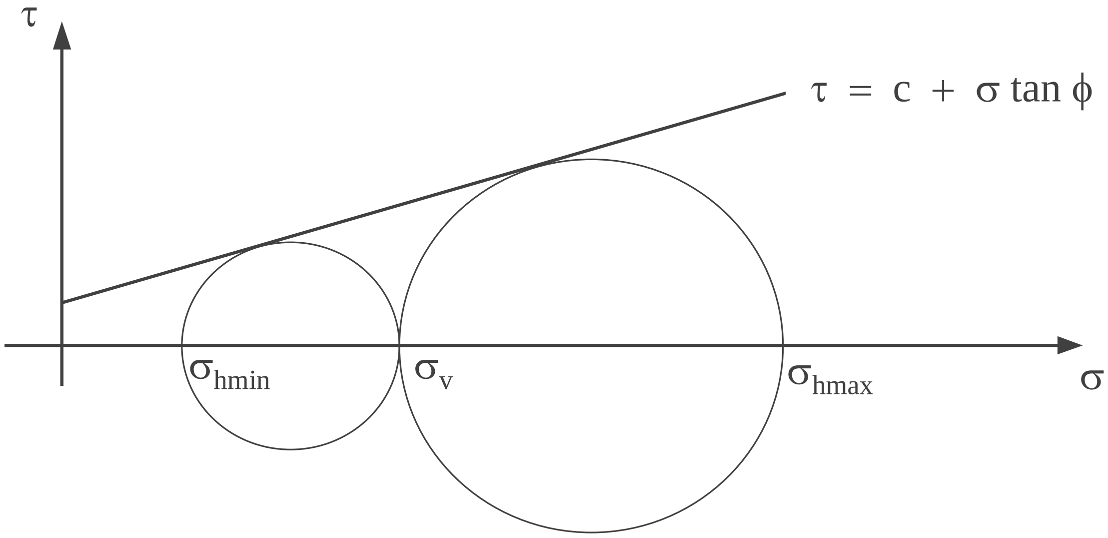


When designing sheet retaining walls it is normal to assume that the effective lateral stresses acting on the wall are given by simple RANKINE active and passive zones. Friction on the wall is usually ignored as this leads to conservative (safe) designs.

6.2.2 Rankine Active and Passive Pressures

The earth pressures acting on the wall are strongly dependent on the deformations in the surrounding soil. When the wall moves away from the soil the stress on the wall drops reaching a minimum, the ACTIVE pressure, with the soil deforming plastically. When the wall moves into the soil the stress increases, finally reaching a maximum, the PASSIVE pressure, when again the soil is deforming plastically.





$$\sigma_1 = N_\phi \sigma_3 + 2c\sqrt{N_\phi}$$

For most retaining walls the long term, fully drained, situation usually governs the wall stability. For the analysis of fully drained conditions the Mohr-Coulomb criterion needs to be expressed in terms of effective stress using the effective strength parameters c´ and ϕ´. For design it is also conservative to use the critical state strength parameters, that is $c' = 0$ and $\phi' = \phi'_{cs}$. The effective lateral stresses on the wall are then

$$\text{ACTIVE} \qquad \sigma'_h = \frac{\sigma'_v}{N_\phi} = \frac{1 - \sin\phi'}{1 + \sin\phi'}\sigma'_v = K_a \sigma'_v$$

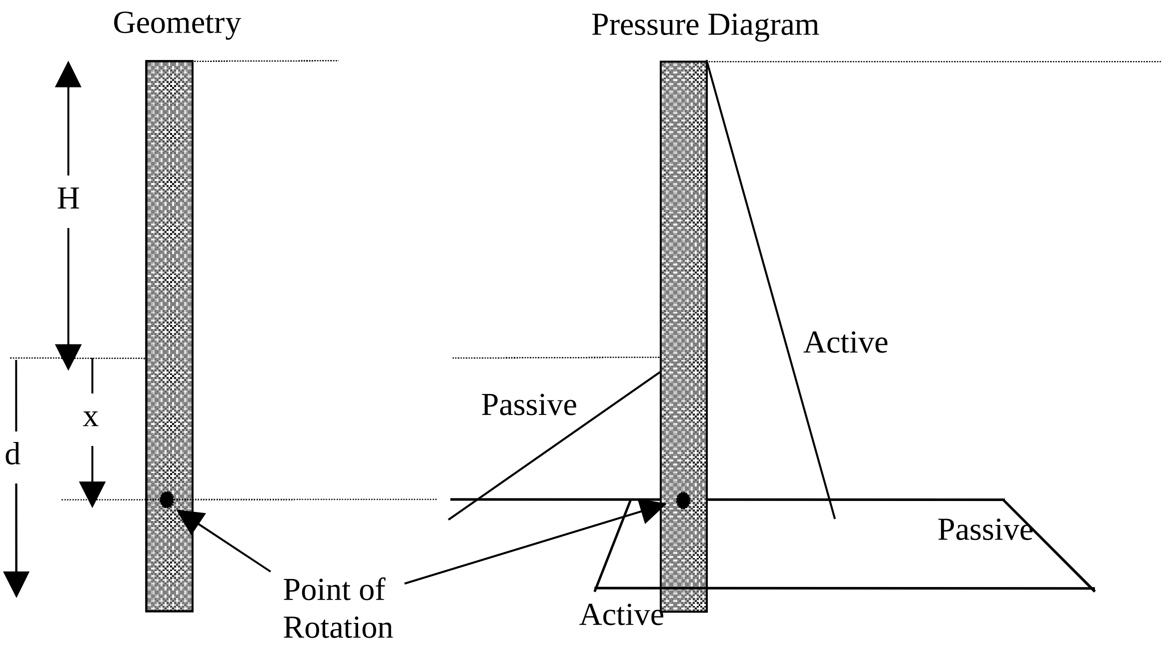
$$\text{PASSIVE} \qquad \sigma'_h = \sigma'_v N_\phi = \frac{1 + \sin\phi'}{1 - \sin\phi'}\sigma'_v = K_p \sigma'_v$$

K_a and K_p are known as the active and passive earth pressure coefficients. For soil at failure the earth pressure coefficients are simply related by $K_a = \frac{1}{K_p}$.

For any vertical wall it is possible to relate the horizontal effective stress to the vertical effective stress, determined from the vertical overburden, by an earth pressure coefficient. The coefficient will depend on the slope of the soil surface and the wall roughness. Published values are available for many situations.

6.2.3 Stability - Limiting Equilibrium

When assessing the stability it is normal to assume triangular pressure distributions, and this is in fact quite realistic if the wall is rigid. For a cantilever wall the stresses acting at failure will then be as shown below, with the wall rotating about a point just above the toe of the wall. The stability of the wall depends mainly on the passive force developed below the excavation.

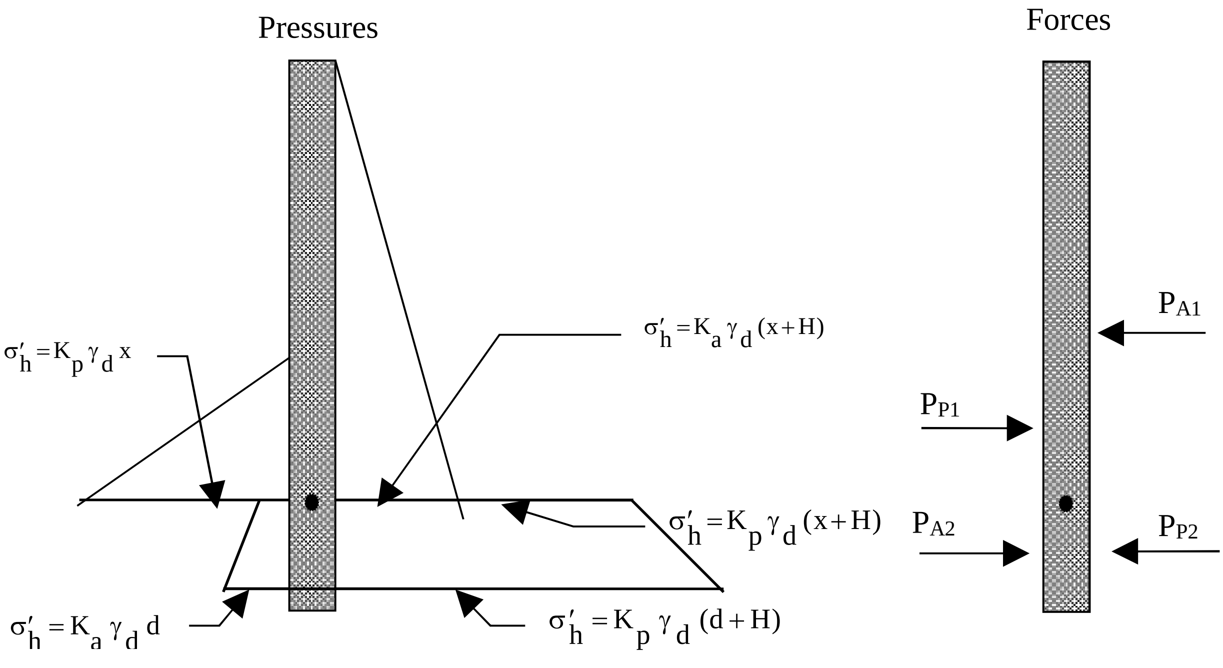


For design we need to determine the required depth of penetration for stability and then to size the wall to resist the maximum moment. To determine the depth of penetration required for a given height H we need to consider both moment and force equilibrium:

$$\Sigma F = 0$$

$$\Sigma M = 0$$

If the soil is dry the pressures and forces are as shown below



Where

$$P_{A1} = \frac{1}{2} K_a \gamma_d (x + H)^2$$

$$P_{P1} = \frac{1}{2} K_P \gamma_d x^2$$

$$P_{A2} = K_a \gamma_d x (d - x) + \frac{1}{2} K_a \gamma_d (d - x)^2$$

$$P_{P2} = K_p \gamma_d (x + H)(d - x) + \frac{1}{2} K_p \gamma_d (d - x)^2$$

From equilibrium

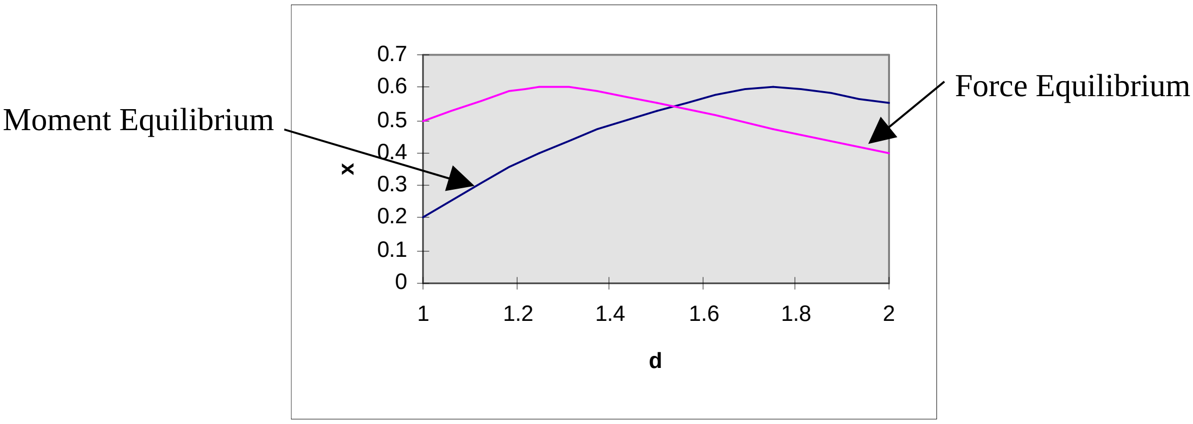
$\Sigma F = 0$: $P_{A1} + P_{P2} - P_{P1} - P_{A2} = 0$
This gives a quadratic equation involving terms in x^2 and d^2

$\Sigma M = 0$: Taking moments about the point of rotation

$$P_{A1}\left[\frac{x + H}{3}\right] + P_{A2}\left[\frac{d - x}{2}\right] \approx P_{P1}\frac{x}{3} + P_{P2}\left[\frac{d - x}{2}\right]$$

This gives a cubic equation involving terms in x^3 and d^3.

We have 2 equations with 2 unknowns, x and d, and hence we can determine the required depth of penetration for the wall. The equations can be solved graphically or by computer. Alternatively simplifying assumptions about the forces below the pivot can be made to enable analytical solutions to be obtained as described in many text books.



As an illustration consider a wall with H = 1.8 m placed in dry soil with γ_d = 19 kN/m^3 and ϕ' = 30º. For ϕ' = 30º K_p = 3, K_a = 0.3333 and the required depth of penetration d = 1.767 m.

6.2.4 Serviceability - Design requirements

By considering the stability we can obtain the limiting stresses on the wall, but the wall would have been considered to have failed from a *serviceability* viewpoint well before this, owing to large settlements in the supported soil. The design approach is to factor the earth pressures.

There are two main design approaches which are both based on the knowledge that the earth pressures acting on the wall are strongly dependent on the deformations in the surrounding soil. The movements required to reach the active and passive conditions depend on the soil type and can be quite different. For example, for retaining walls of height H the movements required are approximately:

SAND		Active	0.001H
		Passive	0.05H - 0.1H
CLAY	Normally Consolidated	Active	0.004H
		Passive	large
	Over-Consolidated	Active	0.025H
		Passive	0.025H

Method 1 - Sands and normally consolidated clay

Assume that sufficient movement occurs to allow active (minimum) pressures to develop, then factor the *effective* passive pressures by 2. Note that where insufficient movement of the wall occurs the active pressures will not reach a minimum and higher pressures will act on the wall. These must be allowed for in design as they can influence the required structural strength.

Consider the same wall as above with H = 1.8, γ_d = 19 kN/m^3, $\phi' = 30^o$

The pressure diagram looks identical but the passive pressures are reduced by using a reduced value of the passive earth pressure coefficient, K^*_p.

Where $K^*_p = K_p/2 = 1.5$ and $K^*_a = K_a = 0.3333$ as before.

Hence d = 2.94 m

The total depth of sheet pile required = 1.8 + 2.94 = 4.74 m

Some texts recommend increasing the depth of penetration by a further 10-20% to allow for uncertainties in the analysis. Alternatively some design codes recommend assuming the top 0.5 m of the soil beneath the excavation provides no restraining effect.

Method 2 - Over-consolidated clays

Here both active and passive pressures are developed for similar movements and both are factored. This is achieved by dividing tan ϕ' by a Factor F_ϕ, and using the reduced angle of friction when calculating the earth pressure coefficients K^*a, K^*p. The factor F_ϕ can be in the range from 1.2 to 1.5, depending on the allowable settlement and soil type, but is usually taken as 1.3.

As for method 1 it is assumed that the shape of the pressure diagram is similar to that at limiting equilibrium, but in this case the passive pressures are reduced and the active pressures increased. Using the same parameters as previously H = 1.8, γ_d = 19 kN/m^3, $\phi' = 30^o$

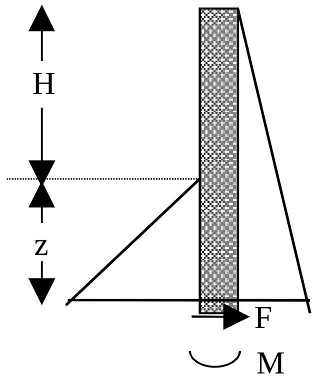
Calculate ϕ^* from $\tan\phi^* = \dfrac{\tan\phi'}{F_\phi} = \dfrac{\tan(30)}{1.3}$

Hence $\phi^* = 23.95^o$ and $K^*_a = 0.423$, $K^*_p = 1/K^*_a = 2.366$
Then the required depth of penetration becomes d = 2.46 m

6.2.4 Structural strength

Having determined the required depth of penetration, the next stage in design is to calculate the maximum moment in the wall so that an appropriate wall thickness and strength can be selected. The position down the wall of the maximum moment can be found by determining where the shear stress in the wall is zero. ($F = \frac{dM}{dz}$)

Consider a free body diagram of a section of the wall



$$F = \frac{1}{2}K_a \gamma_d (z + H)^2 - \frac{1}{2}K_p \gamma_d z^2 = 0$$

$$(K_p - K_a) z^2 - 2 K_a z H - K_a H^2 = 0$$

A quadratic equation that can be solved for z using appropriate (factored) values for K_p, K_a.

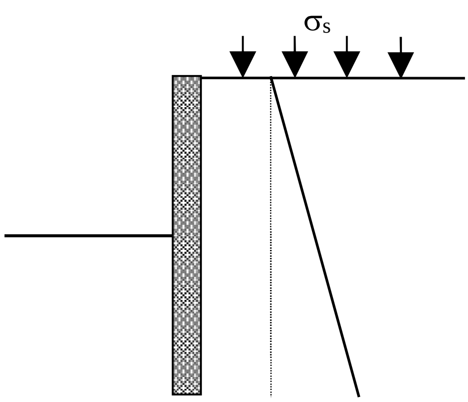
Then taking moments $M = \frac{1}{2}K_p \gamma_d z^2 \frac{z}{3} - \frac{1}{2}K_a \gamma_d (z + H)^2 \frac{(z + H)}{3}$

With H = 1.8, γ_d = 19 kN/m^3, ϕ' = 30^o and using $K^*_p = K_p/2$

z = 1.605 m M = 22.0 kNm/m

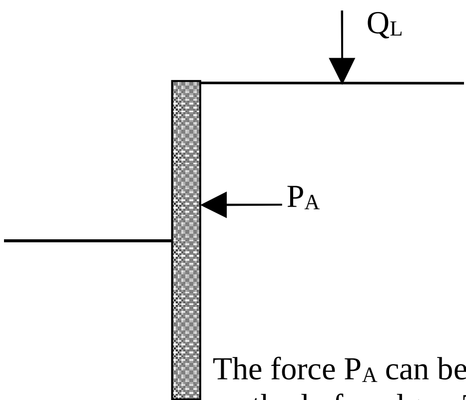
Note that as the factor of safety increases the maximum moment also increases.

The factor of safety can be dramatically reduced by surcharge loadings on the supported ground next to the wall. For a uniform surcharge then the effective active pressure can be increased by $K_a \sigma_s$, while for a concentrated load from a footing the Coulomb method of trial wedges can be used to determine the active force on the wall. In the latter situation allowance must be made for the fact that the point of application of the load will also change.



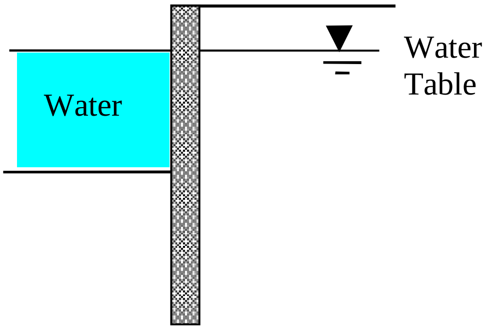
$$\sigma'_v = \sigma_s + \gamma_d z$$

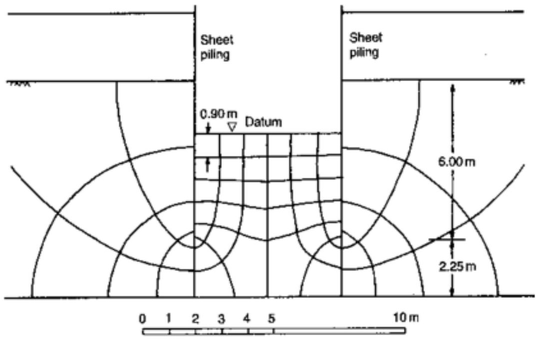
$$\sigma'_h = K_a (\sigma_s + \gamma_d z)$$



The force P_A can be estimated using the method of wedges. The line of action can be estimated using elastic solutions

Consideration must also be given to the water pressures acting on the wall.





Effective stresses must be used in evaluating the lateral stresses from Rankine's method

$$\sigma'_h = K\sigma'_v$$

Pore water pressures are the same on each side of the wall so their effects cancel when considering force and moment equilibrium

Total vertical stress, $\sigma_v = \gamma_{sat}\, z$

Water pressures can be determined from flow net

Hence $\sigma'_v = \sigma_v - u$ and $\sigma'_h = K\sigma'_v$

Forces due to water pressures are different on the two sides of the wall so their effects must be included when considering force and moment equilibrium.

For economic reasons cantilever walls are usually limited to excavations less than 6 m deep.

They are often used to support low banks of free draining sand and gravel soils.

They are not suitable for the long term support of soft clayey soils (clay or silt)

Corrosion can also be a problem with steel sheet piles.

6.3 Anchored Walls

6.3.1 Single anchor (or prop)

When a cantilevered sheet wall is unsuitable, for example because the height is too great or because the deflections need to be limited. Anchors may be used to improve the stability, with the anchors often placed close to the top of the wall.

The anchor force introduces a further unknown into our equations, so that an assumption is required. There are two main methods of analysis:

1. Free earth support method — The base of the pile is assumed to be free to rotate and move laterally

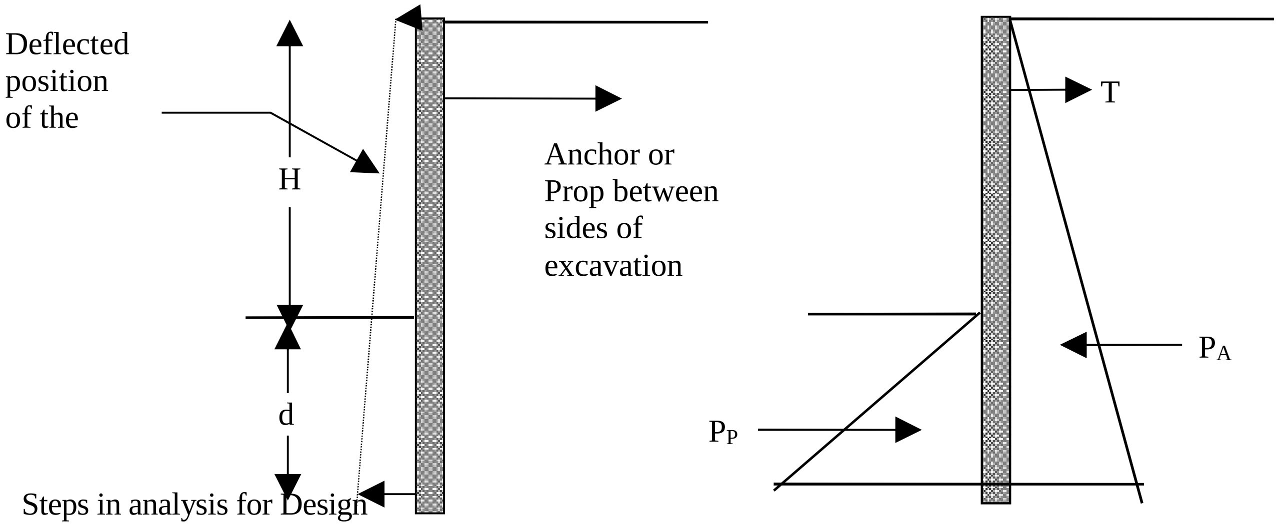
2. Fixed earth support method — The base of the pile is assumed to be fixed in position and direction

The appropriate method depends on the relative stiffness of the wall/soil system. For a relatively rigid system (ie. a heavy wall section in a loose sand) the earth pressure distribution corresponds closely to the triangular active and passive conditions. If the wall is rigid the toe of the wall will be able to move and rotate, and the free earth support method is appropriate.

As the stiffness of the system decreases the pressure distribution alters in such a way as to reduce the bending moment in the sheet pile, and as a consequence, the wall section may be reduced as compared with an infinitely stiff wall. The design procedure is usually to use the free earth support method and then use empirical moment reduction methods to determine the wall section required.

For a very flexible wall the fixed earth support method can be used. The analysis is more complex than the free earth support method and will not be considered here.

6.3.1 The Free Earth Support Method



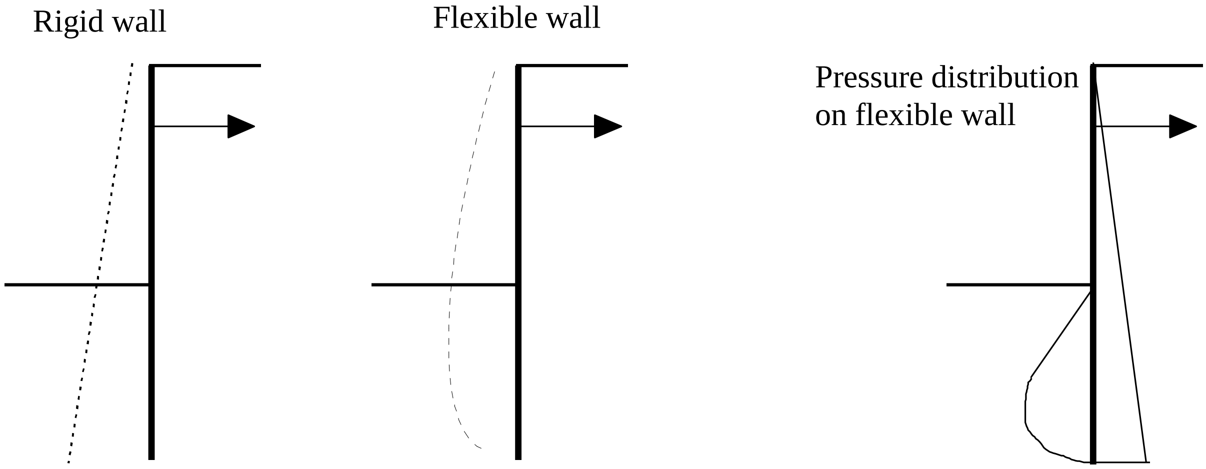
Steps in analysis for Design

- Determine the effective vertical stresses
- Determine the effective lateral stresses assuming Rankine active and passive pressures
- Factor the lateral pressures to limit the deformations - either by factoring K_p or by factoring ϕ'

- Add in water pressures if water levels different on two sides of the wall
- Take moments about the anchor/strut to determine the required depth of the wall
- Use force equilibrium to determine the anchor force
- Design anchor to withstand the force
- Determine maximum moment in the wall and check that section is acceptable

Anchors are typically spaced 2 - 3 m apart, and the load is distributed along the wall by walings running either behind, or in front of the sheet pile walls and bolted to them.

Accurate analysis of sheet walls is complicated by the interaction between the soil and the wall. In practice walls are not perfectly rigid as assumed in the free earth support method and it is important to consider the effects of wall flexibility. If the wall deforms this will influence the pressures mobilised between the soil and the wall and consequently the anchor force and moments in the wall.



Remember that it is important to ensure that the wall movements are compatible with the design assumptions.

6.3.2 Multiple anchors

Where there are relatively deep temporary excavations it is common to support the walls during construction by a system of bracing. This procedure is also used for permanent structures with the struts forming the floors of the basement. Alternatively the walls can be supported by multiple anchors.

A wall with several layers of struts or anchors will have increased restraint as each layer of anchors is added. Consequently the lateral deformations are limited and the retained soil is unlikely to attain failure. The situation is statically indeterminate and analysis is complex. The earth pressure that acts on the wall will depend on:

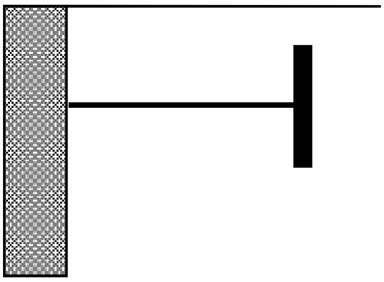
- relative stiffness of soil and wall
- anchor/strut spacing
- load-deformation response of the anchor or strut
- pre-stress (if any) in the anchor/strut during construction

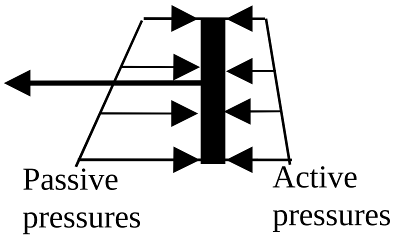
In practice empirical methods are used to estimate the pressures on the wall and forces in the strut, and these methods are based on actual measurements.

6.3.3 Anchor design

The anchor must be able to provide resistance equal to the required anchor force without excessive displacement of the anchorage towards the wall.

There are many anchoring systems used in practice. They rely on a combination of bearing pressures on the faces perpendicular to the anchor, and frictional forces between the anchor and the soil. The simplest is the vertical plate anchor.



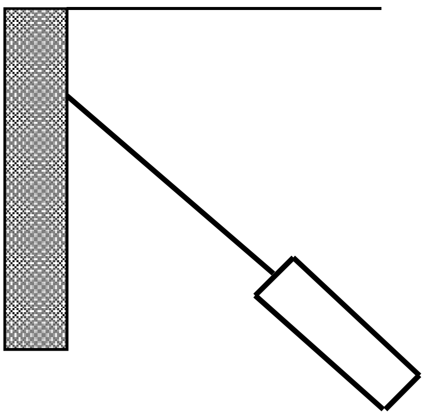


It is assumed that the resistance can be determined simply from the difference between the passive and active pressures on the two sides of the plate. For a plate of area, A, the anchor force is

$$T = (K_p \sigma'_v - K_a \sigma'_v) A \quad \text{/m of the wall}$$

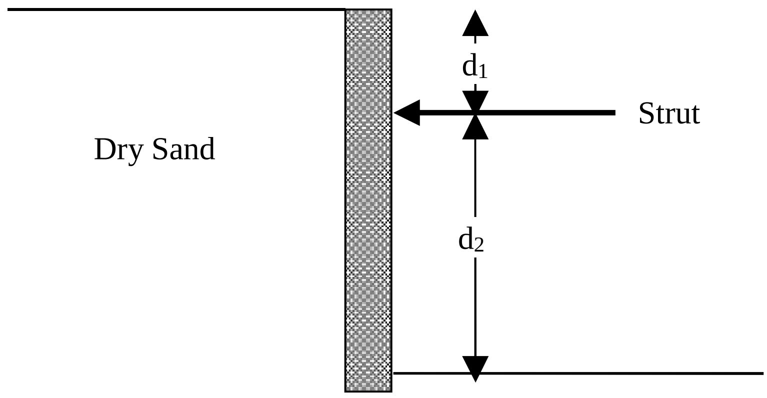
However, to mobilise the full passive pressure significant movement of the plate would be required. To reduce the movement the pressures should be factored as discussed above for the wall.

If the area of the plate anchor is large it will probably be more economic to use raked pile anchors. By installing the anchor at depth the normal stresses and hence the frictional resistance will be much greater than at the surface.



Example 1

Consider the limiting forces acting on a single strut supporting a wall retaining dry sand

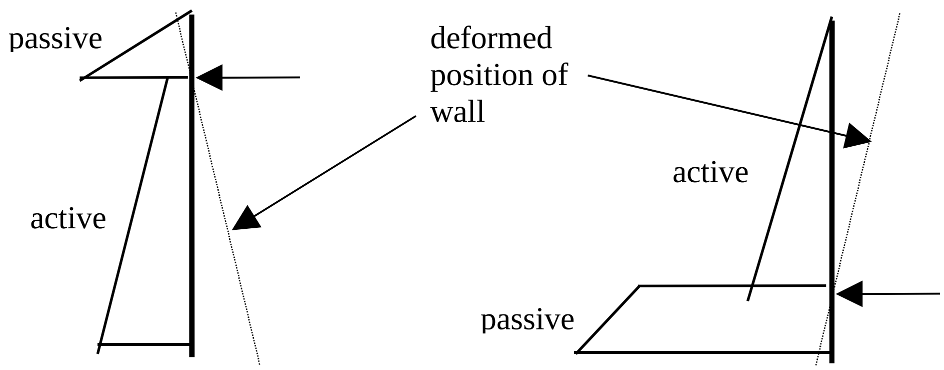


Analysis requires several assumptions

- Rigid wall
- Rigid (unyielding) strut
- Triangular active and passive pressures - no friction
  sufficient wall movements

There are two possible modes of failure depending on the position of the strut.

Strut near the surface — Strut near the base of the wall



Consider the limiting equilibrium of the wall. To eliminate the unknown strut force take moments about the strut.

Strut at surface $$\frac{1}{2}K_p\,\gamma\,d_1^2\,\frac{d_1}{3} = K_a\,\gamma\,d_1\,d_2\,\frac{d_2}{2} + \frac{1}{2}K_a\,\gamma\,d_2^2\,\frac{2d_2}{3}$$

Strut at base $$\frac{1}{2}K_a\,\gamma\,d_1^2\,\frac{d_1}{3} = K_p\,\gamma\,d_1\,d_2\,\frac{d_2}{2} + \frac{1}{2}K_p\,\gamma\,d_2^2\,\frac{2d_2}{3}$$

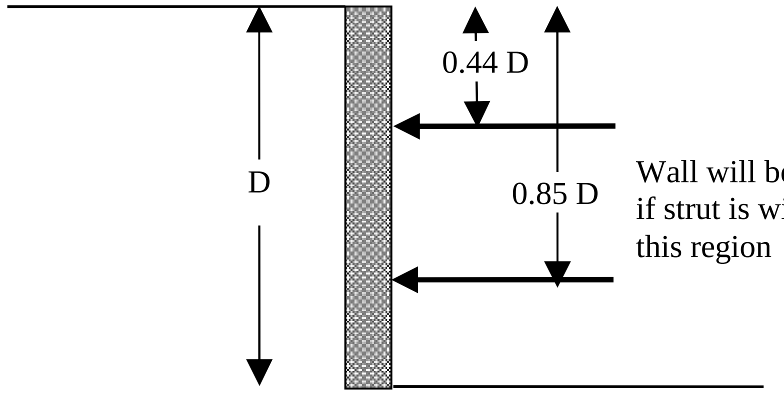
Noting that $K_a = 1 / K_p$ then after rearrangement we obtain for the strut near the surface

$$2\left(\frac{d_2}{d_1}\right)^3 + 3\left(\frac{d_2}{d_1}\right)^2 - K_p^2 = 0$$

and for the strut near the base of the wall

$$2\left(\frac{d_2}{d_1}\right)^3 + 3\left(\frac{d_2}{d_1}\right)^2 - K_a^2 = 0$$

For $\phi' = 30^o$ we obtain solutions for d_2/d_1 of 1.275 and 0.182, or if D is the total height of the wall d_1 / D of 0.44 and 0.85.



In practice several struts would probably be used because of serviceability concerns. As discussed above it is difficult to accurately assess the loads on walls with many struts, and failure of individual struts may occur. This simple analysis indicates where the struts could be positioned to avoid progressive failure. For example, if 3 struts are used and placed at depths of 0.25D, 0.5D and 0.75D then either the top two or the bottom strut can be removed without the wall failing.

In the analysis it has been assumed that the strut is unyielding, so that the wall rotates about the strut. To determine whether this is a reasonable assumption we need to check the force in the strut. For the strut near the surface and the wall at its limiting equilibrium

$$F = \frac{1}{2}K_p \gamma d_1^2 + K_a \gamma d_1 d_2 + \frac{1}{2}K_a \gamma d_2^2$$

For the example with $\phi' = 30^o$ we have found $d_1 = 0.44$ D, $d_2 = 0.56$ D and hence

$$F = 0.425\,\gamma\, D^2$$

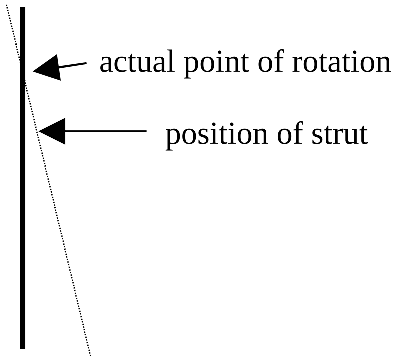
For a retained height of 2 m and $\gamma = 18$ kN/m^3 we find $F \approx 30$ kN/m

Let us assume that we have a 4 m wide trench with struts every 4 m and that the bracing is provided by steel (scaffolding) tubes 50 mm diameter by 5 mm thick.

The force in each strut will be $F = 4 \times 30 = 120$ kN and the strain $$\varepsilon = \frac{F}{A\,E} = 7 \times 10^{-4}$$

and the displacement $$\delta = 2.8 \text{ mm}$$

Now a displacement of this magnitude is sufficient to cause the stress to drop to active conditions at the prop. The wall movements are thus not compatible with our initial assumption. The effect will be for the point of rotation to move up the wall and for premature failure to occur.



This is a simple illustration of the importance of accounting for soil-structure interaction if possible

Example 2

An underground development is planned adjacent to existing structures which can be considered to apply a uniform stress of 100 kPa to the soil surface.

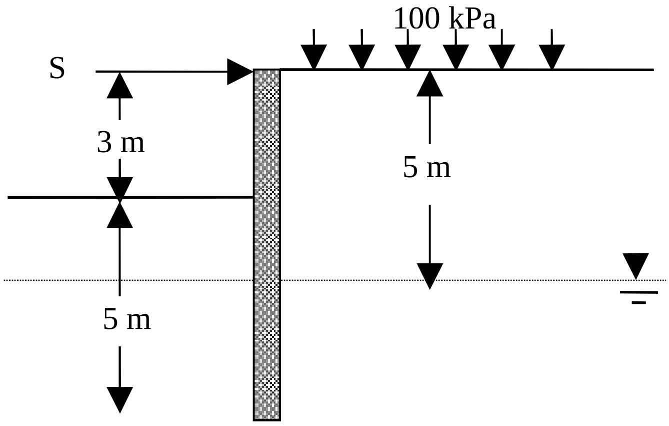
To try to minimise settlement of the surrounding structures it is proposed to construct rigid diaphragm walls, 8 m deep in the uniform sandy soil which has properties $c' = 0$, $\phi' = 30^o$, $\gamma_{dry} = 18$ kN/m^3, $\gamma_{sat} = 20$ kN/m^3. The water table is at a depth of 5 m below the ground surface.

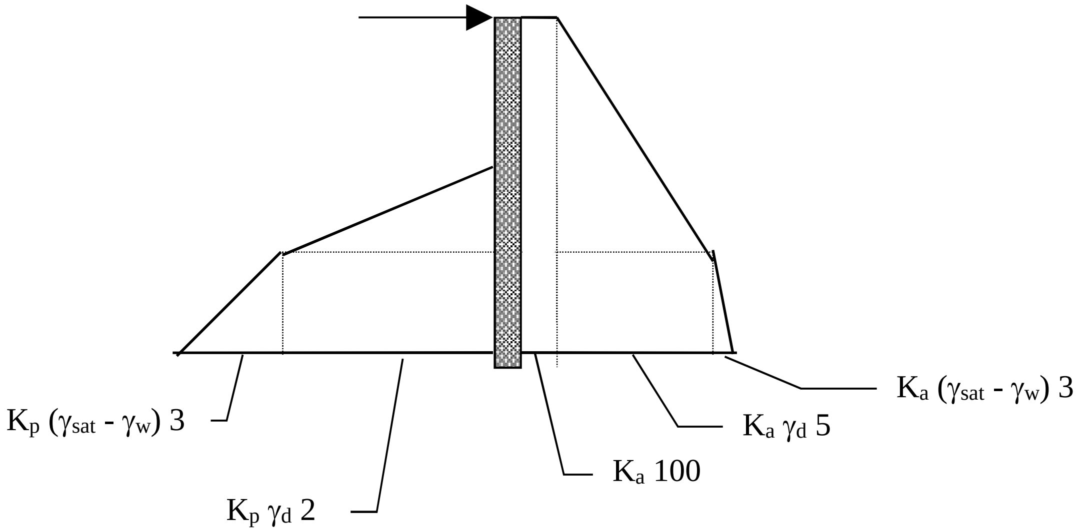
The proposed "top down" construction procedure is to place a slab at ground level to act as a strut to support the wall, and then excavate a further 3 m before placing the next floor slab. Determine the horizontal force per metre in the slab at the ground surface when the excavation has reached 3 m, just before the next slab is placed.

Assumptions:

Rigid wall
Free earth support method
Triangular pressure distributions
Movements sufficient for active pressures to develop

Step 1: Determine the pressures acting on the wall





Step 2 - Take moments about the strut

Moment equilibrium is required to find the factor applied to K_p. The factor is unknown because this is not the limiting case

$$K_a\left[100\times 8\times 4+\frac{1}{2}\gamma_d\,5^2\,\frac{2}{3}5+\gamma_d\,5\times 3\times 6.5+\frac{1}{2}(\gamma_{sat}-\gamma_w)3^2\times 7\right]$$
$$=\frac{K_p}{F}\left[\frac{1}{2}\gamma_d\,2^2\,\frac{13}{3}+\gamma_d\,2\times 3\times 6.5+\frac{1}{2}(\gamma_{sat}-\gamma_w)3^2\times 7\right]$$

Given $\gamma_d = 18$, $\gamma_w = 10$, $\gamma_{sat} = 20$ kN/m^3 and $\phi' = 30^o$, $K_a = 0.3333$, $K_p = 3$

$$2006.7 = \frac{K_p}{F}1173$$

$$F = 1.754$$

Step 3 - Force equilibrium to obtain strut force

$$S \;=\; K_a[100\times 8 + 0.5\times 18\times 5^2 + 18\times 5\times 3 + 0.5\times 10\times 3^2] - \frac{K_p}{F}[0.5\times 18\times 2^2 + 18\times 2\times 3 + 0.5\times 10\times 3^2]$$

$$S \;=\; 123.3 \text{ kN/m}$$

Sheet Retaining Wall problems

1. A quay wall has been built from sheet piling and is to retain 8 m of sand which has strength properties c' = 0, ϕ' = 33°, a bulk unit weight of 16 kN/m^3, and a saturated unit weight of 18 kN/m^3. The wall is anchored 1 m below the top of the wall and has a total length of 15 m. The water table on both sides of the wall is at a level 4 m below the top of the wall.

   Cargo is to be stored on the quay, which may be assumed to apply a uniform surcharge to the surface of the sand. Determine the maximum magnitude of the surcharge loading that can be applied by the cargo so that the factor of safety applied to the passive pressures does not fall below 1.3. It may be assumed that the wall movements are sufficient for the active pressures to be fully mobilised.

   Calculate the maximum moment in the sheet pile wall when this maximum surcharge is applied.

   Explain why such a low factor of safety may give rise to problems with the quay.

2. For the quay wall described in question 1, calculate the factor of safety F_ϕ (applied to tanϕ' affecting both active and passive pressures) if the surcharge is 25 kPa.

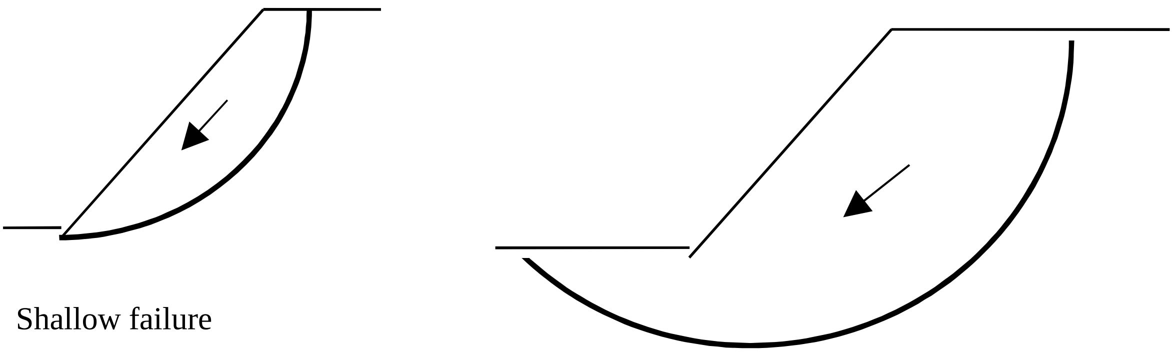
3. A cantilevered wall has been used to retain 2 m of a sandy soil, which has strength properties c' = 0, ϕ' = 35°, and a dry unit weight of 18 kN/m^3. The wall penetrates 4 m below the base of the retained soil, into the same sandy soil.

   It is proposed to raise the level of the retained soil for a new development, by adding fill with a dry unit weight of 14 kN/m^3. Calculate the maximum height of fill that can be added if the factor of safety against passive failure is not to fall below 1.5. The fill may be assumed to apply a uniform surcharge to the retained soil.

Chapter 7: SLOPE STABILITY

7.1 Circular failure mechanisms

When slope failures are investigated it is often found that failure occurs by a rotational slip along an approximately circular failure surface, as shown below. This observation provides a basis for several methods used to assess the stability of slopes.



Shallow failure

Deep-seated failure

7.1.1 Factor of Safety

When performing stability analyses we generally are not interested in failure as such, failure is a final limiting state that we do not want the soil to reach. We are usually more interested in the stability of the unfailed soil, and in determining a factor of safety, F, for the unfailed soil. Factors of safety need to be considered carefully in soils. For example, in the design of retaining walls for active conditions, as the factor of safety increases so will the force that needs to be provided.

To determine the factor of safety we assume that only some part of the frictional and cohesive forces have been mobilised, so that on the assumed failure plane the soil is not at a state of failure.

At failure the stresses are given by the Mohr-Coulomb criterion as

$$\tau = c + \sigma \tan \phi$$

At stress states remote from failure the mobilised shear stress, τ_{mob}, is assumed to be given by

$$\tau_{mob} = \frac{c}{F} + \sigma \left(\frac{\tan \phi}{F} \right)$$

or

$$\tau_{mob} = c_m + \sigma \tan \phi_m$$

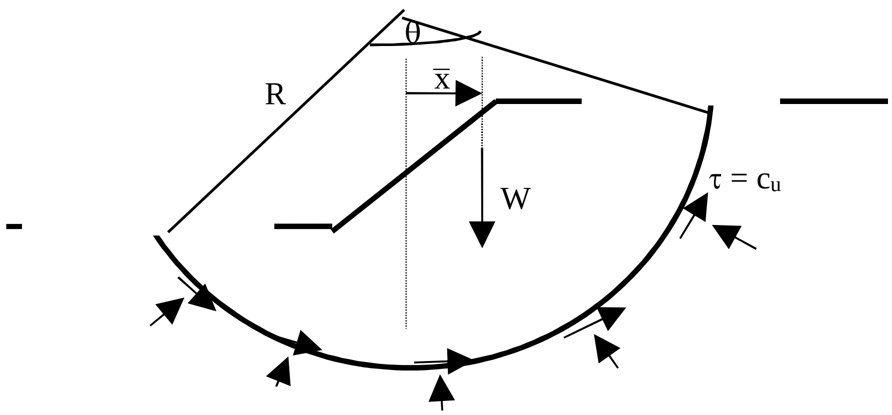
where c_m $(= \frac{c}{F})$ is known as the mobilised cohesion

ϕ_m $\quad$ (= $\tan^{-1}\left[\frac{\tan\phi}{F}\right]$) is known as the mobilised friction angle

Note that it is assumed that both components of strength are divided by the same factor F.

7.1.2 Short term stability of soils with $\phi_u = 0$

For clayey soils that remain undrained in the short term, and that have strength parameters $c = c_u$, $\phi = \phi_u = 0$, the analysis is straightforward. Consider the slope shown below and assume that the shear strength has been reduced by a factor F, so that $c = c_u/F$. Failure will then occur along a circular arc of radius R as indicated in the figure.



If the soil is homogeneous, then by considering moment equilibrium about the centre of the assumed slip circle it can be seen that

$$W\bar{x} = \frac{R^2\,\theta\,c_u}{F}$$

where θ is the angle subtended by the failure circle at its centre
W is the weight of the rotating body
x is the centre of mass of the rotating soil body.

rearranging we obtain

$$F = \frac{R^2\,\theta\,c_u}{W\bar{x}} = \frac{\text{Resisting Moment}}{\text{Disturbing Moment}}$$

- The factor of safety of the slope can then be determined by considering a range of failure surfaces (slip circles) with different centres and radii to find the slip circle that gives the minimum value of F.

- Because this analysis is an undrained, total stress analysis, the possibility that tension cracks may form, and that these cracks may fill with water must be considered. Water in a tension crack will provide an additional disturbing moment and can significantly reduce the factor of safety.

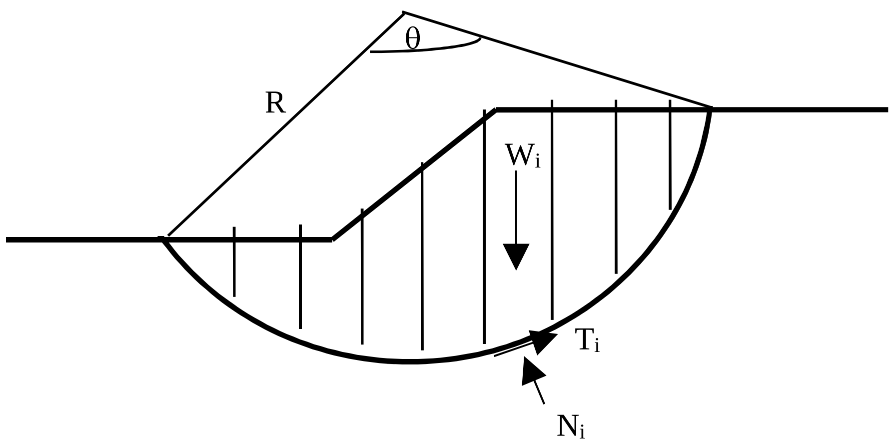
- The analysis can be easily modified to account for non-homogeneous soil deposits.

- To obtain the minimum value of F computer methods are generally used. These methods require the soil to be split into a series of slices. This approach is also used for the more general analysis discussed below.

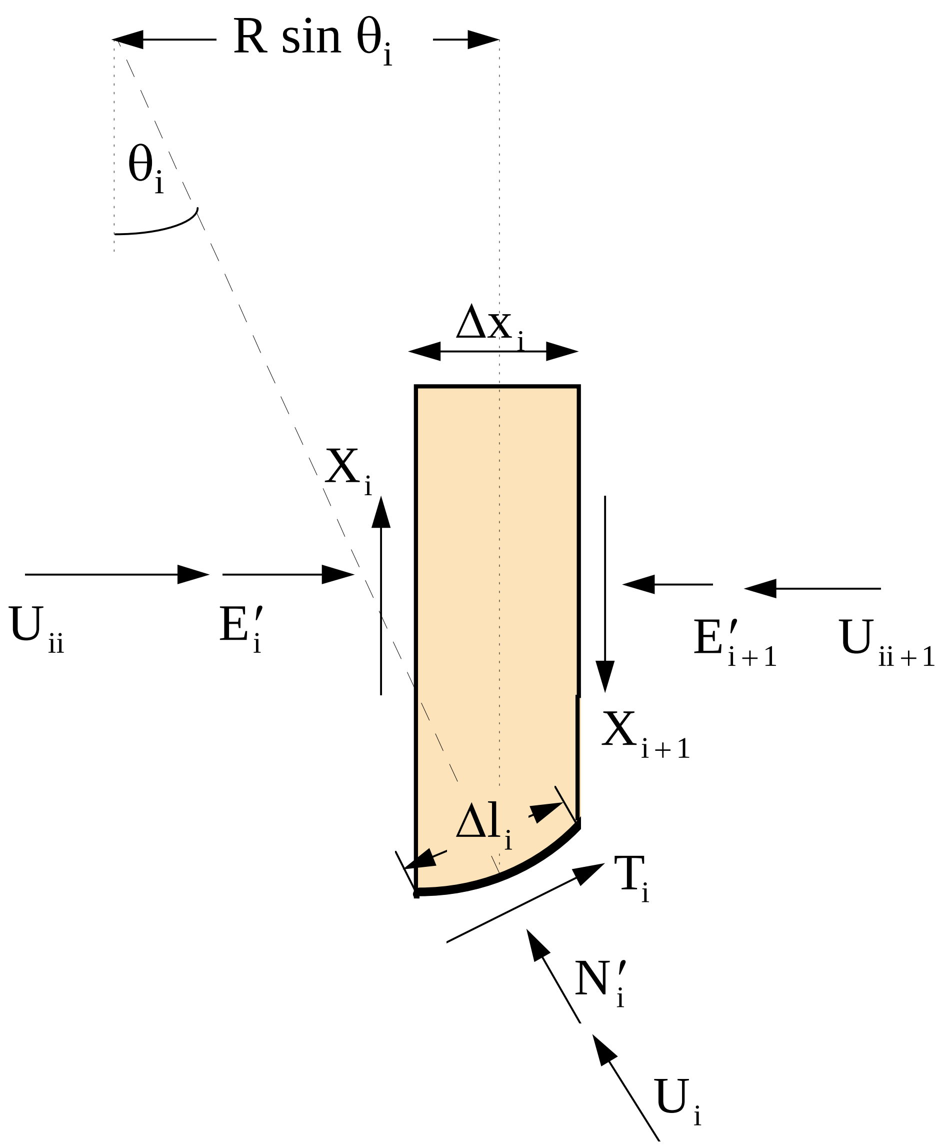
7.1.3 The Method of Slices

For soils which have $\phi \neq 0$ a more elaborate analysis is required. The same general method can be used for both undrained (total stress) and effective stress analysis.

Let us consider the *effective* stress analysis of the slope shown below



The forces acting on the i th slice are as shown below



Noting that the internal forces between the slices will cancel when taking moments we obtain

$$\text{Restoring moment} = R \sum_{i=1}^{n} T_i$$

Assuming a Mohr-Coulomb failure criterion the restoring moment can be written

$$= R \sum_{i=1}^{n} \left[\frac{c'_i \, \Delta l_i}{F} + N'_i \left(\frac{\tan \phi'_i}{F} \right) \right]$$

$$\text{Overturning moment} = R \sum_{i=1}^{n} W_i \sin \theta_i$$

The factor of safety F is then given by

$$F = \frac{\text{Resisting Moment}}{\text{Overturning Moment}} = \frac{\sum_{i=1}^{n}[c'_i \Delta l_i + N'_i \tan\phi'_i]}{\sum_{i=1}^{n} W_i \sin\theta_i}$$

When an undrained (total stress) analysis is being performed there are essentially the same forces acting on the slices. However, in a total stress analysis the forces due to the water pressures U_i, U_{ii} are not required and only the total forces E_i, N_i need to be considered. The shear force on each slice is given by the total stress failure criterion and the restoring moment can be written

$$= R \sum_{i=1}^{n} [\frac{c_{ui} \Delta l_i}{F} + N_i \left(\frac{\tan\phi_{ui}}{F}\right)]$$

To calculate the factor of safety the normal force must be known. By considering the force equilibrium of the slice it can be seen that the force N'_i will depend on the interslice forces X_i and E'_i. Unfortunately N cannot be simply determined from consideration of equilibrium (the slice is statically indeterminate) and it is necessary to make an assumption. There are several methods of determining the factor of safety, each method involving different assumptions. The two simplest and most commonly used methods and their assumptions are considered below.

7.1.3.1 The Swedish method of slices

In this method it is assumed that the resultant of the interslice forces acts in a direction perpendicular to the normal force N.

Then resolving parallel to N we obtain

$$N_i = N'_i + U_i = W_i \cos\theta_i$$

where the force $U_i = u_i \Delta l_i$ and u_i is the pore pressure at the centre of the slice on the assumed failure circle

Substitution of the expression for N_i into the equation for the factor of safety gives

Effective stress analysis

$$F = \frac{\sum_{i=1}^{n} [c'_i \Delta l_i + (W_i \cos\theta_i - U_i) \tan\phi'_i]}{\sum_{i=1}^{n} W_i \sin\theta_i}$$

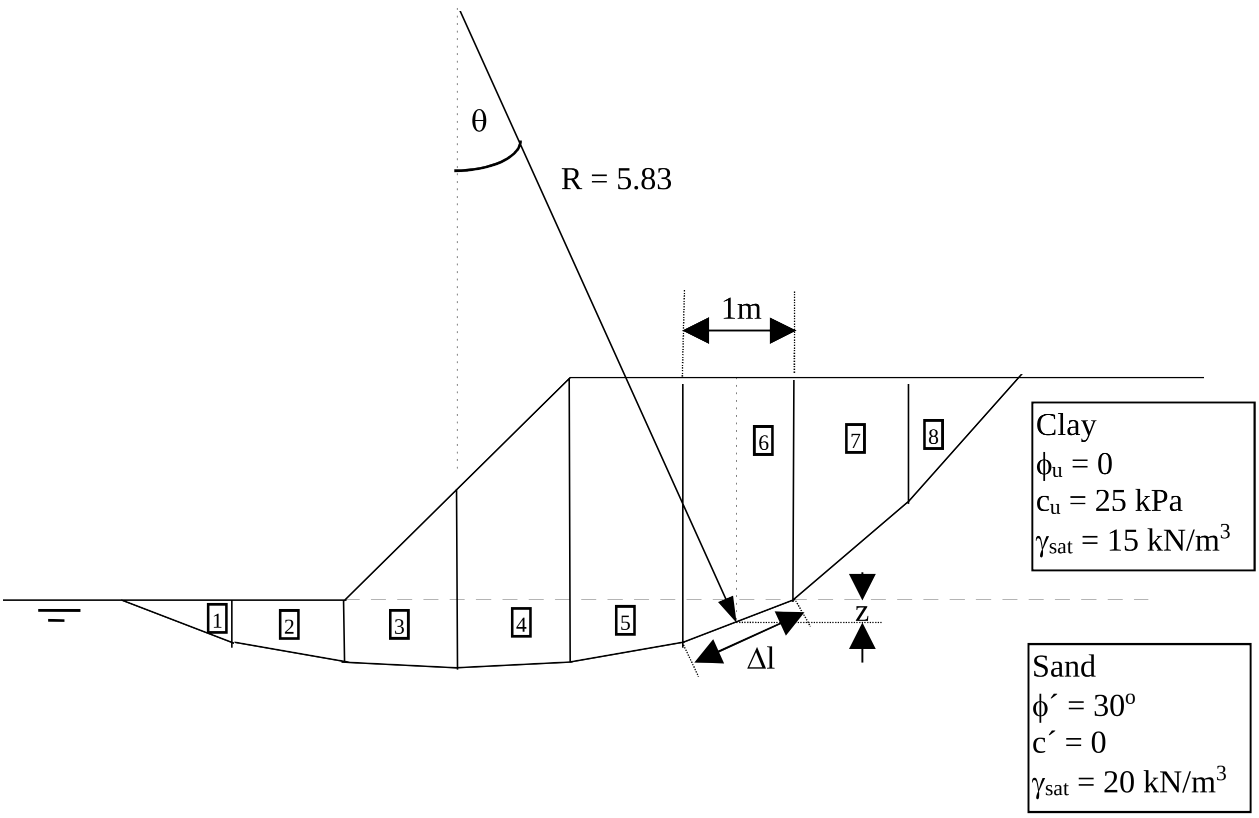
Undrained analysis

$$F = \frac{\sum_{i=1}^{n} [c_{ui} \Delta l_i + W_i \cos\theta_i \tan\phi_{ui}]}{\sum_{i=1}^{n} W_i \sin\theta_i}$$

Example – Swedish method

Determine the short term stability of the slope shown below, given that the slope was initially submerged with water and that the water level has now been drawn down to the level of the top of the sand.

Initially the centre and radius of the failure plane must be assumed. The calculations presented below are for one such assumption. However, to find the factor of safety of the slope, a number of centres and radii will need to be considered to find the combination that gives the minimum factor of safety.



Example calculations for slice 6

1. $\Delta l_i = 1.11$ m measured from figure
2. $x_i = 2.5$ m measured from figure
3. $\theta_i = \sin^{-1}(2.5/5.83) = 25.4^o$ or measure from figure. **Note that θ is positive for slices giving positive overturning moments**
4. $W_i = A\,\gamma = 1 \times 2 \times 15 + 1 \times 0.268 \times 20 = 35.36$ kN/m
5. $N_i = W_i \cos\theta_i = 35.36 \cos(25.4) = 31.94$ kN/m
6. $U_i = \gamma_w\, z\, \Delta l_i = 9.81 \times 0.268 \times 1.11 = 2.92$ kN/m
7. $N'_i = N_i - U_i = 29.02$ kN/m
8. $W_i \sin\theta_i = 35.36 \sin(25.4) = 15.17$ kN/m
9. $T_i = C'_i + N'_i \tan\phi'_i = 0 + 29.02 \tan(30) = 16.75$ kN/m

The results for all the slices can be similarly evaluated and tabulated as shown below

	θ (°)	Δl (m)	u (kPa)	U (kN/m)	W (kN/m)	N (kN/m)	N´ (kN/m)	C (kN/m)	Wsinθ (kN/m)	T (kN/m)
1	-25.4	1.107	2.628	2.910	5.357	4.84	1.93	-	-2.30	1.115
2	-14.9	1.035	6.227	6.646	12.70	12.27	5.822	-	-3.77	3.362
3	-4.93	1.004	7.942	7.974	23.69	23.60	15.63	-	-2.03	9.024
4	4.93	1.004	7.942	7.974	38.69	38.54	30.57	-	3.317	17.65
5	14.89	1.035	6.227	6.646	42.70	41.26	34.81	-	10.98	20.10
6	25.4	1.11	2.628	2.92	35.36	31.94	29.02	-	15.17	16.75
7	36.87	1.250	-	-	24.96	19.96	-	31.25	14.98	31.25
8	50.53	1.572	-	-	10.62	6.755	-	39.30	8.20	39.30

where

$U = u\,\Delta l$ $\qquad$ $N = W\cos\theta$ $\qquad$ $N' = N - U$

$C = c'\,\Delta l$ in the sand (Effective stress analysis)

$C = c_u\,\Delta l$ in the clay (Undrained, Total stress analysis)

For sand $T = C' + N'\tan\phi'$ but $c' = 0$ therefore $T = N'\tan\phi'$

For clay $T = C + N\tan\phi_u$ but $\phi_u = 0$ therefore $T = C$

$$F = \frac{\text{Resisting Moment}}{\text{Disturbing Moment}} = \frac{\sum T}{\sum W\sin\theta} = \frac{138.56}{44.54} = 3.11$$

If a load of 100 kN/m is placed on top of slice 6, only the calculations for slice 6 are affected and these become

$W = 35.36 + 100 \times 1 = 135.36$ $\qquad$ Slice is 1 m wide

$N = W\cos\theta = 122.47$

$N' = N - U = 119.36$

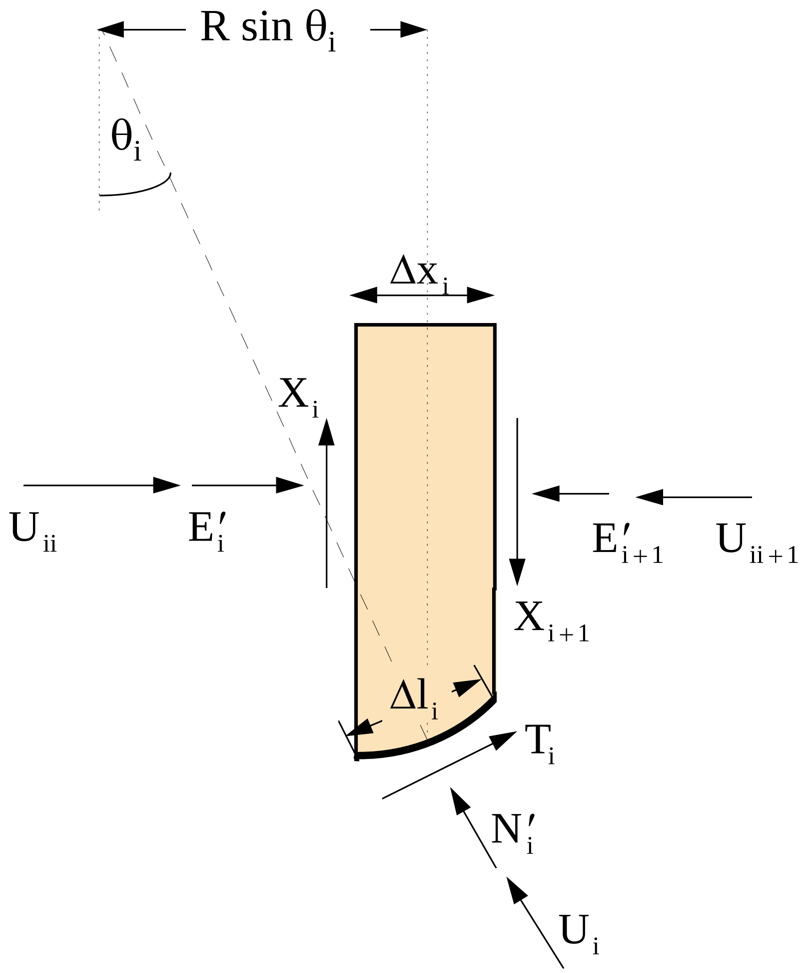
$W\sin\theta = 58.06$

$T = N'\tan\phi' = 68.9$

$$F = \frac{\sum T}{\sum W\sin\theta} = \frac{190.7}{87.44} = 2.18$$

7.1.3.2 Bishop's simplified method of slices

In this method it is assumed that the vertical interslice forces, X_i, X_{i+1}, are equal.



Then resolving vertically we obtain

$$W_i = T_i \sin\theta_i + N'_i \cos\theta_i + u_i \Delta x_i$$

We know that the mobilised strength T_i is given by

$$T_i = \frac{c'_i \Delta l_i}{F} + \frac{N'_i \tan\phi'_i}{F}$$

substituting this into the previous expression, noting that $\Delta x_i = \Delta l_i \cos\theta_i$ and rearranging gives

$$N'_i = \frac{W_i - u_i \Delta x_i - (1/F)\, c'_i \Delta x_i \tan\theta_i}{\cos\theta_i \left[1 + \dfrac{\tan\phi'_i \tan\theta_i}{F} \right]}$$

Let $$M_i(\theta) = \cos\theta_i \left[1 + \tan\theta_i \frac{\tan\phi'_i}{F}\right]$$

Then substitution of the expression for N'_i into the equation for the factor of safety, F, that is

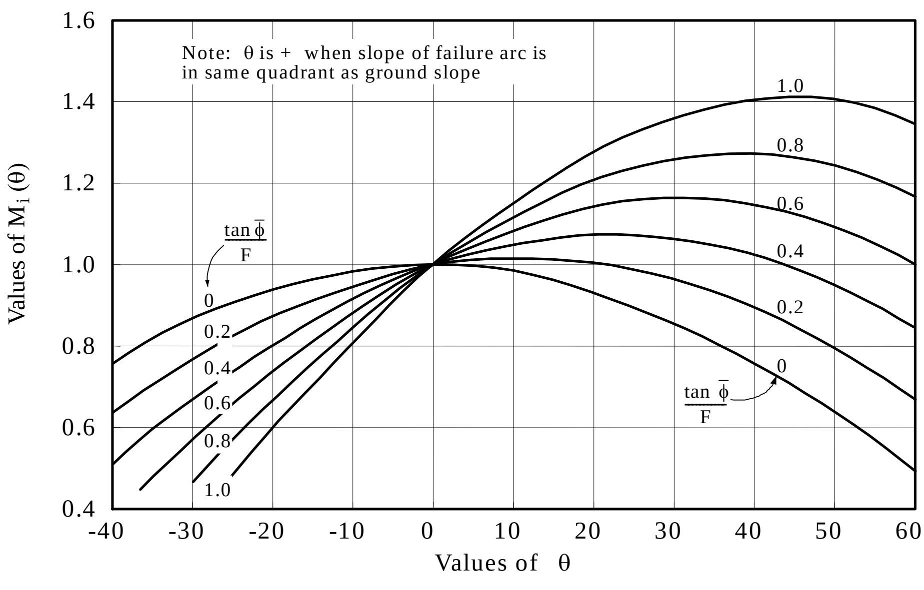
$$F = \frac{\text{Resisting Moment}}{\text{Overturning Moment}} = \frac{\sum_{i=1}^{n} [c'_i \Delta l_i + N'_i \tan\phi'_i]}{\sum_{i=1}^{n} W_i \sin\theta_i}$$

gives

$$F = \frac{\sum_{i=1}^{n} (c'_i \Delta x_i + (W_i - u_i \Delta x_i) \tan \phi'_i) \left[\frac{1}{M_i(\theta)}\right]}{\sum_{i=1}^{n} W_i \sin \theta_i}$$

Note that in the Bishop's simplified method the factor of safety appears in both sides of the equation, as it is included also in the M_i (θ) term. Thus to obtain solutions an iterative approach is needed. This means that you need to assume a value for the factor of safety before evaluating the summations to give a new factor of safety. It is found that the factor of safety converges rapidly.

A chart is shown below (p 183 in Data Sheets) which simplifies hand calculation by giving values for M_i for a range of values of θ and ϕ. Note that the sign of θ is important, as noted above **θ is positive for slices giving positive overturning moments**.



GRAPH FOR DETERMINATION OF $M_i(\theta)$

For undrained (total stress) analysis the procedure is similar and the factor of safety becomes

$$F = \frac{\sum_{i=1}^{n} (c_{ui}\ \Delta x_i + W_i \tan \phi_{ui}) \left[\frac{1}{M_i(\theta)} \right]}{\sum_{i=1}^{n} W_i \sin \theta_i}$$

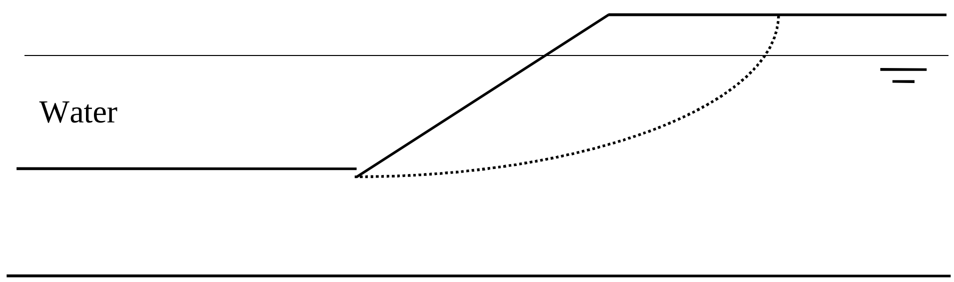
where

$$M_i(\theta) = \cos \theta_i \ [\, 1 + \tan \theta_i \frac{\tan \phi_{ui}}{F} \,]$$

When $\phi_u = 0$, $M_i(\theta) = \cos \theta_i$ and Bishop's simplified method gives an identical answer to the Swedish method. However, in general the methods give different answers. Both methods tend to underestimate the factor of safety estimated by more accurate analyses. Bishop's method is the more (theoretically) accurate and is more widely used.

7.1.4 Important points

- Numerical analyses are required to determine the most critical slip circle
- Both the Swedish and Bishop's methods can be used for undrained (total stress) analysis, and for effective stress (usually drained) analysis. In many situations the slope analysis requires combinations of drained and undrained analyses. For instance, the short term stability of a slope containing layers of clay and sand would require a total stress (undrained) analysis in the clay and an effective stress (drained) analysis in the sand.
- In undrained (total stress) analyses the undrained parameters c_u, ϕ_u must be used in the expressions for the factor F, and the pore pressure term is ignored.
- The effect of vertical surface loads can be included in the analysis by adding the vertical force on a slice to the weight of that slice.
- For submerged slopes, such as shown below, the water must be included in the analysis



There are two basic options

1. Treat the water as a material with no strength, but having a unit weight γ_w. Effectively the water is providing a vertical load onto the underlying slices.

2. Use the submerged unit weight γ' (= γ_{sat} - γ_w) for all the soil below the surface of the water. This approach can only be used in a total stress analysis if $\phi_u = 0$.

- The factor of safety is very sensitive to pore pressures in the ground. The pore pressures may be determined from

1. A piezometric surface. The pore pressures are determined assuming that $u = \gamma_w z$, where z is the distance below the piezometric surface. This is exact when there is no flow and when the flow is horizontal.

2. A flow net. In numerical analyses a grid of pore pressure values can be set up.

Example – Bishop's simplified method

For the same slope and slices as used before the calculations for slice 6 become

Δx_i = 1.0 m measured from figure
x_i = 2.5 m measured from figure
θ_i = $\sin^{-1}(2.5/5.83) = 25.4^o$ or measure from figure. **Note that θ is positive for slices giving positive overturning moments**
$W_i = A\gamma = 1 \times 2 \times 15 + 1 \times 0.268 \times 20 = 35.36$ kN/m
$W_i \sin\theta_i = 35.36 \sin(25.4) = 15.17$ kN/m
$u_i = \gamma_w z = 9.81 \times 0.268 = 2.628$ kN/m
$c_i\Delta x_i + (W_i - u_i\Delta x_i)\tan\phi_i = 0 \times 1 + (35.36 - 2.628 \times 1)\tan 30^o = 18.9$ kN/m **Note that it is ϕ the friction angle, not θ in this calculation**
Now assume a factor of safety, say F = 3
$M_i = \cos\theta_i(1 + \tan\theta_i \tan\phi_i / F) = \cos(25.4)\times(1+\tan(25.4)\times\tan(30)/3) = 0.986$
Or read M_i off the chart for θ = 25.4 and (tan ϕ)/F = tan(30)/3 = 0.19

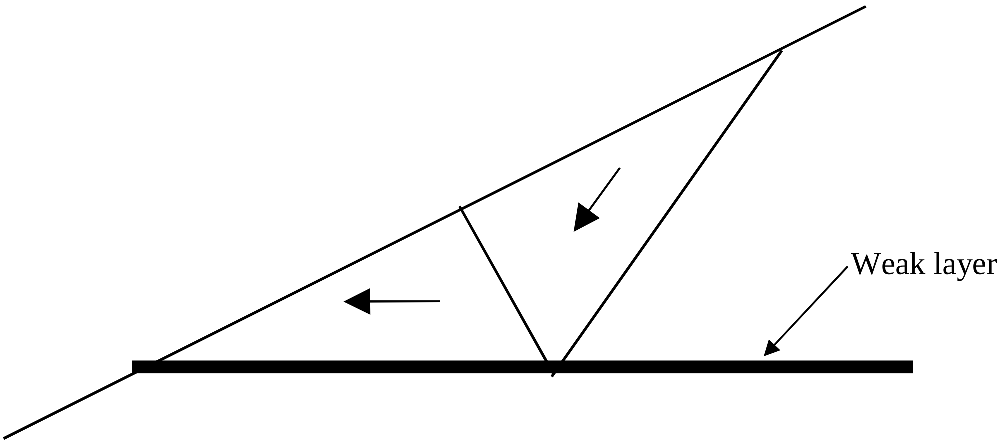
The results for all the slices can be similarly evaluated and tabulated as shown below

	θ (°)	Δx (m)	u (kPa)	W (kN/m)	Wsinθ (kN/m)	cΔx (kN/m)	T* = cΔx + (W-uΔx)tan ϕ (kN/m)	M	T*/M
1	-25.4	1.0	2.628	5.357	-2.30	-	1.58	0.821	1.92
2	-14.9	1.0	6.227	12.70	-3.77	-	3.74	0.917	4.08
3	-4.93	1.0	7.942	23.69	-2.03	-	9.09	0.980	9.28
4	4.93	1.0	7.942	38.69	3.317	-	17.75	1.013	17.52
5	14.89	1.0	6.227	42.70	10.98	-	21.06	1.016	20.73
6	25.4	1.0	2.628	35.36	15.17	-	18.9	0.986	19.17
7	36.87	1.0	-	24.96	14.98	25.0	25	0.800	31.26
8	50.53	1.0	-	10.62	8.20	25.0	25	0.636	39.30

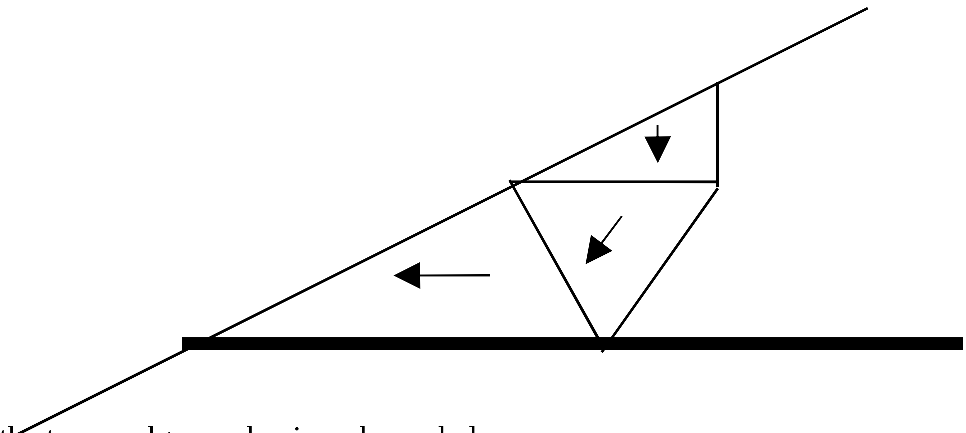
$F = \dfrac{\sum T^*/M}{\sum W\sin\theta} = \dfrac{143.3}{44.54} = 3.22$. Then using the updated F=3.22 re-evaluate M and T*/M until the solution converges. In this problem this gives F = 3.25.

7.2 Multiple wedge failure mechanisms

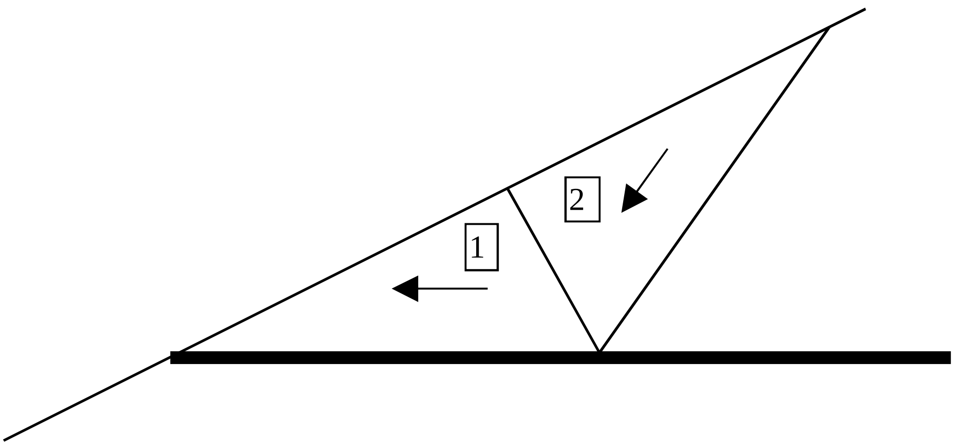
If the soil profile contains weak, usually clay, layers the failure plane may coincide with the weak layer, and analysis of circular failure mechanisms may be inappropriate. In this situation it is often assumed that the failure mechanism consists of wedges of soil moving relative to one another. For example, with a weak horizontal layer the 2 wedge mechanism shown below is a possible failure mechanism:



In some cases more complex mechanisms need to be considered involving 3 or more wedges, for example



Consider the two wedge mechanism shown below



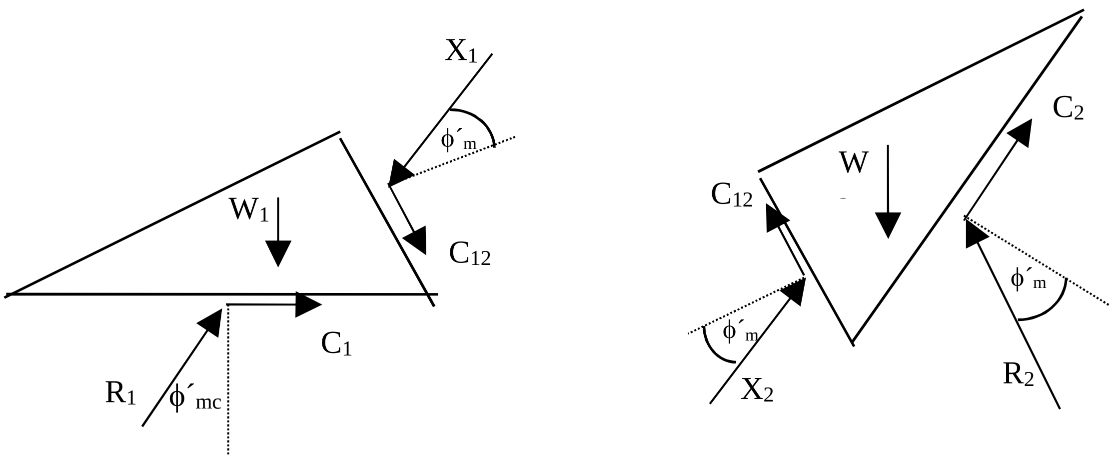
When the slope fails the strength mobilised **between** the two wedges is given by the failure criterion of the soil. However, when the slope is remote from failure the mobilised strength between the two wedges is likely to be different from the mobilised strength on the base of the wedges. The mobilised strength between the wedges may range from zero to that given by the parameters c_m, ϕ_m, giving the mobilised strength on the base of the wedges.

For practical calculations for soil structures that are remote from failure it is often assumed that a median value between 0 and c_m, ϕ_m is appropriate, so that between the wedges

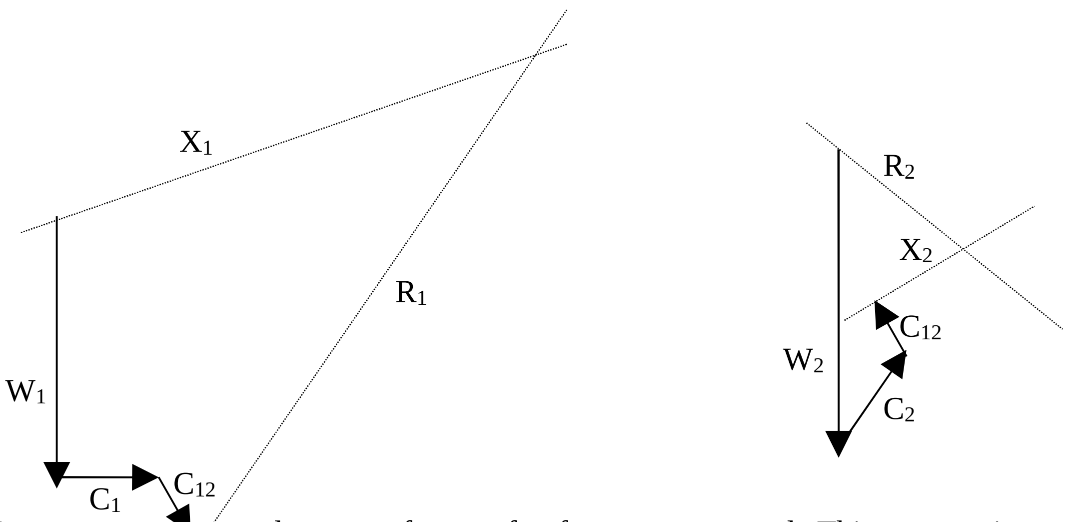
$$c^* = \frac{c_m}{2} \qquad \phi^* = \frac{\phi_m}{2}$$

However, in the limit when F = 1, the mobilised strength must be the same everywhere. It is therefore convenient analytically to assume that the maximum mobilised strength is the same on all the assumed failure planes.

Now if a value of F is assumed the forces acting on the two wedges are as shown below

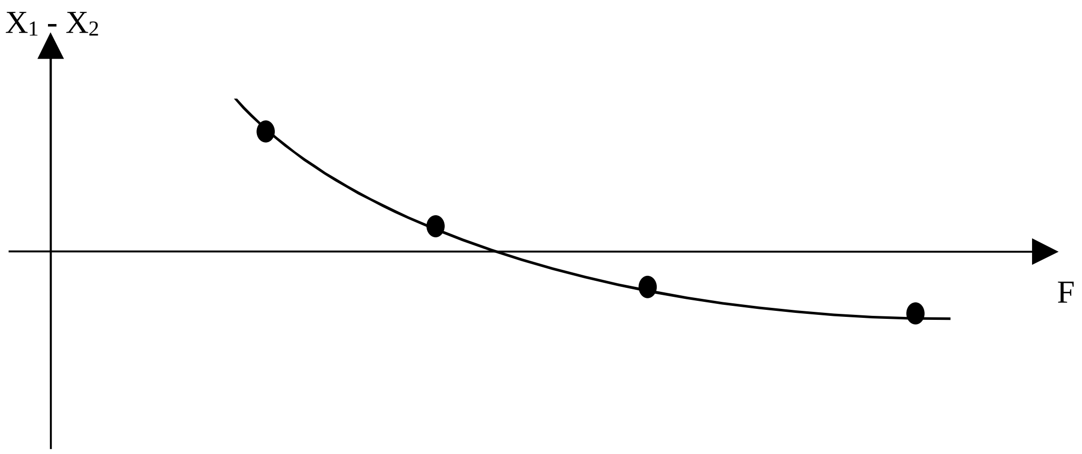


The force polygons can then be constructed



To construct these polygons a factor of safety was assumed. This assumption affects the magnitude of the cohesion forces C_1, C_{12}, C_2 and the mobilized angles of friction.

If the chosen value of the factor of safety is correct the inter-wedge resultant forces (X_1 and X_2) will be equal and opposite, as required for equilibrium. Because the initial value of F was a guess, the inter-wedge forces are unlikely to be equal. To determine the correct factor of safety the calculations must be repeated with different values of F and interpolation used to determine the true factor of safety, *for the assumed mechanism.*

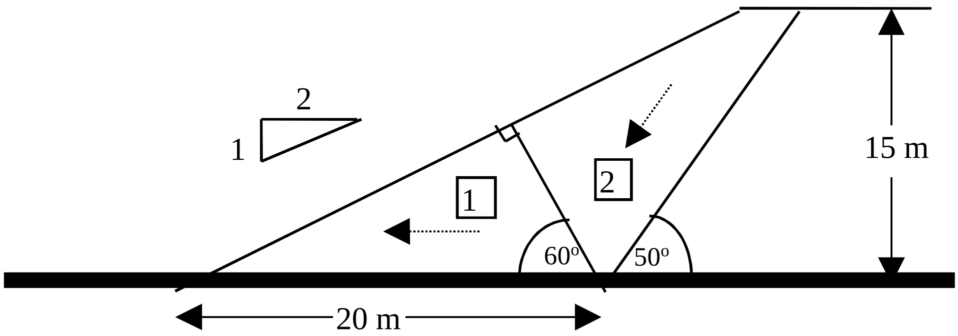


Note:

- the calculated factor of safety is not necessarily the factor of safety of the slope. To determine this all the possible mechanisms must be considered to determine the mechanism giving the lowest factor of safety.
- In any analysis the appropriate parameters must be used for c and ϕ. In an undrained analysis (short term in clays) the parameters are c_u, ϕ_u with total stresses, and in an effective stress analysis (valid any time if pore pressures known) the parameters are c', ϕ' used with the effective stresses.
- In an effective stress analysis if pore pressures are present the forces due to the water must be considered and if necessary included in the inter-wedge forces.

Example – wedge analysis

The figure below shows a slope that has been created by dumping a clayey sand (γ_{bulk} = 18 kN/m^3) onto a soil whose surface has been softened to create a thin soft clay layer. If the shear strength parameters of the clayey sand are $c' = 0$, $\phi' = 30^o$, and the undrained strength of the softened clay layer is 40 kPa, determine the short term factor of safety of the slope. Assume that the failure mechanism is as shown below.



1. Calculate areas:

$A_1 = 86.6\ m^2$ $A_2 = 115.6\ m^2$

2. Assume Factor of Safety

F = 2

3. Calculate c, ϕ parameters

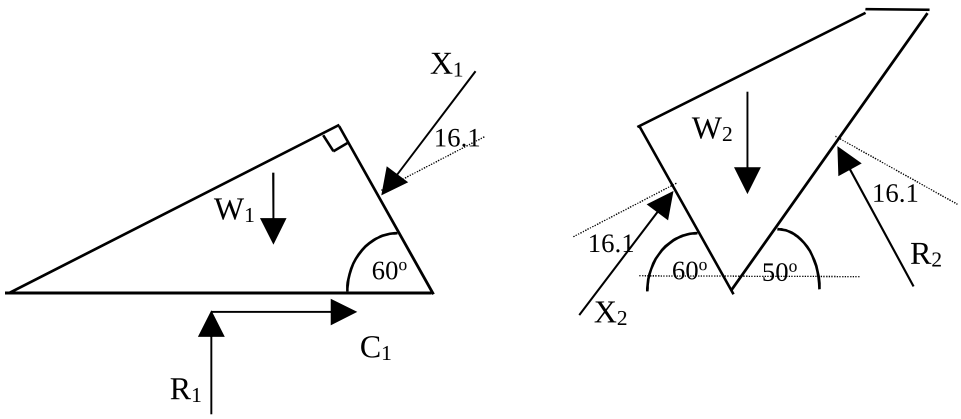
Weak layer $c_m = c_u/F = 40/2 = 20$ kPa, $\phi_m = 0$

Clayey sand $c_m = 0$, $\phi'_m =$

4. Calculate known forces

$W_1 = 86.6 \times 18 = 1558.8$ kN/m $W_2 = 115.6 \times 18 = 2080$ kN/m
$C_1 = 20 \times 20 = 400$ kN/m

5. Draw force diagrams



For Block 1: Resolving horizontally gives $X_1 \cos(16.1+30) = C_1$

$X_1 = 576.9$ kN/m

For Block 2: Resolving horizontally gives $X_2 \cos(16.1+30) = R_2 \cos(16.1+40)$

$X_2 = 0.80\, R_2$

Resolving vertically gives $W_2 = X_2 \sin(46.1) + R_2 \sin(56.1)$

$X_2 = 1186.9$ kN/m

Repeat for F = 1.5 (c_m = 26.67 kPa, ϕ'_m = 21.05°)

$X_1 = 848.5$ kN/m

$X_2 = 0.77\, R_2$

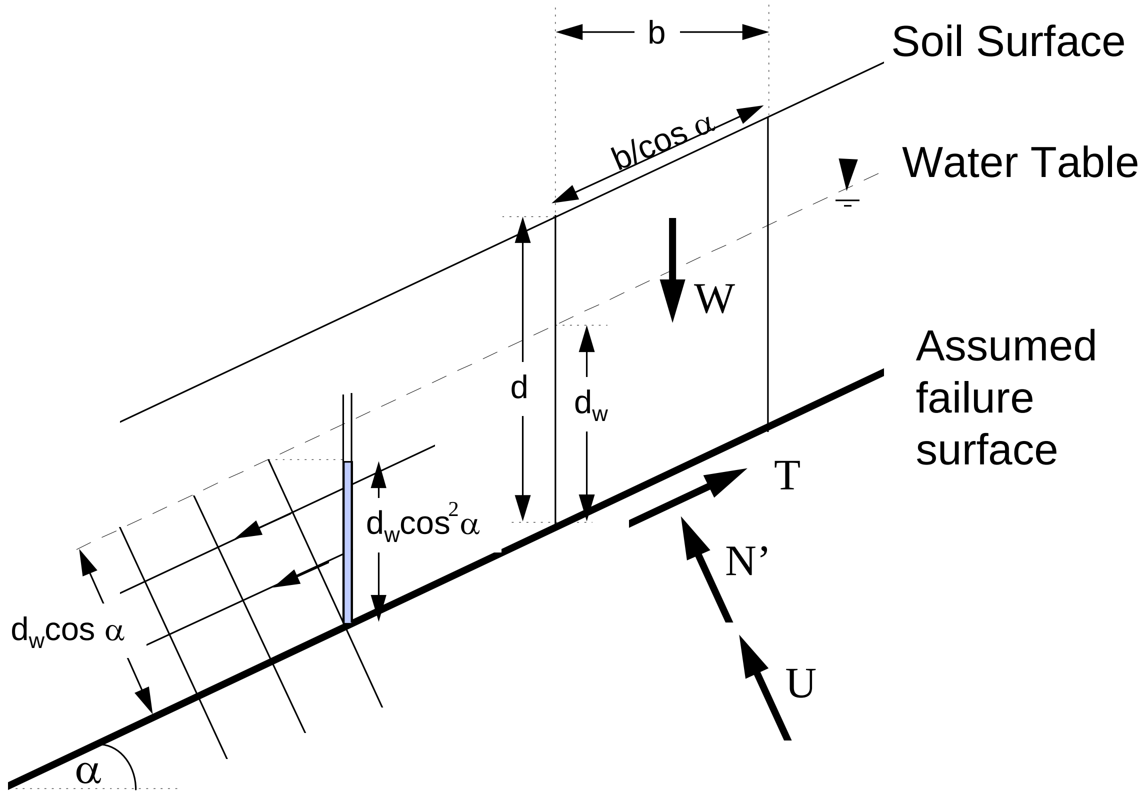
$X_2 = 1086.6$ kN/m

Using linear interpolation/extrapolation

F = 1.18

7.3 Infinite Slopes

For long slopes another potential failure mechanism is a failure plane, usually at relatively small depths, parallel to the soil surface. This situation is demonstrated below.



If the failure surface is very long then the inter-slice forces must cancel out, and then considering equilibrium we can write (assuming the unit weight is the same above and below the water table):

$$N = W \cos\alpha = \gamma\, b\, d \cos\alpha$$

$$T = W \sin\alpha = \gamma\, b\, d \sin\alpha$$

and the normal stress, σ is given by

The normal and shear stresses on the assumed failure plane are thus given by

$$\sigma = \frac{N}{b/\cos\alpha} = \gamma\, d \cos^2\alpha$$

$$\tau = \frac{T}{b/\cos\alpha} = \gamma\, d \sin\alpha \cos\alpha$$

The water pressure can be determined from consideration of the flow (from the flow net)

$$u = \gamma_w\, d_w \cos^2\alpha$$

and the force due to the water pressure on the failure surface is

$$U = u\left(b/\cos\alpha\right) = \gamma_w\, b\, d_w \cos\alpha$$

Because a flow net is being used an effective stress analysis is required and therefore the failure criterion is given by

$$\tau = c' + \sigma' \tan\phi'$$

or in terms of forces by

$$T = C' + N' \tan\phi'$$

and $$\sigma' = \sigma - u = (\gamma d - \gamma_w d_w)\cos^2\alpha$$

If we define a factor of safety F by

$$F = \frac{\tau_f}{\tau} = \frac{\text{shear stress required for failure}}{\text{actual shear stress}}$$

then

$$F = \frac{c' + (\gamma d - \gamma_w d_w)\cos^2\alpha \tan\phi'}{\gamma d \sin\alpha \cos\alpha}$$

It is usually appropriate to use the critical state parameters c' = 0, $\phi' = \phi'_{cs}$, so that

$$F = \frac{(\gamma d - \gamma_w d_w) \tan\phi'_{cs}}{\gamma d \tan\alpha} = \left(1 - \frac{\gamma_w d_w}{\gamma d}\right)\frac{\tan\phi'_{cs}}{\tan\alpha}$$

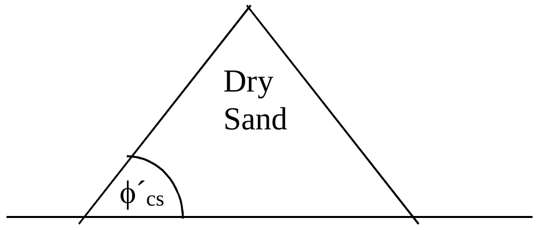
If the soil is dry above the assumed failure plane then the factor of safety becomes

$$F = \frac{\tan\phi'_{cs}}{\tan\alpha}$$

If the soil is failing F = 1 then

$$\alpha = \phi'_{cs}$$

For dry slopes the friction angle is equal to the angle of repose.



If $d_w = d$, that is the soil is saturated and water is flowing parallel to the slope then at failure (F=1)

$$\tan\alpha = \left(1 - \frac{\gamma_w}{\gamma}\right)\tan\phi'_{cs}$$

Typically for sand $\phi'_{cs} = 35^o$ and $\gamma_{sat} = 20$ kN/m^3 which gives $\alpha = 19.3^o$ at failure.

Note that water reduces the stable angle by a factor of about 2.

7.4 Graphical solutions

Solutions are available for some common slope geometries and ground water conditions.

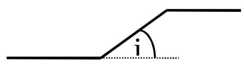
7.4.1 Undrained (total stress) analyses

The stability of homogeneous slopes can be expressed in terms of a dimensionless group known as the stability number, N.

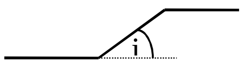
$$N = \frac{c}{\gamma H}$$

Where c = cohesion
γ = bulk unit weight
H = height of the cut

If two slopes are geometrically similar they will have the same factor of safety provided the stability numbers are the same, that is

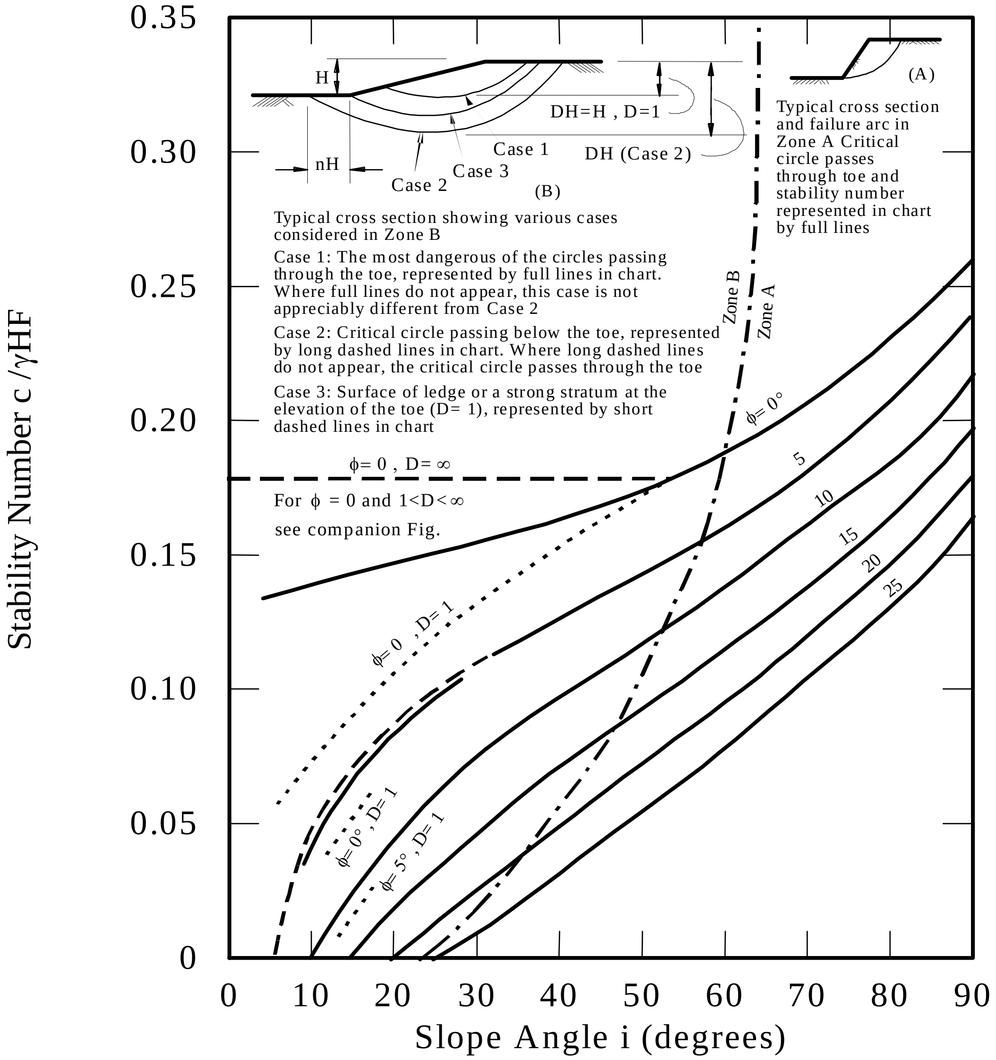


$$\frac{c_1}{\gamma_1 H_1} = \frac{c_2}{\gamma_2 H_2}$$



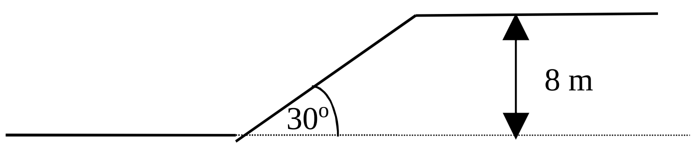
7.4.1.1 Taylors chart – Infinite soil layer

A Chart presented by Taylor is shown below (see also p29 in Data Sheets). The solutions assume circular failure surfaces, and soil strength given by the Mohr-Coulomb criterion. They ignore the possibility of tension cracks.



Example

A slope has an inclination of 30^o and is 8 m high. The soil properties are c_u = 20 kN/m^3, ϕ_u = 5^o, γ_{bulk} = 15 kN/m^3. Determine the short term factor of safety if the clay deposit is infinitely deep.



From the stability chart above for i = 30^o and ϕ = 5^o we obtain

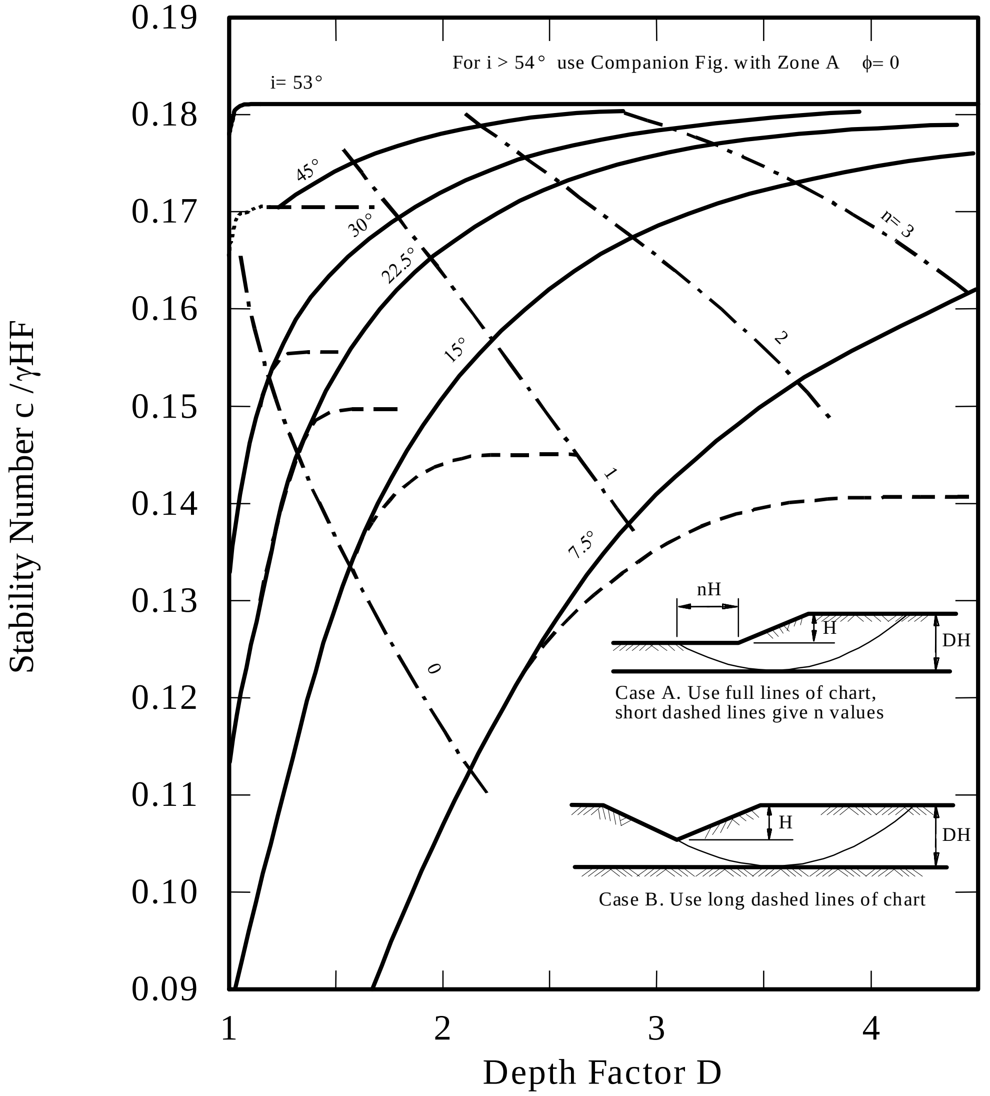
$$\frac{c}{\gamma\, H\, F} = 0.11$$

hence $$F = \frac{c}{\gamma\, H\, N} = \frac{20}{15 \times 8 \times 0.11} = 1.5$$

Regions on the chart indicate the mode of failure; whether it will be shallow or deep-seated. In this example the failure is in zone B, indicating a deep-seated failure mechanism The zone on the chart has no influence on the factor of safety determined provided that the soil layer is sufficiently deep for the implied mechanism to occur.

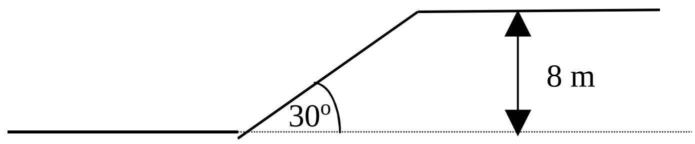
7.4.1.2 Taylor's chart - soil layer of finite depth and $\phi_u = 0$

The influence of a finite depth below the base of the slope can be determined from a second chart produced by Taylor shown below (also on p29 in Data Sheets). This chart is limited to the case of $\phi_u = 0$.



Example

A slope has an inclination of 30^o and is 8 m high. The soil properties are c_u = 20 kN/m^3, ϕ_u = 0^o, γ_{bulk} = 15 kN/m^3. Determine the short term factor of safety if the clay deposit overlies rock which lies 2 m below the base of the slope.



Calculate depth factor D from DH = 10 m, H = 8 m. giving D = 1.25

From chart for D=1.25 and i = 30^o we obtain

$$\frac{c}{\gamma\, H\, F} = 0.155$$

and hence F = 1.075

Note that if ϕ = 0 and D = ∞ then N = 0.181 and F = 0.92

This indicates that for a deep seated failure reductions in the depth of soil below the bottom of the slope result in increases in the factor of safety

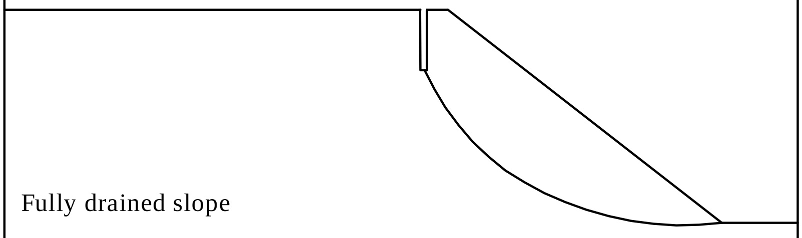
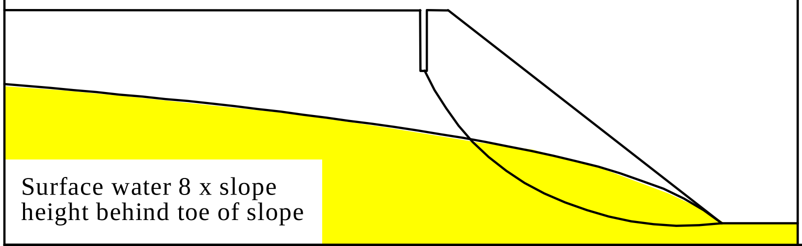
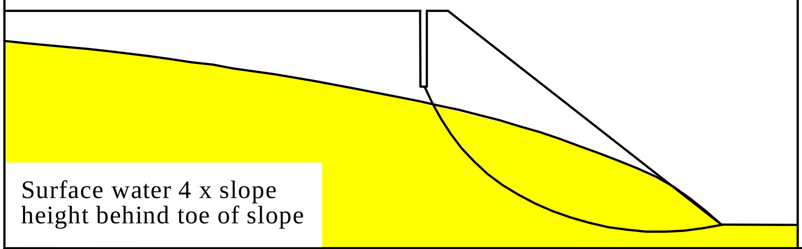
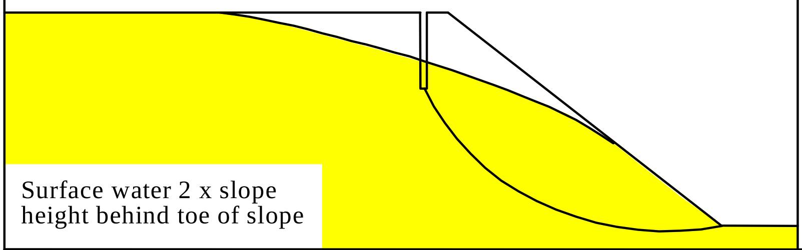
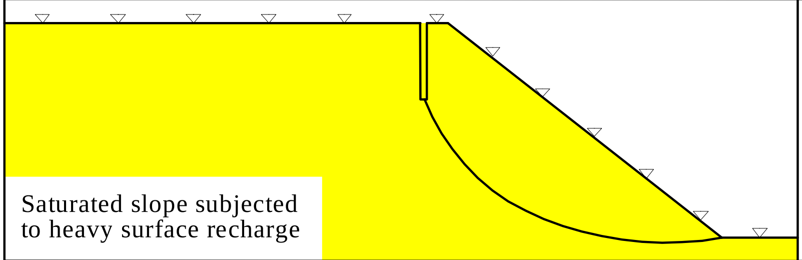
7.4.2 Effective stress analyses

A number of charts have been published for effective stress analyses but they are usually limited to very specific conditions, such as for the construction of large embankments. One of the more useful charts has been presented by Hoek and Bray for a range of relatively common groundwater conditions. In deriving the solutions it is assumed that:

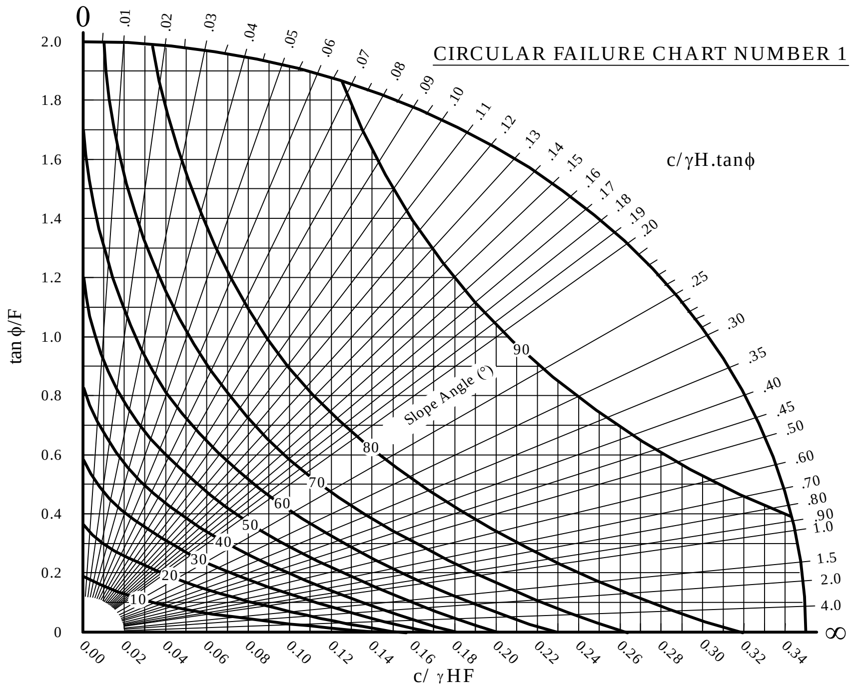
- a circular failure occurs passing through the toe of the slope,
- the soil is homogeneous,
- a vertical tension crack occurs either in the upper surface or in the slope face,
- the soil strength is given by the Mohr-Coulomb criterion.

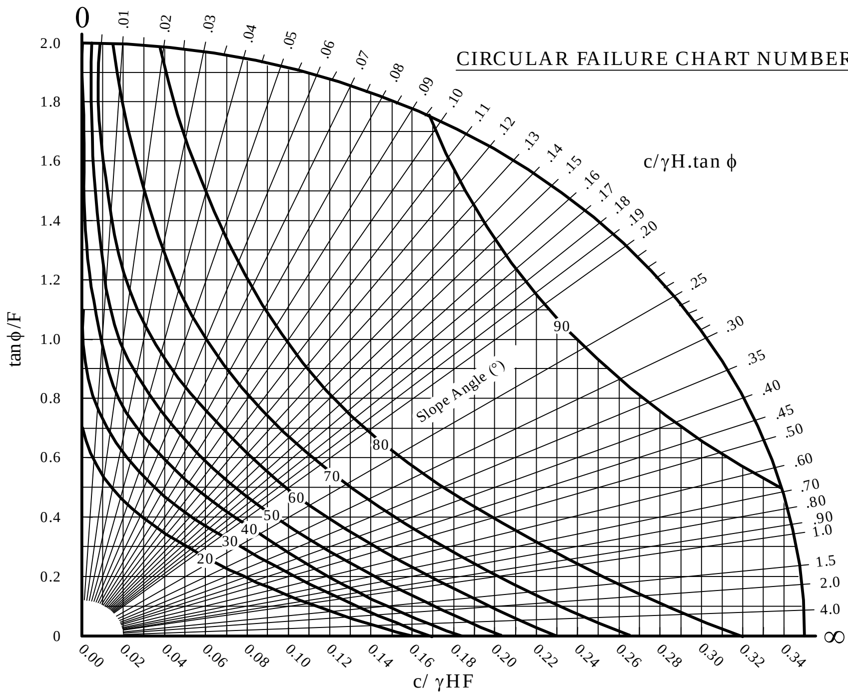
The approach is very similar to that used by Taylor.

Charted solutions are available for the following groundwater conditions

Groundwater Flow Conditions	Chart Number
 Fully drained slope	1
 Surface water 8 x slope height behind toe of slope	2
 Surface water 4 x slope height behind toe of slope	3
 Surface water 2 x slope height behind toe of slope	4
 Saturated slope subjected to heavy surface recharge	5

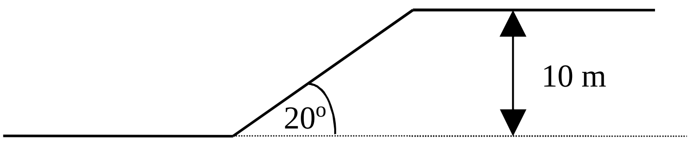
For each groundwater condition a separate chart is available. Two are shown below


CIRCULAR FAILURE CHART NUMBER 1
c/γH.tanϕ
tan ϕ/F
c/ γHF
Slope Angle (°)
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
.11
.12
.13
.14
.15
.16
.17
.18
.19
.20
.25
.30
.35
.40
.45
.50
.60
.70
.80
.90
1.0
1.5
2.0
4.0
∞
10
20
30
40
50
60
70
80
90
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20
0.22
0.24
0.26
0.28
0.30
0.32
0.34


CIRCULAR FAILURE CHART NUMBER 3
c/γH.tan ϕ
tanϕ/F
c/ γHF
Slope Angle (°)
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
.11
.12
.13
.14
.15
.16
.17
.18
.19
.20
.25
.30
.35
.40
.45
.50
.60
.70
.80
.90
1.0
1.5
2.0
4.0
∞
20
30
40
50
60
70
80
90
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20
0.22
0.24
0.26
0.28
0.30
0.32
0.34

Example

To demonstrate the use of the charts consider the case of a slope 10 m high with a slope of 20 degrees in a clayey soil with properties c_u = 20 kN/m^3, ϕ_u = 5°, c′ = 2 kN/m^2, ϕ′ = 25°, γ_{sat} = 16 kN/m^3. In the long term the water table is at the surface for distances greater than 40 m behind the toe of the slope.



When using Hoek and Bray charts it is important that effective strength parameters c´ and ϕ´ are used.

- Determine the appropriate chart from the known position of the water table. In this example it is Chart 3

- Calculate $\dfrac{c}{\gamma\, H \tan\phi} = \dfrac{2}{16 \times 10 \times \tan 25} = 0.027$

- For slope angle 20° read off chart

either $\dfrac{c}{\gamma\, H\, F} = 0.0139$

or $\dfrac{\tan\phi}{F} = 0.518$

- Hence F ≈ 0.9 (The slope would fail)

Note that in practice it is likely in any detailed design that a computer slope stability program will be used. However, the speed and simplicity of using charts such as these make them suitable for checking the sensitivity of the factor of safety to a range of values of the soil parameters and slope geometries.

For instance in the example above if the water table is lowered and chart 2 is appropriate the factor of safety will increase to F ≈ 1.1

Note also that chart 1 which is shown for a fully drained (dry) slope is equivalent to Taylor’s charts. That is chart 1 can be used for a total stress (undrained) analysis. This is because in the analysis of a dry slope the total and effective stresses are the same. The analysis is only concerned with the values of c, ϕ, γ. Solutions will be slightly different to those from Taylor’s chart because slightly different assumptions are made in the two analyses.

Slope Stability Problems

1. Use Taylor's curves to determine the maximum height of a 70 ° slope in homogeneous soil for which γ = 16 kN/m^3 and c = 20 kN/m^2 if
   a) ϕ = 25 ° b) ϕ = 10 ° c) ϕ = 0
   What would be the answer in each case using Hoek and Bray's charts
2. Use Taylor's curves to determine the factor of safety and depth of critical circle of a wide cutting 12 m deep of 7.5 ° slope in a clay for which ϕ_u = 0, c_u = 40 kN/m^2 and γ = 16 kN/m^3. Assume
   a) The clay extends to a great depth
   b) There is a hard stratum at 36 m below the top of the cutting
   c) A hard stratum at 22 m
   d) A hard stratum at 12 m
   e) A hard stratum at 6 m
   Repeat cases a to e for a narrow cutting where the toes of the two slopes coincide

3 Determine the factor of safety against immediate shear failure along the slip circle shown in Figure 1 below:
(a) when the tension crack of depth z = 4.32 m is empty of water
(b) when the tension crack is full of water
The soil properties are c_u = 40 kN/m^2, ϕ_u = 0. The weight of the sliding mass of soil, W = 1325 kN/m, and the horizontal distance of the centroid of this mass from the centre of the circle, d = 5.9 m. The radius of the slip circle, R = 17.4 m, and the angle θ = 67.4°. (You do not need to use the method of slices).

4 A wide cutting of slope 45° is excavated in a silt of unit weight γ_{sat} = 19 kN/m^3. When the cut is 12 m deep a rotational slip occurs which is estimated to have a radius of 17 m and to pass through the toe and a point 5.5 m back from the upper edge of the slope. Shear tests on undisturbed samples give variable values for c_u. Assuming ϕ_u = 10° estimate an average value of c_u round the failure surface by using
(a) the Swedish method of slices
(b) Bishop's simplified method of slices

5 Shown in Figure 2 is the cross-section of a cutting that is to be made in a partially saturated clayey sand which contains a weak clay seam that will be intersected by the face of the cut.
Calculate the factor of safety that the slope would have against a wedge type failure by using the two wedges that are shown in the figure.
Properties of the materials are as follows:
Clayey sand: γ_{bulk} = 18 kN/m^3, c′ = 0, ϕ' = 26°
Clay seam: c_u = 45 kN/m^2, ϕ_u = 0

6 Determine the factor of safety of a long (infinite) slope as a function of the slope angle, α, if the water flows horizontally out of the slope. Take c' = 0.
Calculate the limiting value of α if ϕ' = 30°, and γ_{sat} = 20 kN/m^3.